(1)

1. Solve $y'' = 9xe^{3x}$ with $y'(0) = 2$, $y(0) = \frac{4}{3}$□
2. Solve $(x^2 + x - 12)dy = (y^2 - 3y)dx$□
3. Solve $(2y + \tan 2x)dy + \left(\frac{1}{x} + 2y\sec^2 2x\right)dx = 0$ □
4. Solve $x^2\frac{dy}{dx} + 3xy = \frac{\cos 3x}{x}$ □

(2) □

1. Solve $y'' = 4xe^{2x}$ with $y'(0) = \frac{3}{2}$, $y(0) = 3$
2. Solve $(y^2 + y - 12)dx = (x^2 - 2x)dy$
3. Solve $(2x + \tan 3y)dx + \left(\frac{1}{y} + 3x\sec^2 3y\right)dy = 0$ □
4. Solve $x^3\frac{dy}{dx} + 2x^2y = x\sin 2x$ □

(3) □

1. Solve $y'' = 6xe^{3x}$ with $y'(0) = \frac{4}{3}$, $y(0) = \frac{5}{9}$□
2. Solve $(x^2 - x - 12)dy = (y^2 + 2y)dx$□
3. Solve $(2y + \tan 3x)dy + \left(\frac{1}{x} + 3y\sec^2 3x\right)dx = 0$ □
4. Solve $x^2\frac{dy}{dx} + 3xy = \frac{\sin 2x}{x}$ □

(4) □

1. Solve $y'' = 6xe^{2x}$ with $y'(0) = \frac{3}{2}$, $y(0) = \frac{5}{2}$□
2. Solve $(y^2 - y - 12)dx = (x^2 + 3x)dy$
3. Solve $(3x^2 + \tan 2y)dx + \left(\frac{4}{y} + 2x\sec^2 2y\right)dy = 0$ □
4. Solve $x^2\frac{dy}{dx} + 2xy = \cos 3x$ □

الاجوبة النموذجية (1)

1. Solve $y'' = 9xe^{3x}$ with $y'(0) = 2\,,\quad y(0) = \dfrac{4}{3}$ □

$$y' = \int 9xe^{3x}dx = 3xe^{3x} - e^{3x} + C$$ □

$$y'(0) = 2 \Rightarrow 2 = 0 - 1 + C \Rightarrow C = 3$$

$$y' = 3xe^{3x} - e^{3x} + 3$$

x & $D.$	$9e^{3x}$ & I.
x $+$	$9e^{3x}$
1 $-$	$3e^{3x}$
0	e^{3x}

$$y = \int (3xe^{3x} - e^{3x} + 3)dx = xe^{3x} - \frac{1}{3}e^{3x} - \frac{1}{3}e^{3x} + 3x + K$$ □

$$y(0) = \frac{4}{3} \quad \Rightarrow \quad \frac{4}{3} = 0 - \frac{1}{3} - \frac{1}{3} + 0 + K \quad \Rightarrow \quad K = 2$$ □

$$y = xe^{3x} - \frac{2}{3}e^{3x} + 3x + 2$$ □

2. Solve $(x^2 + x - 12)dy = (y^2 - 3y)dx$ □

$$\frac{dy}{(y^2 - 3y)} = \frac{dx}{(x^2 + x - 12)} \quad \Rightarrow \quad \frac{dy}{y(y-3)} = \frac{dx}{(x-3)(x+4)}$$ □

$$\frac{1}{y(y-3)} = \frac{A}{y} + \frac{B}{(y-3)}\ ;\ y = 0 \Rightarrow A = -\frac{1}{3}\ \&\ y = 3 \Rightarrow B = \frac{1}{3}$$ □

$$\frac{1}{y(y-3)} = \frac{-1}{3y} + \frac{1}{3(y-3)}$$ □

$$\frac{1}{(x-3)(x+4)} = \frac{C}{(x-3)} + \frac{D}{(x+4)}\ ; x = 3 \Rightarrow C = \frac{1}{7}\ \&\ x = -4 \Rightarrow D = -\frac{1}{7}$$ □

