Kinetics of Particles

A- Force, Mass, and Acceleration

Newton's Second Law of Motion:

Kinetics is a branch of dynamics that deals with the relationship between the change in motion of a body and the forces that cause this change.

The basis for kinetics is ***Newton's second law***, which states that when an unbalanced force acts on a particle, the particle will accelerate in the direction of the force with a magnitude that is proportional to the force.

If the mass of the particle is m, *Newton's second law of motion* may be written in mathematical form as:

$$\boldsymbol{F = ma}$$

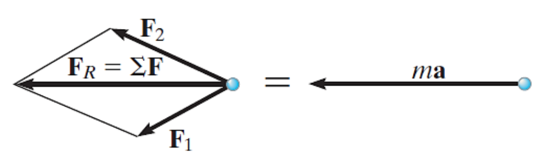
Constrained and Unconstrained Motions:

- Unconstrained motion: No mechanical guides or linkages to constrain its motion. Example: airplanes, rockets, etc.
- Constrained motion: Motion is limited by some mechanical guide or linkages. Example: mechanisms

1- Equation of Motion: Rectangular coordinates $(x - y)$

$$\sum \boldsymbol{F} = m\mathbf{a}$$

$$\sum F_x\,\mathbf{i} + \sum F_y\,\mathbf{j} = m(a_x\mathbf{i} + a_y\mathbf{j})$$



Equating $\mathbf{i}$ and $\boldsymbol{j}$ terms

$$\boxed{\begin{aligned} \sum F_x &= ma_x \\ \sum F_y &= ma_y \end{aligned}}$$

Notes:

1- If a moving particle contacts a *rough surface,* it may be necessary to use the *frictional equation:*

$$\boldsymbol{F_f = \mu_k\, N}$$

$\boldsymbol{F_f}$: Friction force , $\boldsymbol{\mu_k}$**:** kinetic friction coefficient, $\boldsymbol{N}$: normal force

2- If the particle is connected to an *elastic spring* having negligible mass, the spring force $\boldsymbol{F_s}$ is:

$$\boldsymbol{F_s = k\, s}$$

Where $\boldsymbol{s = l - l_0}$

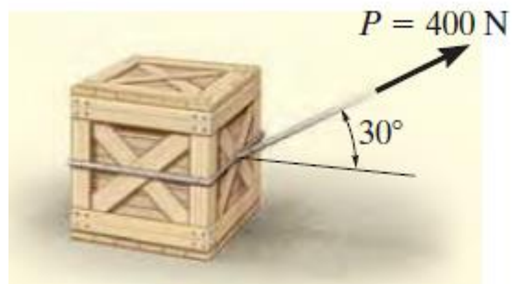
$\boldsymbol{k}$: spring's stiffness (constant) N/m , lb/ft
$\boldsymbol{s}$ **:** is the stretch or compression mm, ft
$\boldsymbol{l}$ **:** deformed length
$\boldsymbol{l_0}$**:** undeformed length

3- If $\boldsymbol{a = f(t)}$, use [$\boldsymbol{a = dv/dt}$ and $\boldsymbol{v = ds/dt}$] which, when integrated, yield the particle's velocity and position, respectively.

4- If $\boldsymbol{a} = constant$ ($\boldsymbol{a_c}$), use [$\boldsymbol{v = v_0 + a_c t}$, $\boldsymbol{v^2 = v_0^2 + 2a_c(s} - s_0)$ and $\boldsymbol{s = s_0 + v_0 t + \frac{1}{2} a_c t^2}$] to determine the velocity or position of the particle.

5- If $\boldsymbol{a}$ is a function of displacement ($\boldsymbol{a = f(s)}$), use $\boldsymbol{a\,ds = v\,dv}$

Ex. (1): The 50-kg crate shown in figure, rests on a horizontal surface for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate is subjected to a 400N towing force as shown, determine the velocity of the crate in 3 s starting from rest.



