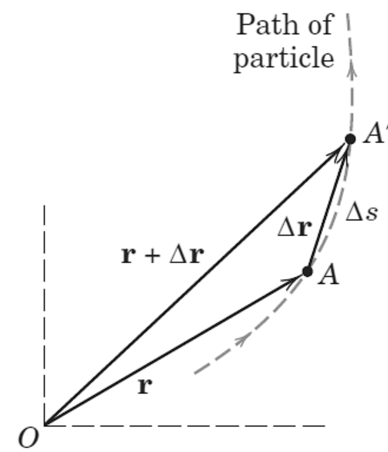
2. Plane Curvilinear Motion:

Curvilinear motion occurs when a particle moves along a curved path. Curvilinear motion can cause changes in both the magnitude and direction of the position, velocity, and acceleration vectors

- **Displacement**

The displacement represented the change in the particle's positions.

$\Delta \mathbf{r} = \mathbf{r}_{\mathbf{A'}} - \mathbf{r}_{\mathbf{A}} = \mathbf{r}' - \mathbf{r}$



- **Velocity**

Average velocity, $\mathbf{v}_{avg} = \frac{\Delta \boldsymbol{r}}{\Delta \boldsymbol{t}}$

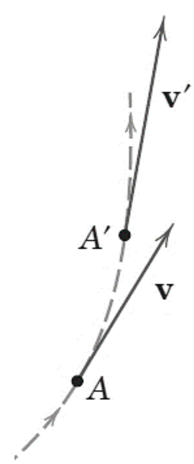
Instantaneous velocity, $\text{v} = \lim_{\Delta t \to 0} \left(\frac{\Delta \boldsymbol{r}}{\Delta \boldsymbol{t}}\right)$

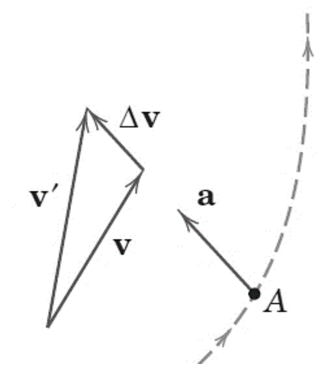
Or, $$\mathbf{v} = \frac{\mathbf{dr}}{\mathbf{dt}} = \dot{\boldsymbol{r}}$$

The magnitude of v is called speed (scalar).

$$v = \lim_{\Delta t \to 0} \left(\frac{\Delta \boldsymbol{s}}{\Delta \boldsymbol{t}}\right)$$

Or, $$v = |\mathbf{v}| = \frac{\mathbf{ds}}{\mathbf{dt}} = \dot{\boldsymbol{s}}$$





- **Acceleration**

Average accel., $\mathbf{a}_{avg} = \frac{\Delta \mathbf{v}}{\Delta \boldsymbol{t}}$, $\Delta \mathbf{v} = \mathbf{v}' - \mathbf{v}$

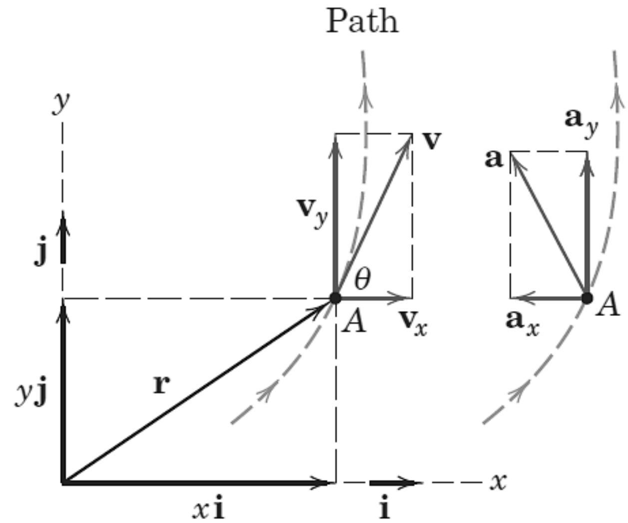
Instantaneous accel. , $\mathbf{a} = \mathbf{lim}_{\Delta t \to \mathbf{0}} \left(\frac{\Delta \mathbf{v}}{\Delta \boldsymbol{t}}\right)$, or

$$\mathbf{a} = \frac{\mathbf{dv}}{\mathbf{dt}}$$

Note: The velocity (**v**) is always a vector tangent to the path, while the acceleration (**a**) is not tangent to the path.

