University of Babylon

College of Engineering

Department of Electrical Engineering

Class :2^{st}

Lectures of

Electromagnetic Field Theory

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References :

1-Bhag Singh Guru and Huseyin R. Hiziroglu ***Electromagnetic Field Theory Fundamentals*** ,second edition copyright@2004

2-William H. Hayt and John A .Buck ***Engineering Electromagnetic*** ,sixth edition

3-***schaum's outline of theory and problems of Electromagnetic*** , second edition

Material Syllabus :

First term
1- Vectors , coordinate system , Vector algebra
2- Coulomb's Law and electric field intensity
3- Gauss's law and divergence
4- Vector operator and divergence theorem
5- Energy density in electrostatic field
Second term
6- Capacitance
7- The steady magnetic field
8- The scalar and vector magnetic potential
9- Materials and inductance
10-The nature of magnetic material

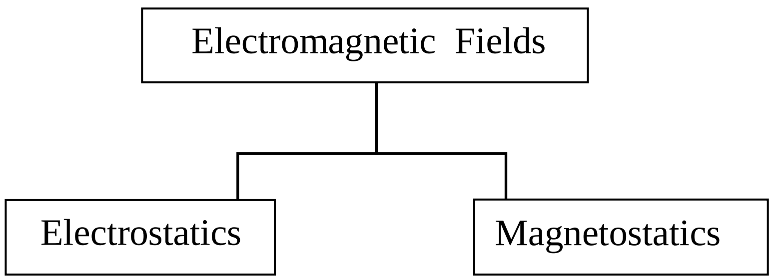
<u>Review :</u>

What is a field? Is it a scalar field or a vector field?

It is defined as *the behavior of a quantity* in a given region in terms of a set of values, one for each point in that region. The value at each point of a field can be either measured experimentally or predicted mathematically. It can be scalar or vector field.
Also it can be define the field as follows :
It is a function that characterizes a physical entity at all points in a region.
A **scalar field** is specified by a single number at each point in the region.
A **vector field** demands the knowledge of both the magnitude and direction for its specification at each point in the region.

There are two *Static field* and *Time –varying field* . our study will be concentrated up on the *Static field***.**

<u>Electromagnetic field theory:</u>
the study of electromagnetic field theory is vital to understanding many phenomena that take place in electrical engineering.



In the study of electrostatics, or static electric fields, we assume that:-
(a) all charges are fixed in space, (b) all charge densities are constant in time, and (c) the charge is the source of the electric field.
Our interest is to determine :-
(a) the electric field intensity at any point, (b) the potential distribution, (c) the forces exerted by the charges on other charges, and(d) the electric energy distribution in the region.

In the study of magnetostatics, or static magnetic fields ,our interest is to determine :-(a) magnetic field intensity, (b) magnetic flux density, (c) magnetic flux, and (d) the energy stored in the magnetic field.

Chapter 1
Vectors analysis

Scalar:
A physical quantity that can be completely described by its magnitude is called a scalar. For example: mass , time , temperature , work , and electric charge.

Vector:
A physical quantity that having a magnitude as well as a direction. For example: force, velocity, torque, acceleration , and electric field .

A vector quantity is graphically depicted by a line segment equal to its ***magnitude*** according to a convenient scale, and the ***direction*** is indicated by means of an arrow, as shown in Figure 1, where $\overrightarrow{\mathrm{R}}$ represents a vector directed from point 0 toward point *p* .

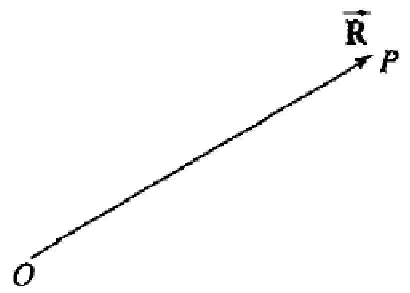


Figure 1 Graphical representation of a vector

Important note :
- A vector of magnitude zero is called a ***null vector*** or a ***zero vector***.
- A vector of unit magnitude (length) is called a ***unit vector***.