Sol.

equations of motion.

$$+\uparrow \sum F_y = ma_y : \quad N_C - 490.5 + 400 \sin 30 = 50\ (0)$$

$$\therefore \quad N_C = 290.5 \text{ N}$$

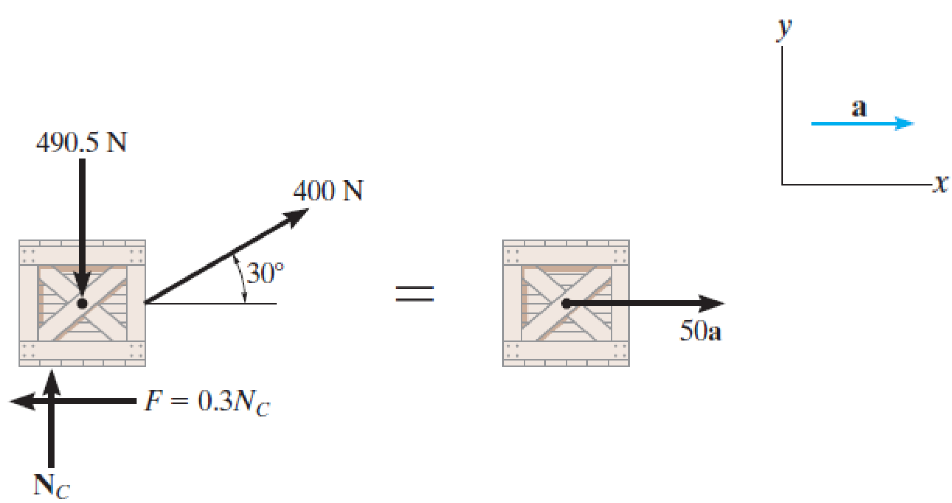
$$\overset{+}{\rightarrow} \quad \sum F_x = ma_x : \quad 400 \cos 30 - 0.3N_C = 50\ a$$

$$\therefore \quad a = 5.185 \text{ m/s}^2$$

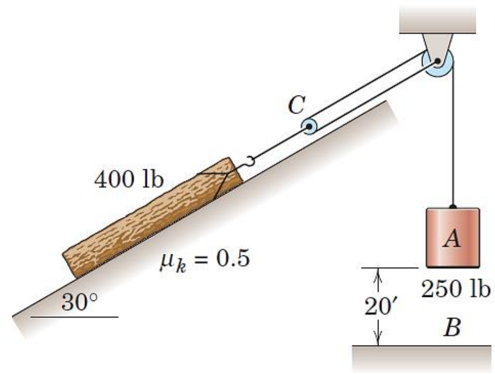
Kinematics: $a = 5.185 \text{ m/s}^2$ is constant

$$v = v_0 + a_c t$$

$v = 0 + 5.185(3) = 15.6 \text{ m/s}$ **Ans.**



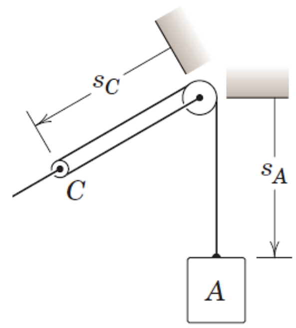
Ex.(2): The 250-lb concrete block *A* is released from rest in the position shown and pulls the 400-lb log up the 30^o ramp. If the coefficient of kinetic friction between the log and the ramp is 0.5, determine the velocity of the block as it hits the ground at *B*.



Sol.:

The total length of cable is

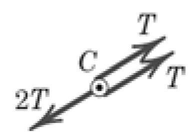
$$L = 2s_C + s_A + constant$$



Differentiating twice with respect to time gives

$$0 = 2a_C + a_A \qquad (1)$$

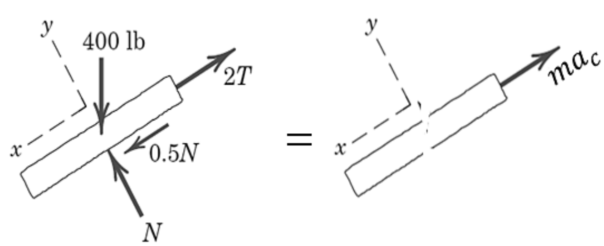
For the log:



$F_f = \mu_k N = 0.5N$

$\sum F_y = ma_y$

$N - 400\cos 30 = 0 \quad \Longrightarrow \quad N = 346\,\text{lb}$



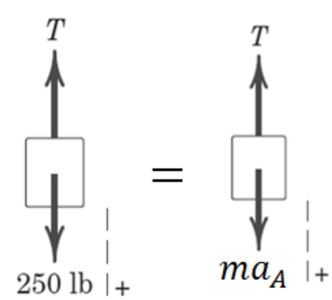
$\sum F_x = ma_x$

$$0.5N + 400\sin 30 - 2T = \frac{400}{32.2}a_C$$

$373 - 2T = 12.42a_C$

$$T = 186.5 - 6.211a_C \qquad (2)$$

For the block A:



$+\downarrow \quad \sum F = ma$

$$250 - T = \frac{250}{32.2}a_A$$

$$250 - (186.5 - 6.211a_C) = 7.764a_A$$

$$63.5 + 6.211a_C = 7.764a_A \qquad (3)$$

Solving the three equations gives us

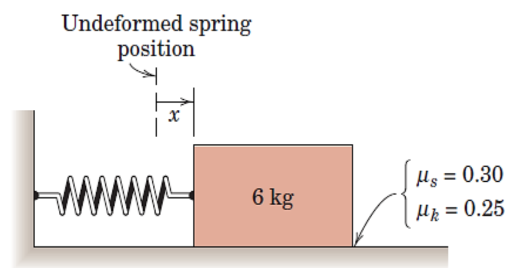
$$a_A = 5.83 \text{ ft}/s^2 \,, \quad a_C = -2.92 \text{ ft}/s^2 \quad \text{and} \quad T = 205 \text{ lb}$$

Since a_A is constant

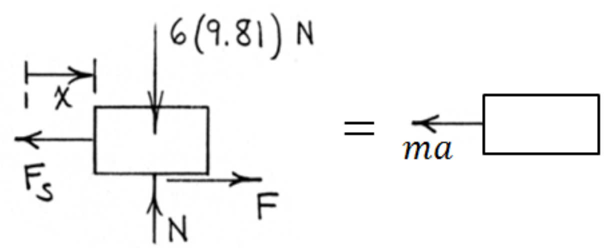
$$v_A^2 = v_A^2)_0 + 2a_A(y - y_0)_A$$

$$v_A^2 = 0 + 2(5.83)(20 - 0)_A \implies v_A = 15.27 \text{ ft/s}$$ *Ans.*

Ex.(3): The nonlinear spring has a tensile force-deflection relationship given by $F_s = 150x + 400x^2$, where x is in meters and F_s is in Newton. Determine the acceleration of the 6 kg block if it is released from rest at (a) $x = 50$ mm and (b) $x = 100$ mm



<u>Sol:</u>



$$+\uparrow \ \sum F_y = ma_y = 0$$

$$N = 6(9.81) = 58.9 \text{ N}$$

$$F_{max} = \mu_s N = 0.3(58.9) = 17.66 \text{ N}$$

(a) At $x = 50 \text{ mm} = 0.05 \text{ m}$

$$F_s = 150(0.05) + 400(0.05)^2$$

$$F_s = 8.5 \text{ N} < F_{max} \implies a = 0$$ No motion (the block at rest) **Ans.**

(b) At $x = 0.01 \text{ m}$

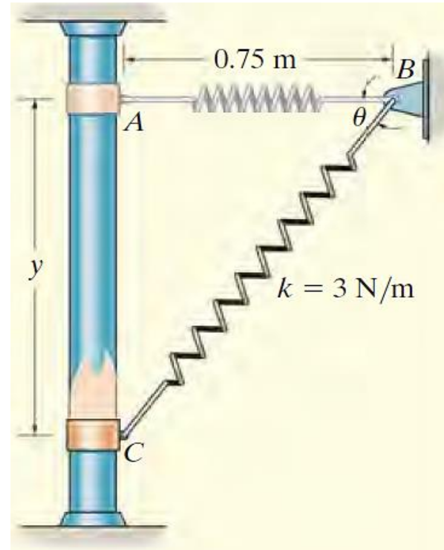
$$F_s = 150(0.01) + 400(0.01)^2$$

$$F_s = 19 \text{ N} > F_{max}$$ { the block moves (accelerates) }

$$\overset{+}{\rightarrow} \ \sum F_x = ma_x \ : \quad -F_s + \mu_k N = 6a$$

$$-19 + 0.25(58.9) = 6a \implies a = -0.714 = 0.714 \text{ m/s}^2 \leftarrow$$ **Ans.**

Ex.(4): A smooth 2-kg collar, shown in Figure below, is attached to a spring having a stiffness $k = 3$ N/m and an unstretched length of 0.75 m. If the collar is released from rest at *A*, determine its acceleration and the normal force of the rod on the collar at the instant $y = 1$ m.



Sol.:

equations of motion.

