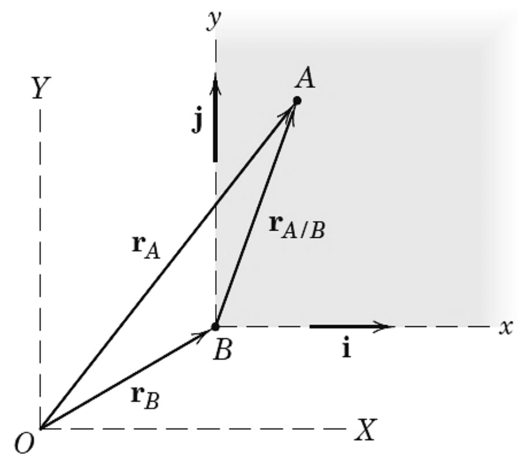
3. Relative Motion of Two Particles using Translating Axes(*x*-y):

If two particles *A* and *B* undergo independent motions then these motion can be related to their relative motion using a translating set of axis attached to one of the particle *B*.



Vector Representation

1. ***Position***

$$\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$$
$$\mathbf{r}_{A/B} = x\mathbf{i} + y\mathbf{j}$$

Where:

$\mathbf{r}_A$: position vector of particle *A*

$\mathbf{r}_B$: position vector of particle ***B***

$\mathbf{r}_{A/B}$: relative position vector

2. ***Velocity***

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B} \quad \text{or} \quad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$
$$\dot{\mathbf{r}}_{A/B} = \mathbf{v}_{A/B} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

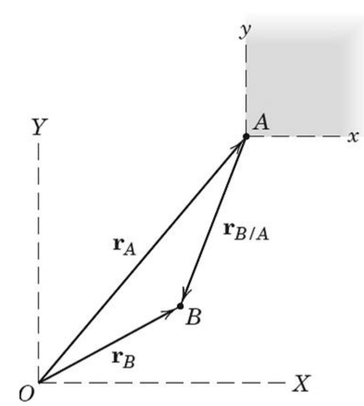
3. ***Acceleration***

$$\ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B} \quad \text{or} \quad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$
$$\ddot{\mathbf{r}}_{A/B} = \dot{\mathbf{v}}_{A/B} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

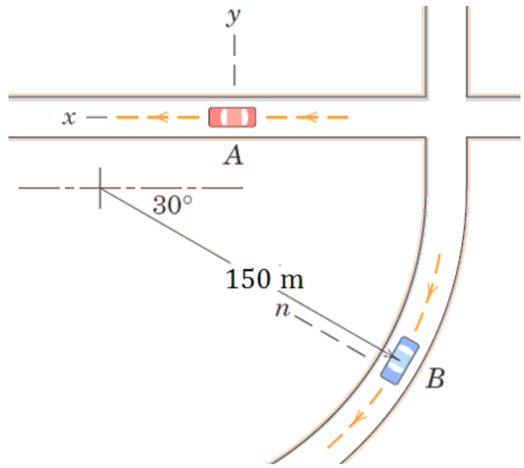
Additional Considerations

The relative-motion equations for position, velocity, and acceleration are:

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A} \qquad \mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A} \qquad \mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$



Ex. (17): Car A is accelerating in the direction of its motion at the rate of $1.2\ \mathrm{m/s^2}$. Car B is rounding a curve of 150 m radius at a constant speed of $54\ \mathrm{km/h}$. Determine the velocity and acceleration which car B appears to have to an observer in car A if car A has reached a speed of $72\ \mathrm{km/h}$ for the positions represented.

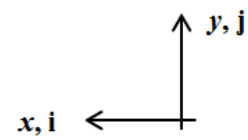


***Solution I**: Use Vector solution*

The relative-velocity equation is

$$\mathrm{v}_B = \mathrm{v}_A + \mathrm{v}_{B/A}$$

$$v_A = \frac{72}{3.6} = 20\ \mathrm{m/s}\ ,\quad v_B = \frac{54}{3.6} = 15\ \mathrm{m/s}$$