We can always represent a vector in terms of its unit vector. For example, vector $\overrightarrow{\mathrm{A}}$ can be written as

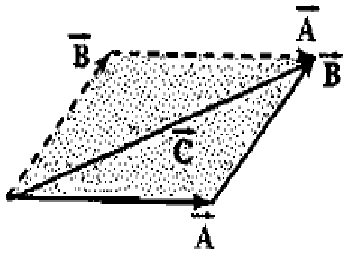
$$\vec{\mathrm{A}} = A\vec{\mathrm{a}}_A$$

where A is the magnitude of $\overrightarrow{\mathrm{A}}$ and $\vec{\mathrm{a}}_A$ is the *unit vector* in the same direction of $\overrightarrow{\mathrm{A}}$ such that

$$\vec{\mathrm{a}}_A = \frac{\overrightarrow{\mathrm{A}}}{A}$$

Vector operations

If $\vec{A}$ and $\vec{B}$ are two vectors , then



Vector addition :

$\vec{A} + \vec{B} = \vec{C}$

$\vec{C}$ represents a vector

Therefore , *The sum of two vectors is a vector.*

Properties :

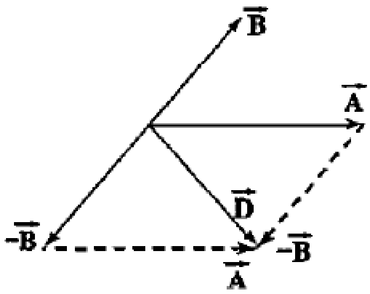
commutative law of addition: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

associative law of addition : $\vec{A} + (\vec{B} + \vec{C}) = (\vec{B} + \vec{A}) + \vec{C}$

Vector subtraction

$\vec{A} + (-\vec{B}) = \vec{D}$

$-\vec{B}$ (minus B) is also a vector with the same magnitude as $\vec{B}$ but in the opposite direction.



Multiplication of a vector by a scalar

If we multiply a vector $\vec{A}$ by a scalar k, we obtain a vector $\vec{B}$ such that

$\vec{B} = k\,\vec{A}$

The magnitude of $\vec{B}$ is simply equal to $|k|$ times the magnitude of $\vec{A}$.

If $k < 0$ then $\vec{B}$ in the opposite direction from $\vec{A}$

if $k > 0$ then $\vec{B}$ in the same direction as $\vec{A}$

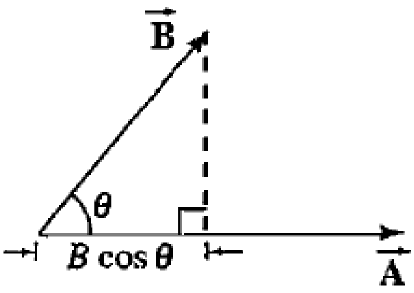
So vector $\vec{B}$ is said to be a *dependent vector*

Product of two vectors

There are two useful definitions for the product of two vectors they are *dot product*, and *cross product.*

The Dot product or (scalar) product

The dot product of two vectors $\vec{A}$ and $\vec{B}$ is written as $\vec{A} \cdot \vec{B}$ and is read as " $\vec{A}$ dot $\vec{B}$ "



$\vec{A} \cdot \vec{B} = A\ B\ \cos\theta$

The dot product is maximum when the two vectors are parallel and zero if the two vectors are orthogonal .

Properties

Commutative : $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Distributive : $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$

Scaling : $k(\vec{A} \cdot \vec{B}) = (k\vec{A}) \cdot \vec{B} = \vec{A} \cdot (k\vec{B})$

The component of $\vec{B}$ along $\vec{A}$ is called as the ***scalar projection*** of $\vec{B}$ on $\vec{A}$ and equal to " $B\ \cos\theta$ ".

$$B\ \cos\theta = \frac{\vec{A} \cdot \vec{B}}{A} = \vec{B} \cdot \vec{a}_A$$

The ***vector projection*** of $\vec{B}$ on $\vec{A}$ is equal to " $B\ \cos\theta\ \vec{a}_A$ ".

$$B\ \cos\theta\ \vec{a}_A = (\ B.\ \vec{a}_A)\ \vec{a}_A$$

$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$$

$$A = \sqrt{\vec{A} \cdot \vec{A}}$$

Example : If $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C}$, does this imply that B must always be equal to $\vec{C}$?

Solution :

Because $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C}$, we can rewrite it as $\vec{A} \cdot (\vec{B} - \vec{C}) = 0$

We can now make the following conclusions:

a) Either $\vec{A}$ is perpendicular to $(\vec{B} - \vec{C})$, or

b) $\vec{A}$ is a null vector, or

c) $(\vec{B} - \vec{C}) = 0$

Thus, only when $(\vec{B} - \vec{C})$ is equal to zero does $\vec{B} = \vec{C}$.

Thus , $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C}$ does not always mean that $\vec{B} = \vec{C}$.