$\overset{+}{\rightarrow} \quad \sum F_x = m a_x \ : \quad F_s \cos\theta - N_C = 2(0)$ (1)

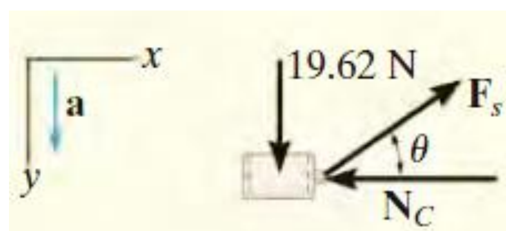
$+\uparrow \ \sum F_y = m a_y : \quad 19.62 - F_s \sin\theta = 2a$ (2)

Spring force $F_s = k\ s$

$s =$ deformed (stretched) length $-$ undeformed (unstretched) length

$s = CB - AB \quad , \ AB = 0.75 \ , \ CB = \sqrt{y^2 + (0.75)^2}$

$\therefore \quad s = \sqrt{y^2 + (0.75)^2} - 0.75$



$F_s = 3\ \sqrt{y^2 + (0.75)^2} - 0.75$

$\theta = tan^{-1}\left(\frac{y}{0.75}\right)$

At $y = 1$ m $\Longrightarrow$ $F_s = 1.5$ N $, \theta = 53.1^{\text{o}}$ sub. In eq. (1) & (2) we obtained

$N_C = 0.9$ N Ans.

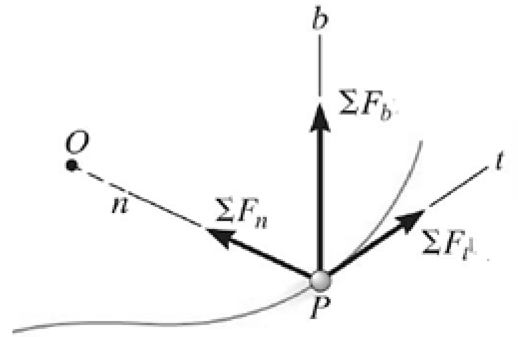
$a = 9.21\ \text{m/s}^2 \ \downarrow$ Ans.

2- Equations of Motion: Normal and Tangential Coordinates (*n*- *t*)

When a particle moves along a **curved path** which is known, the equation of motion for the particle may be written in the **tangential**, **normal**, and **binormal** directions. *Note that there is no motion of the particle in the binormal direction*, since the particle is constrained to move along the path.

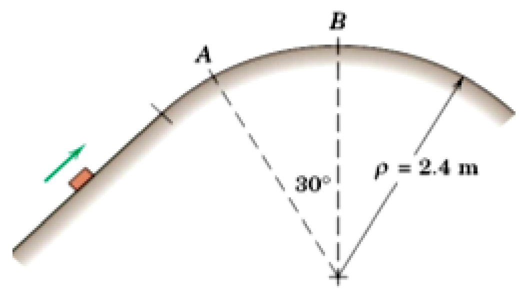
The equations of motion are:

$$\boxed{\begin{aligned} \sum \boldsymbol{F_t} &= \boldsymbol{m a_t} \\ \sum \boldsymbol{F_n} &= \boldsymbol{m a_n} \\ \sum \boldsymbol{F_b} &= \boldsymbol{0} \end{aligned}}$$



Where $\boldsymbol{a_t = \frac{dv}{dt} = \dot{v}}$, $\boldsymbol{a_n = \frac{v^2}{\rho} = v\dot{\beta}}$, and $\boldsymbol{v = \rho\dot{\beta}}$

<u>Ex. (5):</u> If the 2-kg block passes over the top B of the circular portion of the path with a speed of 3.5 m/s, calculate the magnitude of the normal force exerted by the path on the block. Determine the maximum speed v which the block can have at A without losing contact with the path.



<u>Sol. :</u>

$$\Sigma F_n = m a_n = m \frac{v^2}{\rho}$$

$$2(9.81) - N = 2 \frac{3.5^2}{2.4}$$

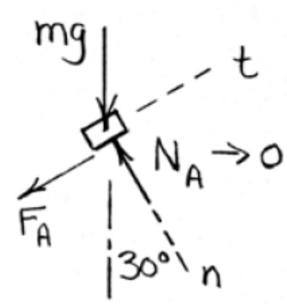
$$\underline{N_B = 9.41 \text{ N}}$$

Loss of contact at A: $N_A \to 0$

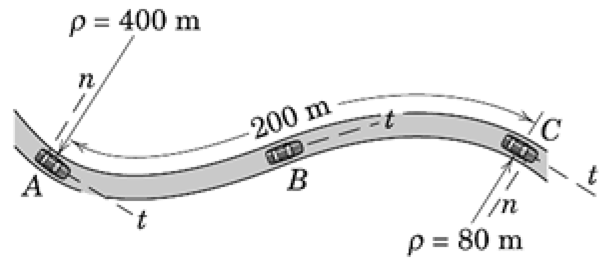
$$\Sigma F_n = m a_n = m \frac{v^2}{\rho} :$$