2.1. Rectangular Coordinates (*x*-*y*):

- Vector Representation



Position vector

$$\mathrm{r} = x\mathbf{i} + y\mathbf{j}$$

velocity vector

$$\mathrm{v} = \dot{\mathrm{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

$$\mathrm{v} = \mathrm{v}_x\mathbf{i} + \mathrm{v}_y\mathbf{j}$$

acceleration vector

$$\mathrm{a} = \dot{\mathrm{v}} = \ddot{\mathrm{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

$$\mathrm{a} = \mathrm{a}_x\mathbf{i} + \mathrm{a}_y\mathbf{j}$$

The scalar values of the components of **v** and **a** are;

$$\boldsymbol{v_x} = \dot{\boldsymbol{x}} \ , \ \boldsymbol{v_y} = \dot{\boldsymbol{y}} \ , \quad \text{and} \quad a_x = \dot{\boldsymbol{v}}_{\boldsymbol{x}} = \ddot{x} \ , \ a_y = \dot{\boldsymbol{v}}_{\boldsymbol{y}} = \ddot{y}$$

Where: $x = x(t)$, $y = y(t)$, $\dot{x} = \frac{dx}{dt}$, $\dot{y} = \frac{dy}{dt}$

The *magnitude* of the velocity is : $\boldsymbol{v} = \sqrt{\boldsymbol{v_x^2} + \boldsymbol{v_y^2}}$ (16)

The *direction* of v is : $\boldsymbol{\theta} = \boldsymbol{tan^{-1}}\left(\frac{v_y}{v_x}\right)$ (17)

The *magnitude* of acceleration is : $\boldsymbol{a} = \sqrt{\boldsymbol{a_x^2} + \boldsymbol{a_y^2}}$ (18)

Ex. (7): The curvilinear motion of a particle is defined by $v_x = 50 - 16\text{t}$ and $y = 100 - 4t^2$, where v_x is in meters per second, *y* is in meters, and *t* is in seconds. It is also known that $x = 0$ when $t = 0$. Determine velocity and acceleration of the particle when the position $y = 0$ is reached.

Sol.:

$$\left[\int dx = \int v_x\, dt\right] \qquad \int_0^x dx = \int_0^t (50 - 16t)\, dt \qquad x = 50t - 8t^2 \text{ m}$$

$$[a_x = \dot{v}_x] \qquad a_x = \frac{d}{dt}(50 - 16t) \qquad a_x = -16 \text{ m/s}^2$$

The *y*-components of velocity and acceleration are

$$[v_y = \dot{y}] \qquad v_y = \frac{d}{dt}(100 - 4t^2) \qquad v_y = -8t \text{ m/s}$$

$$[a_y = \dot{v}_y] \qquad a_y = \frac{d}{dt}(-8t) \qquad a_y = -8 \text{ m/s}^2$$

When $y = 0$

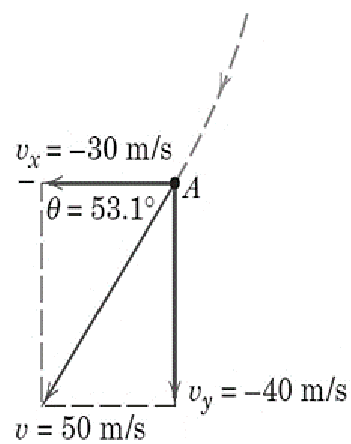
$$0 = 100 - 4t^2 \qquad \therefore\ t = 5\ s$$

$$\therefore\ v_x = 50 - 16t = 50 - 16(5) = -30 \text{ m/s}$$

$$v_y = -8t = -8(5) = -40 \text{ m/s}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-30)^2 + (-40)^2} = 50 \text{ m/s}$$

