Bernoulli Equation and Applications

INTRODUCTION

In chapter five flow of ideal fluids was discussed. The main idea was the study of flow pattern. The determination of equal flow paths and equal potential lines was discussed. No attempt was made to determine the numerical value of these quantities. In this chapter the method of determination of the various energy levels at different locations in the flow is discussed. In this process first the various forms of energy in the fluid are identified. Applying the law of conservation of energy the velocity, pressure and potential at various locations in the flow are calculated. Initially the study is limited to ideal flow. However the modifications required to apply the analysis to real fluid flows are identified. The material discussed in this chapter are applicable to many real life fluid flow problems. The laws presented are the basis for the design of fluid flow systems.

Energy consideration in fluid flow:

Consider a small element of fluid in flow field. The energy in the element as it moves in the flow field is conserved. This principle of conservation of energy is used in the determination of flow parameters like pressure, velocity and potential energy at various locations in a flow. The concept is used in the analysis of flow of ideal as well as real fluids. Energy can neither be created nor destroyed. It is possible that one form of energy is converted to another form. The total energy of a fluid element is thus conserved under usual flow conditions. If a stream line is considered, it can be stated that the total energy of a fluid element at any location on the stream line has the same magnitude.

FORMS OF ENERGY ENCOUNTERED IN FLUID FLOW

Energy associated with a fluid element may exist in several forms. These are listed here and the method of calculation of their numerical values is also indicated.

Kinetic Energy

This is the energy due to the motion of the element as a whole. If the velocity is V, then the kinetic energy for m kg is given by

$KE = mV^2/2go$ Nm

The unit in the SI system will be Nm also called Joule (J)

The learner should be familiar with both forms of the equation and should be able to choose and use the proper equation as the situation demands. When different forms of the energy of a fluid element is summed up to obtain the total energy, all forms should be in the same unit.

Potential Energy

This energy is due to the position of the element in the gravitational field. While a zero value for *KE* is possible, the value of potential energy is relative to a chosen datum. The value of potential energy is given by

$PE = m\,Z\,g/go$ Nm

Where *m* is the mass of the element in kg, Z is the distance from the datum along the gravitational direction, in *m*. The unit will be (kg m m/s2) × (Ns2/kgm) *i.e.*, Nm. The specific potential energy (per kg) is obtained by dividing equation 6.1.3 by the mass of the element. $PE = Z\,g/g0$ Nm/kg
This gives the physical quantity of energy associated with 1 kg due to the position of the fluid element in the gravitational field above the datum. As in the case of the kinetic energy, the value of *PE* also is expressed as head of fluid, *Z.*

$PE = Z\,(g/go)\,(go/g) = Z\,m.$

This form will be used in equations, but as in the case of KE, one should be familiar with both the forms and choose the suitable form as the situation demands.

Pressure Energy (Also Equals Flow Energy)

The element when entering the control volume has to flow against the pressure at that location. The work done can be calculated .
The boundary of the element of fluid considered is shown by the dotted line,
Force = $P1$
A, distance to be moved = L, work done = $P1AL = P1\,mv$ as AL = volume = mass × specific
volume, v. ∴ flow work = $P\,mv$.
The pressure energy per kg can be calculated using $m = 1$. The flow energy is given by
$FE = P.v = P/\rho$, Nm/kg
$FE = P/\gamma$, m
It is important that in any equation, when energy quantities are summed up consistent forms of these set of equations should be used, that is, all the terms should be expressed either as head of fluid or as energy (J) per kg. These are the three forms of energy encountered more often in flow of incompressible fluids.

Internal Energy

This is due to the thermal condition of the fluid. This form is encountered in compressible fluid flow. For gases (above a datum temperature) $IE = cv\ T$ where T is the temperature above the datum temperature and cv is the specific heat of the gas at constant volume. The unit for internal energy is J/kg (Nm/kg). When friction is significant other forms of energy is converted to internal energy both in the case of compressible and incompressible flow.

Electrical and Magnetic Energy

These are not generally met with in the study of flow of fluids. However in magnetic pumps and in magneto hydrodynamic generators where plasma flow in encountered, electrical and magnetic energy should also be taken into account.