$$\cancel{m} g \cos 30^\circ = \cancel{m} \frac{v^2}{2.4}$$

$$\underline{v = 4.52 \text{ m/s}}$$



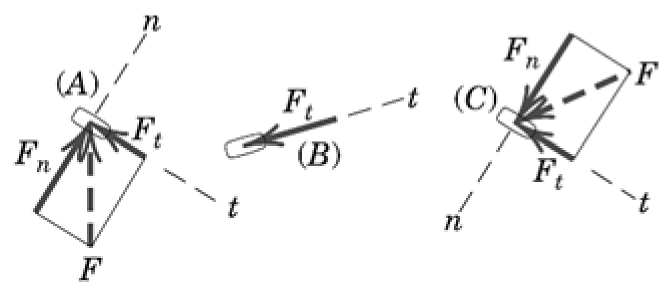
Ex. (6): A 1500-kg car enters a section of curved road in the horizontal plane and slows down at a uniform rate from a speed of 100 km/h at *A* to a speed of 50 km/h as it passes *C*. The radius of curvature ρ of the road at *A* is 400 m and at *C* is 80 m. Determine the total horizontal force exerted by the road on the tires at positions *A*, *B*, and *C*. Point *B* is the inflection point where the curvature changes direction.



Sol.:

$v_C = \frac{50000}{3600} = 13.889\ \text{m/s}$, $v_A = \frac{100000}{3600} = 27.777\ \text{m/s}$

The constant tangential acceleration is in the negative t-direction, and its magnitude is given by



$v_C^2 = v_A^2 + 2a_t\Delta s$

$a_t = \frac{(13.889^2 - 27.777^2}{2(200)} = 1.447\ \text{m/s}^2$

The a_t is equal at each point

$a_t = a_t)_A = a_t)_B = a_t)_C = 1.447\ \text{m/s}^2$

The normal components of acceleration at *A*, *B*, and *C* are

$[a_n = \frac{v^2}{\rho}]$ at A $a_n = \frac{v_A^2}{\rho_A} = \frac{27.777}{400} = 1.929\ \text{m/s}^2$

at *B* $a_n = \frac{v_B^2}{\rho_B = \infty} = 0$

at *C* $a_n = \frac{v_C^2}{\rho_C} = \frac{13.889}{80} = 2.41\ \text{m/s}^2$

$\sum F_t = ma_t$; $F_t = 1500(1.447) = 2170$ N

$\sum F_n = ma_n$; at *A* $F_n = 1500(1.929) = 2890$ N at B $F_n = 1500(0) = 0$

at C $F_n = 1500(2.41) = 3620$ N

Thus, the total horizontal force acting on the tires becomes

at A $F = \sqrt{F_n^2 + F_t^2} = \sqrt{2890^2 + 2170^2} = 3620$ N **Ans.**

at B $F = \sqrt{F_n^2 + F_t^2} = F_t = 2170$ N **Ans.**

at C $F = \sqrt{F_n^2 + F_t^2} = \sqrt{3620^2 + 2170^2} = 4220$ N **Ans.**

3- Equations of Motion: Polar Coordinates $(r - \theta)$

$$\boxed{\begin{aligned} \sum \boldsymbol{F_r} &= \boldsymbol{m a_r} \\ \sum \boldsymbol{F_\theta} &= \boldsymbol{m a_\theta} \end{aligned}}$$

where $a_r = \ddot{r} - r\dot{\theta}^2$ and $a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$\boldsymbol{\dot{\theta} = \frac{d\theta}{dt} = \omega}$, $\boldsymbol{\omega = \frac{v}{r}}$

$\boldsymbol{v}$: tangential velocity (m/s)