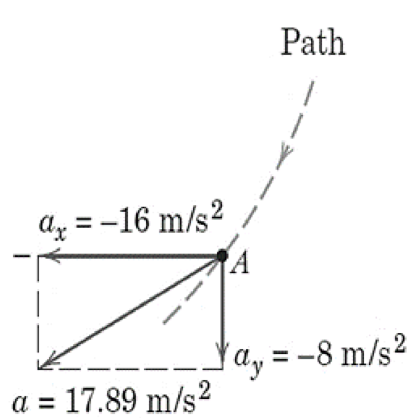
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-16)^2 + (-8)^2} = 17.89 \text{ m/s}^2$$



The velocity and acceleration as a vectors are

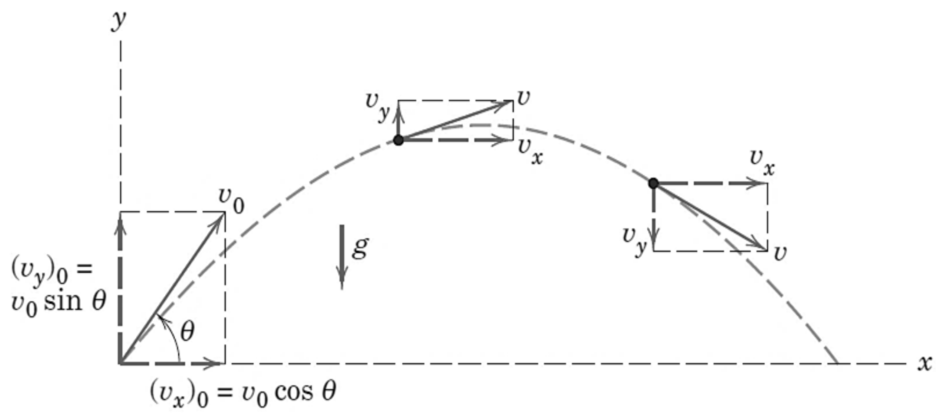
$\text{v} = v_x\mathbf{i} + v_y\mathbf{j} = -30\mathbf{i} - 40\mathbf{j}$ m/s ***Ans.***

$\text{a} = a_x\mathbf{i} + a_y\mathbf{j} = -16\mathbf{i} - 8\mathbf{j}$ m/s^2 ***Ans.***



Projectile Motion:

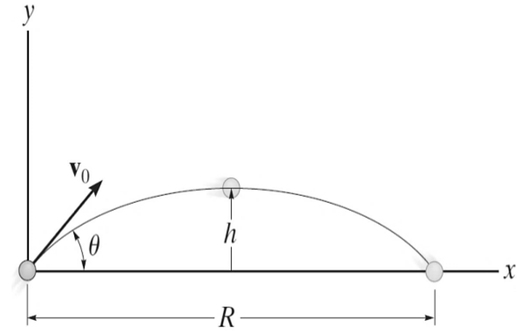
An important application of two-dimensional kinematic theory is the problem of projectile motion. The free- flight motion of projectile is studied in terms of its rectilinear components.



The projectile have a constant downward acceleration (gravity acceleration) is ($\boldsymbol{a = g = 9.81 m/s^2}$ or $\boldsymbol{g = 32.2\ ft/s^2}$).

Horizontal motion	Vertical motion
$a = a_x = 0$	$a = a_y = -g$
$(\overset{+}{\rightarrow})$ $v = v_o + at$ $v_x = (v_x)_o = v_o \cos\theta$	$(+\uparrow)$ $v = v_o + at$ $v_y = (v_y)_o - gt$
$(\overset{+}{\rightarrow})$ $v^2 = v_o^2 + 2a(x - x_o)$ $v_x = (v_x)_o = v_o \cos\theta$	$(+\uparrow)$ $v^2 = v_o^2 + 2a(y - y_o)$ $v_y^2 = (v_o^2)_y - 2g(y - y_o)$
$(\overset{+}{\rightarrow})$ $x = x_o + v_o t + \frac{1}{2}at^2$ $x = x_o + (v_x)_o t$	$y = y_o + v_o t + \frac{1}{2}at^2$ $y = y_o + (v_y)_o t - \frac{1}{2}gt^2$

Ex. (8): The projectile is launched with a velocity v_o. Determine the range *R*, the maximum height *h* attained, and the time of flight. Express the results in terms of the angle and .The acceleration due to gravity is g.