$$\mathrm{v}_A = v_A\mathbf{i} = 20\mathbf{i}\ \ \mathrm{m/s}$$

$$\mathrm{v}_B = (v_B\cos 60)\mathbf{i} - (v_B\sin 60)\,\boldsymbol{j}$$

$$\mathrm{v}_B = (15\cos 60)\mathbf{i} - (15\sin 60)\,\boldsymbol{j}$$

$$\mathrm{v}_B = 7.5\mathbf{i} - 13\boldsymbol{j}\quad \mathrm{m/s}$$

$$\therefore\ \ \mathrm{v}_{\boldsymbol{B}} = \mathrm{v}_{\boldsymbol{A}} + \mathrm{v}_{\boldsymbol{B/A}}$$

$$7.5\mathbf{i} - 13\boldsymbol{j} = 20\mathbf{i} + \mathrm{v}_{\boldsymbol{B/A}}$$

$$\mathrm{v}_{\boldsymbol{B/A}} = -12.5\mathbf{i} - 13\boldsymbol{j}\ \ \mathrm{m/s}$$

the magnitude of $\mathrm{v_{B/A}}$ is: $v_{\boldsymbol{B/A}} = \sqrt{(-12.5)^{\mathbf{2}} + (-13)^{\mathbf{2}}} = 18.034\ \mathrm{m/s}$ ***Ans.***

And the direction of $\mathrm{v_{B/A}}$ is : $\theta = tan^{-1}\left(\frac{-13}{-12.5}\right) = 46.12^{o}$ ***Ans***.

$$(a_n)_B = \frac{v_B^2}{\rho} = \frac{15^2}{150} = 1.5\ \mathrm{m/s^2}\quad ,\qquad (a_t)_B = \dot{v}_B = 0$$

$$\mathrm{a}_B = \mathrm{a}_A + \mathrm{a}_{B/A}$$

$$1.5\cos 30\ \mathbf{i} + 1.5\sin 30\ \boldsymbol{j} = 1.2\ \mathbf{i} + \mathrm{a}_{B/A}$$

$$\therefore \quad \mathrm{a}_{B/A} = 0.099\mathbf{i} + 0.75\mathbf{j}$$

the magnitude of $\mathrm{a}_{B/A}$ is: $a_{B/A} = \sqrt{(0.099)^2 + (0.75)^2} = 0.756 \text{ m/s}^2$ ***Ans.***

the direction of $\mathrm{a}_{B/A}$ is : $\theta_a = tan^{-1}\left(\frac{0.75}{0.099}\right) = 82.48^o$ ***Ans.***

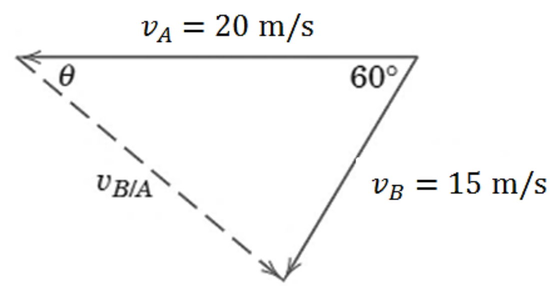
Solution II: ***Use Trigonometric solution***

$$v_A = \frac{72}{3.6} = 20 \text{ m/s} \;, \quad v_B = \frac{54}{3.6} = 15 \text{ m/s}$$

cosine law; $v_{B/A} = \sqrt{v_A^2 + v_B^2 - 2v_A v_B \cos 60} = 18.03 \text{ m/s}$ ***Ans.***