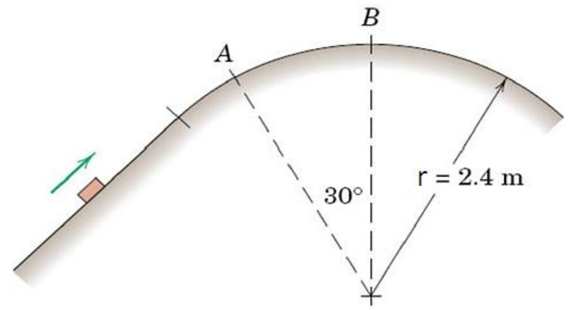
$\boldsymbol{\dot{\theta} = \frac{d\theta}{dt} = \omega}$: angular velocity (rad/s)

$\boldsymbol{\ddot{\theta} = \frac{d_\theta^2}{dt^2}}$: angular acceleration (rad/s^2)

$\boldsymbol{a_r}$: radial component of acceleration

$\boldsymbol{a_\theta}$: transverse component of acceleration

Ex. (7): If the 2-kg block passes over the top B of the circular portion of the path with a speed of 3.5 m/s, calculate the magnitude of the normal force exerted by the path on the block. Determine the maximum speed v which the block can have at A without losing contact with the path.



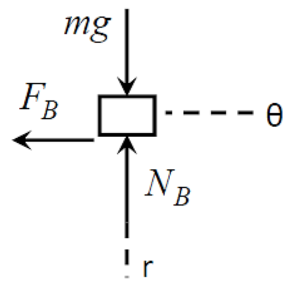
Sol.:

$r = 2.4\ \mathrm{m}$

$\therefore \quad \dot{r} = 0 \quad , \quad \ddot{r} = 0$

$\dot{\theta} = \frac{v}{r}$

$\sum F_r = m(\ddot{r} - r\dot{\theta}^2) = -m\frac{v^2}{r}$

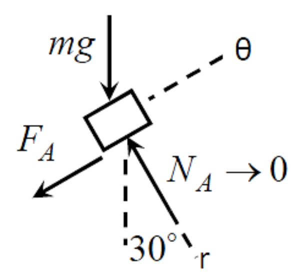


$-2(9.81) + N_B = -2\frac{(3.5)^2}{2.4}$

$N_B = 9.41\ \mathrm{N}$

Loss of contact at $A:\ N_A \to 0$

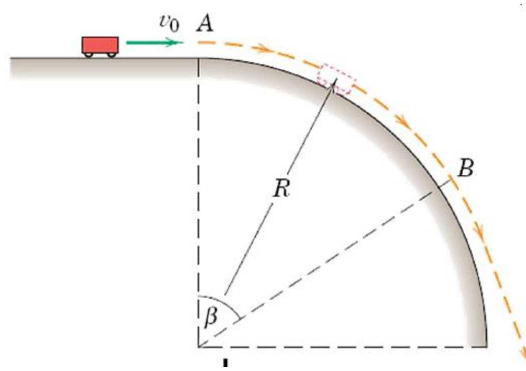
$\sum F_r = -m\frac{v^2}{r}$



$-mg\cos 30 = -m\frac{v^2}{r}$

$v = 4.52\ \mathrm{m/s}$ **Ans.**

Ex. (8): A small vehicle enters the top A of the circular path with a horizontal velocity υ_0 and gathers speed as it moves down the path. Determine an expression for the angle β which locates the point where the vehicle leaves the path and becomes a projectile. Evaluate your expression for $\upsilon_0 = 0$. Neglect friction



Sol.:

$$\sum F_\theta = ma_\theta, \quad mg\sin\theta = ma_\theta, \quad a_\theta = g\sin\theta$$

$$\int \upsilon d\upsilon = \int a_\theta ds, \quad \int_{\upsilon_0}^{\upsilon} \upsilon d\upsilon = \int_0^\theta g\sin\theta(Rd\theta)$$

$$\upsilon^2 = \upsilon_0^2 + 2gR(1-\cos\theta)$$

$$\sum F_r = ma_r, \quad -mg\cos\theta + N = -m\frac{\upsilon^2}{R}$$

$$N = mg\cos\theta - m\frac{\upsilon_0^2}{R} - 2mg(1-\cos\theta)$$

$$= mg(3\cos\theta - 2 - \frac{\upsilon_0^2}{gR})$$

When $N = 0$, so $3\cos\beta = 2 + \frac{\upsilon_0^2}{gR}$ $\beta = \cos^{-1}(\frac{2}{3} + \frac{\upsilon_0^2}{3gR})$

For $\upsilon_0 = 0$, $\beta = \cos^{-1}(\frac{2}{3}) = 48.2^\circ$ ***Ans.***