$$\frac{1}{(x-3)(x+4)} = \frac{1}{7(x-3)} - \frac{1}{7(x+4)}$$ □

$$\int \left(\frac{-1}{3y} + \frac{1}{3(y-3)}\right)dy = \int \left(\frac{1}{7(x-3)} - \frac{1}{7(x+4)}\right)dx$$

$$\frac{1}{3}(-\ln y + \ln(y-1)) = \frac{1}{7}(\ln(x-3) - \ln(x+4)) + c$$

3. Solve $(2y + \tan 2x)dy + \left(\frac{1}{x} + 2y\sec^2 2x\right)dx = 0$

$$\frac{\partial M}{\partial x} = 2\sec^2 2x \quad , \quad \frac{\partial N}{\partial y} = 2\sec^2 2x \quad \Rightarrow \quad \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

Which implies that the differential equation is exact.

$$F = \int (2y + \tan 2x)dy$$

$$F = y^2 + y\tan 2x + \mathrm{g}(x)$$

Similarly

$$F = \int \left(\frac{1}{x} + 2y\sec^2 2x\right)dx$$

$$F = \ln x + y\tan 2x + \omega(y)$$

$$\mathrm{g}(x) = \ln x \quad \& \quad \omega(y) = y^2$$

$$\ln x + y\tan 2x + y^2 = C$$

4. Solve $x^2\frac{dy}{dx} + 3xy = \frac{\cos 3x}{x}$

$$y' + \left(\frac{3}{x}\right)y = \frac{\cos 3x}{x^3}$$

Standard form, $y' + P(x)y = Q(x)$

$P(x) = \frac{3}{x}$ and $Q(x) = \frac{\cos 3x}{x^3}$. So, the integrating factor is

$$u(x) = e^{\int P(x)dx} = e^{\int 3/x dx} = e^{3\ln x} = e^{\ln x^3} = x^3$$

$$y = \frac{1}{u(x)}\int u(x)\ Q(x)dx$$

$$y = \frac{1}{x^3}\int x^3 \times \frac{\cos 3x}{x^3}dx = \frac{1}{x^3}\int \cos 3x\,dx$$

$$y = \frac{1}{x^3}\left(\frac{1}{3}\sin 3x + c\right)$$

الاجوبة النموذجية (2)

1. Solve $y'' = 4xe^{2x}$ with $y'(0) = \frac{3}{2}$, $y(0) = 3$ □

x & D.	$4e^{2x}$ & I.
x	$4e^{2x}$
1	$2e^{2x}$
0	e^{2x}

$$y' = \int 4xe^{2x}dx = 2xe^{2x} - e^{2x} + \mathrm{C} \quad □$$

$$y'(0) = \frac{3}{2} \quad \Rightarrow \frac{3}{2} = -1 + C \quad \Rightarrow \quad C = \frac{5}{2}□$$

$$y' = 2xe^{2x} - e^{2x} + \frac{5}{2} \qquad □$$

$$y = \int \left(2xe^{2x} - e^{2x} + \frac{5}{2}\right)dx = xe^{2x} - \frac{1}{2}\,e^{x} - \frac{1}{2}\,e^{2x} + \frac{5}{2}x + K \qquad □$$

$$y(0) = 3 \quad \Rightarrow \quad 3 = 0 - \frac{1}{2} - \frac{1}{2} + 0 + K \quad \Rightarrow \quad K = 4□$$

$$y = xe^{2x} - e^{x} + \frac{5}{2}x + 4 \qquad □$$

2. Solve $(y^2 + y - 12)dx = (x^2 - 2x)dy$

$$\frac{dy}{(y^2 + y - 12)} = \frac{dx}{(x^2 - 2x)} \quad \Rightarrow \quad \frac{dy}{(y-3)(y+4)} = \frac{dx}{x(x-2)}□$$

