Forces on Surfaces Immersed in Fluids

INTRODUCTION

In the previous chapter the pressure distribution in fluids in static and dynamic condition was discussed. **When a fluid is in contact with a surface is exerts a normal force on the surface.** The walls of reservoirs, sluice gates, flood gates, oil and water tanks and the hulls of ships are exposed to the forces exerted by fluids in contact with them. The fluids are generally under static condition. For the design of such structures **it is necessary to determine the total force** on them. It is **also necessary to determine the point of action of this force.** The point of action of the total force is known as **center of pressure or pressure center**. From the basic hydrodynamic equation it is known that the force depends on the pressure at the depth considered.

i.e.,

$P = \gamma h$. Force on an elemental area dA as a depth, h, will be

$dF = \gamma h dA$

The total force is obtained by integrating the basic equation over the area

$$F = \gamma \int_A hdA$$

From the definition of center of gravity or centroid

$$\int_A hdA = h^{-} A$$

where h^{-} is the depth of the Centre of gravity of the area.

To determine the point of action of the total force, moment is taken of the elemental forces with reference to an axis and equated to the product of the total force and the distance of the Centre of pressure from the axis namely hcp

$$F.\ hcp = \int_A hdF = \gamma \int_A h^2 \ dA$$

The integral over the area is nothing but the second moment or the moment of inertia of the area about the axis considered. Thus there is a need to know the Centre of gravity and the moment of inertia of areas.

FORCE ON AN ARBITRARILY SHAPED PLATE IMMERSED IN A LIQUID

Case 1 : Surface exposed to gas pressure : For plane surface, force = area × pressure The contribution due to the weight of the gas column is negligible. The resultant acts at the centroid of the area as the pressure at all depths are the same.
Case 2 : Horizontal surface at a depth y.
$P = -y \times \gamma$ and as y is – ve, force = $Ay\gamma$ in which y may also be expressed as head of the fluid. The resultant force acts vertically through the centroid of the area, Here also the pressure at all locations are the same.
Case : Plane inclined at angle θ with horizontal.

Centre of Pressure for Immersed Vertical Planes

Case 1: A rectangle of width b and depth d, the side of length b being horizontal.
Case 2: A circle of diameter d.
Case 3: A triangle of height h with base b, horizontal and nearer the free surface.

COMPONENT OF FORCES ON IMMERSED INCLINED RECTANGLES

Consider a case of a rectangle of $a \times d$, with side d inclined at θ to the horizontal, immersed in a fluid with its centroid at a depth of h^{-} m. For this case it can be shown that
(*i*) **The horizontal component of the resultant force equals the force on the vertical projection of the area and**
(*ii*) **The vertical component equals the weight of the fluid column above this area.**

FORCES ON CURVED SURFACES

(i) Vertical forces : The vertical force on a curved surface is given by the weight of the liquid enclosed by the surface and the horizontal free surface of the liquid. The force acts along the Centre of gravity of the volume. In case there is gas pressure above the

surface, the force due to gas pressure equals the product of horizontal projected area and the
gas pressure and acts at the centroid of the projected area.
If the other side of the surface is exposed to the same gas pressure, force due to the gas
pressure cancels out. This applies to doubly curved surfaces and inclined plane surfaces.
(ii) Horizontal forces: The horizontal force equals the force on the projected area of the curved surface and acts at the centre of pressure of the projected area.
The value can be calculated