sine law ; $\dfrac{v_B}{\sin\theta} = \dfrac{v_{B/A}}{\sin 60}$

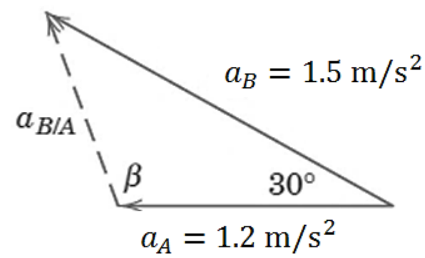
$$\therefore \quad \theta = 46.1^o$$



$$a_B = a_n = \frac{v_B^2}{\rho} = \frac{15^2}{150} = 1.5 \text{ m/s}^2$$

$$\mathrm{a}_B = \mathrm{a}_A + \mathrm{a}_{B/A}$$

$$\therefore \quad \mathrm{a}_{B/A} = \mathrm{a}_B - \mathrm{a}_A$$



$$\left(a_{B/A}\right)_x = 1.5\cos 30 - 1.2 = 0.099 \text{ m/ } s^2$$

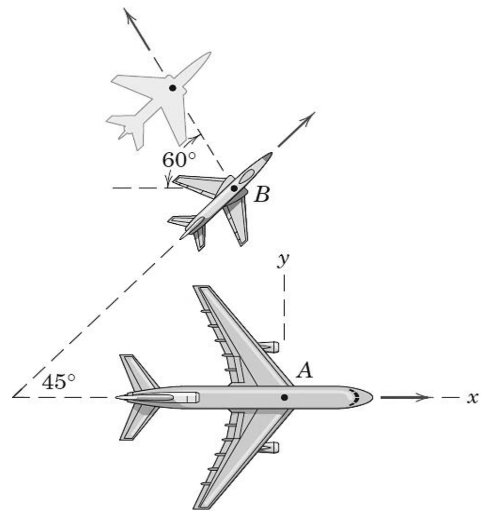
$$\left(a_{B/A}\right)_y = 1.5\sin 30 = 0.75 \text{ m/ } s^2$$

The magnitude of $a_{B/A}$ is: $a_{B/A} = \sqrt{\left(a_{B/A}\right)_x^2 + \left(a_{B/A}\right)_y^2} = \sqrt{0.099^{\,2} + 0.75^2} = 0.756 \text{ m/ } s^2$

The direction of $\mathrm{a}_{B/A}$ is determined from sine law

$$\frac{a_B}{\sin\beta} = \frac{a_{B/A}}{\sin 30} \quad \Rightarrow \quad \frac{1.5}{\sin\beta} = \frac{0.756}{\sin 30} \quad \Rightarrow \quad \beta = 97.52^o$$ ***Ans.***

Ex. (18): Passengers in the jet transport A flying east at a speed of 800 km/h observe a second jet plane B that passes under the transport in horizontal flight. Although the nose of B is pointed in the 45^o northeast direction, plane B appears to the passengers in A to be moving away from the transport at the 60^o angle as shown. Determine the true velocity of B.

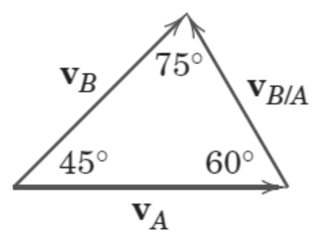


Sol. I: Use ***Trigonometric.***

Sine law;

$$\frac{v_B}{\sin 60} = \frac{v_A}{\sin 75} \quad \Longrightarrow \quad v_B = v_A\left(\frac{\sin 60}{\sin 75}\right)$$

$$v_B = 8000(\left(\frac{\sin 60}{\sin 75}\right) = 717 \text{ km/h}$$ ***Ans.***



Sol. II: ***Vector Algebra***

Using unit vectors **i** and **j**, we express the velocities in vector form as

$$\mathbf{v}_A = 800\mathbf{i} \text{ km/h} \qquad \mathbf{v}_B = (v_B \cos 45°)\mathbf{i} + (v_B \sin 45°)\mathbf{j}$$

$$\mathbf{v}_{B/A} = (v_{B/A} \cos 60°)(-\mathbf{i}) + (v_{B/A} \sin 60°)\mathbf{j}$$

$$\therefore \quad \mathrm{v}_{\boldsymbol{B}} = \mathrm{v}_{\boldsymbol{A}} + \mathrm{v}_{\boldsymbol{B/A}}$$

$$v_B \cos 45\mathbf{i} + v_B \sin 45\,\boldsymbol{j} = 8000\mathbf{i} + (v_{B/A} \cos 60)(-\mathbf{i}) + (v_{B/A} \sin 60)\,\boldsymbol{j}$$

i-terms: $v_B \cos 45 = 8000 + v_{B/A} \cos 60$ (1)

j-terms: $v_B \sin 45 = v_{B/A} \sin 60$ (2)

Solving the equation (1) & (2) gives;

$v_B = 586$ km/h and $v_{B/A} = 717$ km/h ***Ans.***