Sol.:

$(\overset{+}{\rightarrow})\qquad s = s_0 + v_0 t$

$$R = 0 + (v_0 \cos\theta)t$$

$(+\uparrow)\qquad s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$$0 = 0 + (v_0 \sin\theta)\, t + \frac{1}{2}(-g)t^2$$

$$0 = v_0 \sin\theta - \frac{1}{2}(g)\left(\frac{R}{v_0 \cos\theta}\right)$$

$$R = \frac{v_0^2}{g}\sin 2\theta \qquad \textbf{Ans.}$$

$$t = \frac{R}{v_0\cos\theta} = \frac{v_0^2\,(2\sin\theta\cos\theta)}{v_0\, g\cos\theta}$$

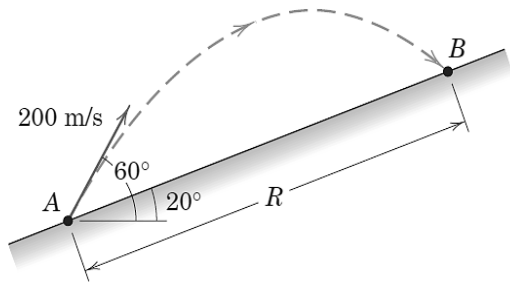
$$= \frac{2v_0}{g}\sin\theta \qquad \textbf{Ans.}$$

$(+\uparrow)\qquad v^2 = v_0^2 + 2a_c(s - s_0)$

$$0 = (v_0 \sin\theta)^2 + 2(-g)(h - 0)$$

$$h = \frac{v_0^2}{2g}\sin^2\theta \qquad \textbf{Ans.}$$

Ex. (9): A projectile is launched with an initial speed of 200 m/s at an angle of 60^o with respect to the horizontal. Compute the range R as measured up the incline.



Sol.:

$v_{x_0} = 200\cos 60 = 100 \text{ m/s}$,

$v_{y_0} = 200\sin 60 = 30 \text{ m/s}$

At B: $x = x_0 + v_{x_0}t$

$R\cos 20 = 0 + 100t_f \Longrightarrow t_f = 0.00940R$, (t_f : the time duration of the flight)

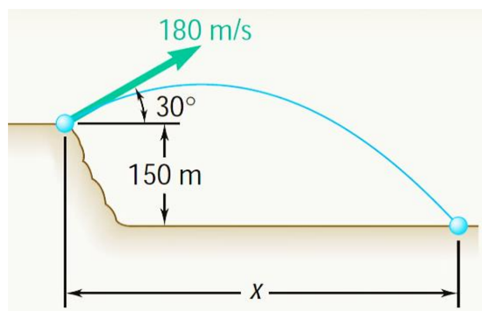
$y = y_0 + v_{y_0}t - \frac{1}{2}gt^2$

$R\sin 20 = 173.2\, t_f - \frac{9.81}{2}t_f^2$

$R\sin 20 = 173.2(0.00940R) - \frac{9.81}{2}(0.00940R)^2$

$\therefore\ R = 2970 \text{ m/s}$ ***Ans.***

Ex. (10): A projectile is fired from the edge of a 150-m cliff with an initial velocity of 180 m/s at an angle of 30° with the horizontal. Neglecting air resistance, find (*a*) the horizontal distance from the gun to the point where the projectile strikes the ground, (*b*) the greatest elevation above the ground reached by the projectile



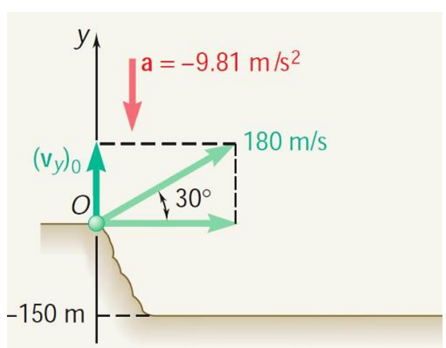
Sol.:

$(v_x)_0 = 180\cos 30 = 155.9 \text{ m/s}$

$(v_y)_0 = 180\sin 30 = 90 \text{ m/s}$, $a_y = a = g = 9.81 \text{ m/s}^2$

$\left(\overset{+}{\rightarrow}\right)$ $x = x_0 + v_{x_0}t$

$x = 0 + 155.9\, t$ (1)



$(+\uparrow)$ $y = y_0 + (v_y)_0 t - \frac{1}{2}gt^2$

(a). When the projectile strikes the ground; $y = -150$ m

$-150 = 0 + 90t - 4.9t^2$

$t = 19.91$ sec.