$$\frac{1}{x(x-2)} = \frac{A}{x} + \frac{B}{(x-2)} \;;\; x = 0 \Rightarrow A = -\frac{1}{2} \;\&\; x = 2 \Rightarrow B = \frac{1}{2}□$$

$$\frac{1}{x(x-2)} = \frac{-1}{2x} + \frac{1}{2(x-2)}□$$

$$\frac{1}{(y-3)(y+4)} = \frac{C}{(y-3)} + \frac{D}{(y+4)} \;; y = 3 \;\Rightarrow C = \frac{1}{7} \;\&\; y = -4 \;\Rightarrow D = -\frac{1}{7}□$$

$$\frac{1}{(y-3)(y+4)} = \frac{1}{7(y-3)} - \frac{1}{7(y+4)}□$$

$$\int \left(\frac{-1}{2x} + \frac{1}{2(x-2)}\right)dx = \int \left(\frac{1}{7(y-3)} - \frac{1}{7(y+4)}\right)dy$$

$$\frac{1}{2}(-\ln x + \ln(x-2)) = \frac{1}{7}(\ln(y-3) - \ln(y+4)) + c□$$

3. Solve $(2x + \tan 3y)dx + \left(\frac{1}{y} + 3x\sec^2 3y\right)dy = 0$

$$\frac{\partial M}{\partial y} = 3\sec^2 3y \quad , \quad \frac{\partial N}{\partial x} = 3\sec^2 3y \quad \Rightarrow \quad \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

Which implies that the differential equation is exact.

$$F = \int (2x + \tan 3y)dx$$

$$F = x^2 + x\tan 3y + \omega(y)$$

Similarly

$$F = \int \left(\frac{1}{y} + 3x\sec^2 3y\right)dy$$

$$F = \ln y + x\tan 3y + g(x)$$

$$g(x) = x^2 \;\; \& \;\; \omega(y) = \ln y$$

$$x^2 + x\tan 3y + \ln y = C$$

4. Solve $x^3\frac{dy}{dx} + 2x^2y = x\sin 2x$

$$y' + \left(\frac{2}{x}\right)y = \frac{\sin 2x}{x^2}$$

Standard form, $y' + P(x)y = Q(x)$

$P(x) = \frac{2}{x}$ and $Q(x) = \frac{\sin 2x}{x^2}$. So, the integrating factor is

$$u(x) = e^{\int P(x)dx} = e^{\int 2/x dx} = e^{2\ln x} = e^{\ln x^2} = x^2$$

$$y = \frac{1}{u(x)}\int u(x)\; Q(x)dx$$

$$y = \frac{1}{x^2}\int x^2 \times \frac{\sin 2x}{x^2}dx = \frac{1}{x^2}\int \sin 2x\, dx$$

$$y = \frac{1}{x^2}\left(-\frac{1}{2}\cos 2x + c\right)$$

الاجوبة النموذجية (3)

1. Solve $y'' = 6xe^{3x}$ with $y'(0) = \frac{4}{3}$, $y(0) = \frac{5}{9}$□

$$y' = \int 6xe^{3x}dx = 2xe^{3x} - \frac{2}{3}e^{3x} + C$$

x&D.	$6e^{3x}$& I.
x	$6e^{3x}$
1	$2e^{3x}$
0	$(2/3)e^{3x}$

$$y'(0) = \frac{4}{3} \Rightarrow \frac{4}{3} = 0 - \frac{2}{3} + C \Rightarrow C = 2$$

$$y' = 2xe^{3x} - \frac{2}{3}e^{3x} + 2$$

$$y = \int \left(2xe^{3x} - \frac{2}{3}e^{3x} + 2\right)dx = \frac{2}{3}xe^{3x} - \frac{2}{9}e^{3x} - \frac{2}{9}e^{3x} + 2x + K$$ □

$$y(0) = \frac{5}{9} \Rightarrow \frac{5}{9} = 0 - \frac{2}{9} - \frac{2}{9} + 0 + K \Rightarrow K = 1$$ □