The cross product or (vector) product

The cross product of two vectors $\vec{A}$ and $\vec{B}$ is written as $\vec{A} \times \vec{B}$ and is read as "$\vec{A}$ cross $\vec{B}$." The cross product is a vector that is directed normal to the plane containing $\vec{A}$ and $\vec{B}$. That is ,

$$\vec{A} \times \vec{B} = A\, B \sin\theta\, \vec{a}_n$$

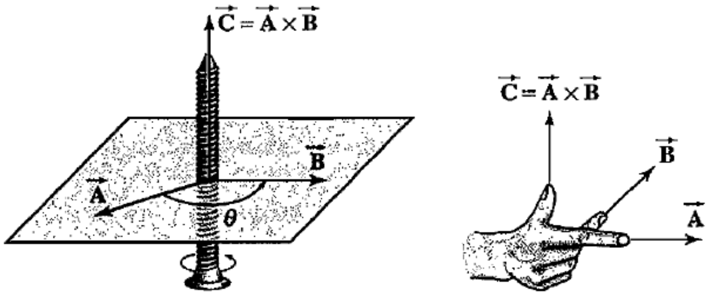
where $\vec{a}_n$ is a unit vector normal to the plane of $\vec{A}$ and $\vec{B}$.

If $\vec{C}$ represents the cross product of two vectors $\vec{A}$ and $\vec{B}$ such that

$$\vec{C} = \vec{A} \times \vec{B} \quad \text{,then}$$

$$C\, \vec{a}_n = (A\, \vec{a}_A) \times (B\, \vec{a}_B) = (\vec{a}_A \times \vec{a}_B)\, A\, B$$

$$\vec{a}_n = \frac{(\vec{a}_A \times \vec{a}_B)}{\sin\theta}$$



a) Right-handed screw rule b) Right-hand rule

Properties :

the cross product is not commutative because : $\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$

Distributive: $\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$

Scaling: $(k\vec{A}) \times \vec{B} = k(\vec{A} \times \vec{B}) = \vec{A} \times (k\vec{B})$

<u>**Note**</u> :It should be noted that the cross product will be zero for two nonzero **parallel vectors**.

Example : Derive the law of sines for a triangle using vectors.

Solution :

From Figure , we have

$\vec{B} = \vec{C} - \vec{A}$

Because $\vec{B} \times \vec{B} = 0$, we can write

$\vec{B} \times (\vec{C} - \vec{A}) = 0$

or

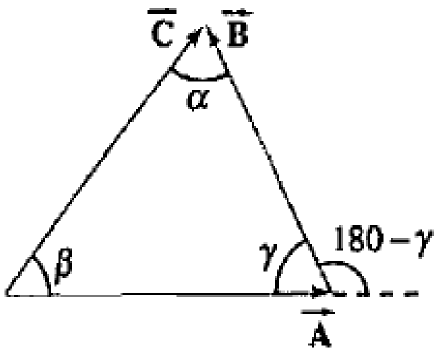
$\vec{B} \times \vec{C} = \vec{B} \times \vec{A}$

Therefore,

$B\ C\ \sin\alpha = B\ A \sin(\pi - \lambda)$

or

$$\frac{A}{\sin\alpha} = \frac{C}{\sin\lambda}$$



Similarly, we can show that

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta}$$

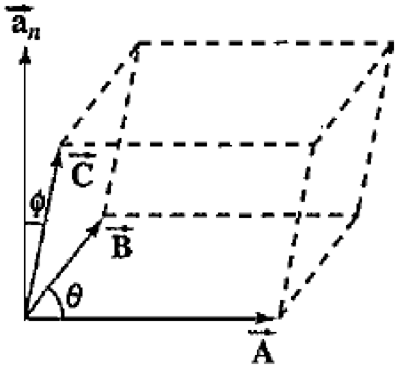
Thus, we can state the law of sines for a triangle as

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\lambda}$$

Scalar triple product

The scalar triple product of three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ is a scalar and is computed as

$$\vec{C} \cdot (\vec{A} \times \vec{B}) = ABC \ \sin\theta \cos\emptyset$$



The scalar triple product yields to the volume of **parallelepiped** if the three vectors represent of its sides , as shown in Figure.

associative

$$\vec{C} \cdot (\vec{A} \times \vec{B}) = \vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A})$$

Vector triple product

The vector triple product of three vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ is a vector and is written as $\vec{A} \times (\vec{B} \times \vec{C})$. We can show that the vector triple product is **not associative**. That is

$$\vec{A} \times (\vec{B} \times \vec{C}) \neq (\vec{A} \times \vec{B}) \times \vec{C}$$