$\therefore \quad x = 155.9\ (19.91) = 3100$ m **Ans.**

(b). at the greatest elevation; $v_y = 0$

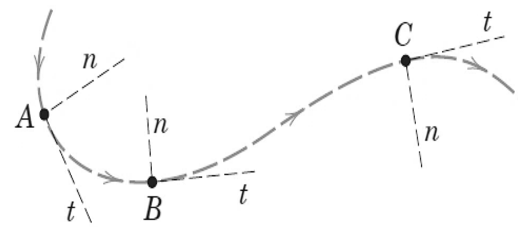
$$(+\uparrow) \qquad v_y^2 = \left(v_y\right)_0^2 + 2a(y - y_o)$$

$$0 = 8100 + 2(-9.81)(y - 0) \quad , \quad y = 413 \text{ m}$$

$\therefore$ The greatest elevation above the ground $= 150 + 413 = 563$ m **Ans.**

2.2. Normal and Tangential Coordinates (*n-t*):

One of the common descriptions of the curvilinear motion uses the *path variables*, which are measurements made along the normal(n) and tangent(t) to the path of particle.



$ds = \rho d\beta$

ρ : radius of curvature (m, ft)

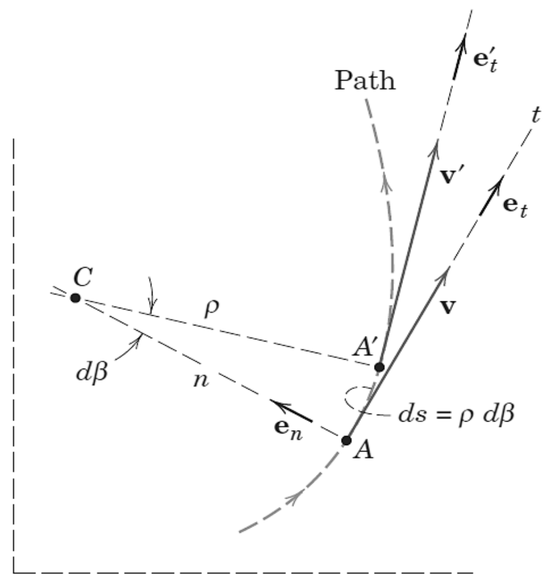
$d\beta$: angle in (radian)