$$y = \frac{2}{3}xe^{3x} - \frac{4}{9}e^{3x} + 2x + 1$$ □

2. Solve $(x^2 - x - 12)dy = (y^2 + 2y)dx$□

$$\frac{dy}{(y^2 + 2y)} = \frac{dx}{(x^2 - x - 12)} \Rightarrow \frac{dy}{y(y+2)} = \frac{dx}{(x+3)(x-4)}$$ □

$$\frac{1}{y(y+2)} = \frac{A}{y} + \frac{B}{(y+2)} \ ; \ y = 0 \Rightarrow A = \frac{1}{2} \ \& \ y = -2 \Rightarrow B = -\frac{1}{2}$$ □

$$\frac{1}{y(y+2)} = \frac{1}{2y} - \frac{1}{2(y+2)}$$ □

$$\frac{1}{(x+3)(x-4)} = \frac{C}{(x+3)} + \frac{D}{(x-4)} \ ; x = -3 \Rightarrow C = -\frac{1}{7} \ \& \ x = 4 \Rightarrow D = \frac{1}{7}$$ □

$$\frac{1}{(x+3)(x-4)} = \frac{1}{7(x-4)} - \frac{1}{7(x+3)}$$ □

$$\int \left(\frac{1}{2y} - \frac{1}{2(y+2)}\right)dy = \int \left(\frac{1}{7(x-4)} - \frac{1}{7(x+3)}\right)dx$$

$$\frac{1}{2}(\ln y - \ln(y+2)) = \frac{1}{7}(\ln(x-4) - \ln(x+3)) + c$$

3. Solve $(2y+\tan 3x)dy+\left(\frac{1}{x}+3y\sec^2 3x\right)dx=0$ □

$$\frac{\partial M}{\partial x}=3\sec^2 3x \quad , \quad \frac{\partial N}{\partial y}=3\sec^2 3x \quad \Rightarrow \quad \frac{\partial N}{\partial x}=\frac{\partial M}{\partial y} \quad □$$

Which implies that the differential equation is exact.

$$F=\int\left(\frac{1}{x}+3y\sec^2 3x\right)dx □$$

$$F=\ln x+y\tan 3x+\omega(y) □$$

Similarly

$$F=\int(2y+\tan 3x)dy □$$

$$F=y^2+y\tan 3x+g(x) □$$

$$g(x)=\ln x \quad \& \quad \omega(y)=y^2 □$$

$$y^2+y\tan 3x+\ln x=C □$$

4. Solve $x^2\frac{dy}{dx}+3xy=\frac{\sin 2x}{x}$ □

$$y'+\left(\frac{3}{x}\right)y=\frac{\sin 2x}{x^3}$$ Standard form, $y'+P(x)y=Q(x)$ □

$P(x)=\frac{3}{x}$ and $Q(x)=\frac{\sin 2x}{x^3}$. So, the integrating factor is □

$$u(x)=e^{\int P(x)dx}=e^{\int 3/x dx}=e^{3\ln x}=e^{\ln x^3}=x^3 □$$

$$y=\frac{1}{u(x)}\int u(x)\ Q(x)dx \quad □$$

$$y=\frac{1}{x^3}\int x^3\times\frac{\sin 2x}{x^3}dx=\frac{1}{x^3}\int \sin 2x\, dx □$$

$$y=\frac{1}{x^3}\left(-\frac{1}{2}\cos 2x+c\right) \quad □$$

الاجوبة النموذجية (4)

