

Vectors

Examples

1) Derive the general law for the distance between two parallel planes (using vectors).

Solution

Assume the general plane equations are

$$Ax_1 + By_1 + Cz_1 = D_1 \quad \dots 1$$

And

$$Ax_2 + By_2 + Cz_2 = D_2 \quad \dots 2$$

The normal vector is

$$\vec{N} = Ai + Bj + Ck$$

Assume

$x_1 = y_1 = 0$ from equation (1) we get $z_1 = D_1/C$
then $P_1(0, 0, D_1/C)$

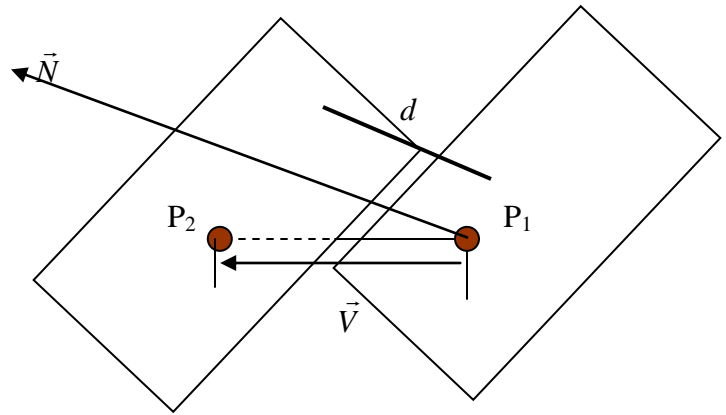
Assume

$x_2 = y_2 = 0$ from equation (2) we get $z_2 = D_2/C$
then $P_2(0, 0, D_2/C)$

$$\vec{V} = P_2P_1 = 0i + 0j + 1/C(D_2 - D_1)k$$

The general law is

$$d = \frac{\vec{V} \cdot \vec{N}}{|\vec{N}|} = \frac{|D_2 - D_1|}{\sqrt{A^2 + B^2 + C^2}}$$



2) Derive the general law for the distance between any point in space and any plane.

Solution

The general plane equation is

$$Ax + By + Cz = D$$

And the normal vector is

$$\vec{N} = Ai + Bj + Ck$$

Assume $x = y = 0$ sub. in plane equation

Then $z = D/C$

$$P_0(0, 0, D/C)$$

$$\vec{V} = P_0P_1 = x_1i + y_1j + (z_1 - D/C)k$$

The general law is the projection of vector V on to vector N

$$d = \frac{\vec{V} \cdot \vec{N}}{|\vec{N}|} = \frac{|Ax_1 + By_1 + Cz_1 - D|}{\sqrt{A^2 + B^2 + C^2}}$$