1. Solve $y'' = 6xe^{2x}$ with $y'(0) = \frac{3}{2}$, $y(0) = \frac{5}{2}$□

x&$D.$	$6e^{2x}$&$I.$
x	$6e^{2x}$
1	$3e^{2x}$
0	$(3/2)e^{2x}$

$$y' = \int 6xe^{2x}dx = 3xe^{2x} - \frac{3}{2}e^{2x} + \mathrm{C}$$

$$y'(0) = \frac{3}{2} \Rightarrow 3 = 0 - \frac{3}{2} + C \Rightarrow C = 3$$

$$y' = 3xe^{2x} - \frac{3}{2}e^{2x} + 3$$

$$y = \int \left(3xe^{2x} - \frac{3}{2}e^{2x} + 3\right)dx = \frac{3}{2}xe^{2x} - \frac{3}{4}e^{2x} - \frac{3}{4}e^{2x} + 3x + K \quad □$$

$$y(0) = \frac{5}{2} \quad \Rightarrow \quad \frac{5}{2} = 0 - \frac{3}{4} - \frac{3}{4} + 0 + K \quad \Rightarrow \quad K = 4□$$

$$y = \frac{3}{2}xe^{2x} - \frac{3}{2}e^{2x} + 3x + 4 \quad □$$

2. Solve $(y^2 - y - 12)dx = (x^2 + 3x)dy$

$$\frac{dy}{(y^2 - y - 12)} = \frac{dx}{(x^2 + 3x)} \quad \Rightarrow \quad \frac{dy}{(y+3)(y-4)} = \frac{dx}{x(x+3)}□$$

$$\frac{1}{x(x+3)} = \frac{A}{x} + \frac{B}{(x+3)} \; ; \; x = 0 \Rightarrow A = \frac{1}{3} \;\&\; x = -3 \Rightarrow B = -\frac{1}{3}□$$

$$\frac{1}{x(x-3)} = \frac{1}{3x} - \frac{1}{3(x-3)}□$$

$$\frac{1}{(y+3)(y-4)} = \frac{C}{(y+3)} + \frac{D}{(y-4)} \; ; y = -3 \Rightarrow C = -\frac{1}{7} \;\&\; y = 4 \Rightarrow D = \frac{1}{7}□$$

$$\frac{1}{(y+3)(y-4)} = \frac{1}{7(y-4)} - \frac{1}{7(y+3)}□$$

$$\int \left(\frac{1}{3x} - \frac{1}{(x-3)}\right)dx = \int \left(\frac{1}{7(y-4)} - \frac{1}{7(y+3)}\right)dy$$

$$\frac{1}{3}(\ln x - \ln(x-3)) = \frac{1}{7}(\ln(y-4) - \ln(y+3)) + c□$$

3. Solve $(3x^2 + \tan 2y)dx + \left(\frac{4}{y} + 2x\sec^2 2y\right)dy = 0$

$$\frac{\partial M}{\partial y} = 2\sec^2 2y \quad , \quad \frac{\partial N}{\partial x} = 2\sec^2 2y \quad \Rightarrow \quad \frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

Which implies that the differential equation is exact.

$$F = \int (3x^2 + \tan 2y)dx$$

$$F = x^3 + x\tan 2y + \omega(y)$$

Similarly

$$F = \int \left(\frac{4}{y} + 2x\sec^2 2y\right)dy$$

$$F = 4\ln y + x\tan 2y + \mathrm{g}(x)$$

$$\mathrm{g}(x) = x^3 \;\; \& \;\; \omega(y) = 4\ln y$$

$$x^3 + x\tan 2y + 4\ln y = C$$

4. Solve $x^2\frac{dy}{dx} + 2xy = \cos 3x$

$$y' + \left(\frac{2}{x}\right)y = \frac{\cos 3x}{x^2}$$

Standard form, $y' + P(x)y = Q(x)$

$P(x) = \frac{2}{x}$ and $Q(x) = \frac{\cos 3x}{x^2}$. So, the integrating factor is

$$u(x) = e^{\int P(x)dx} = e^{\int 2/x dx} = e^{2\ln x} = e^{\ln x^2} = x^2$$

$$y = \frac{1}{u(x)}\int u(x)\ Q(x)dx$$

$$y = \frac{1}{x^2}\int x^2 \times \frac{\cos 3x}{x^2}dx = \frac{1}{x^2}\int \cos 3x\,dx$$

$$y = \frac{1}{x^2}\left(\frac{1}{3}\sin 3x + c\right)$$