- *Velocity.*

$$\dot{s} = \frac{ds}{dt} \quad , \quad \dot{\beta} = \frac{d\beta}{dt}$$

$$v = \frac{ds}{dt} = \rho \frac{d\beta}{dt} = \rho\dot{\beta}$$

$$\therefore \quad V = ve_t = \rho\dot{\beta}e_t$$



- *Acceleration.*

$$\mathrm{a} = \frac{d\mathrm{v}}{dt} = \frac{d(ve_t)}{dt}$$

$$\mathrm{a} = \dot{v}e_t + v\dot{e}_t$$

for small angle $\tan d\beta = d\beta = de_t/e_n$

$$\therefore \quad de_t = e_n d\beta$$

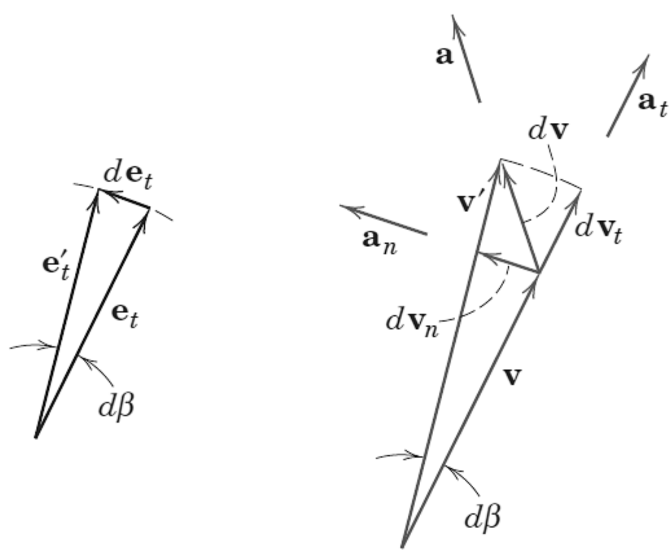
e_t , e_n : unit vectors

$$\frac{de_t}{d\beta} = e_n$$

Divided by (de_t) gives;

$$\frac{de_t}{dt} = \frac{d\beta}{dt}e_n$$

$$\therefore \quad \dot{e}_t = \dot{\beta}e_n$$



(b) (c)

The acceleration becomes

$$a = \frac{v^2}{\rho} e_n + \dot{v} e_t$$

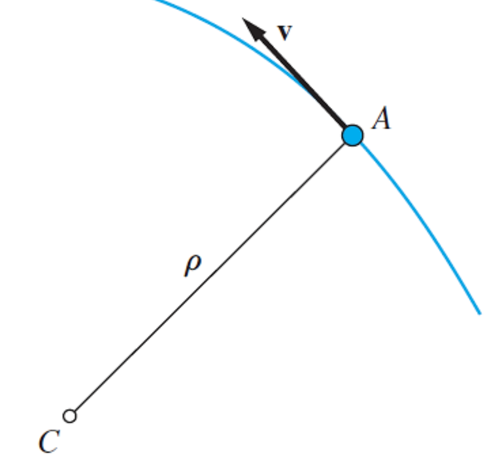
$a = a_n e_n + a_t e_t$ (as a vector)

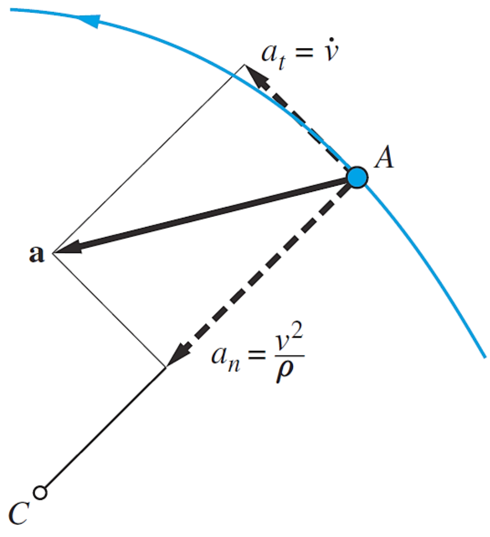
where:

$$\boxed{\boldsymbol{a_n = \frac{v^2}{\rho} = \rho\dot{\beta}^2 = v\dot{\beta} \quad , \quad a_t = \dot{v} = \ddot{s}}} \tag{19}$$

the magnitude of acceleration is :

$$\boldsymbol{a = \sqrt{a_n^2 + a_t^2}} \tag{20}$$



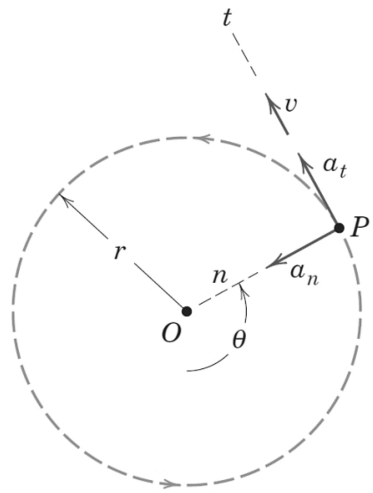


Circular Motion:

In this case the radius of curvature (ρ) is replaced by constant radius (r) of the circle and the angle (β) is replaced by angle (θ).

$$\rho \rightarrow r \quad and \quad \beta \rightarrow \theta$$

The velocity and acceleration components for the circular motion become:



$$\boxed{\begin{aligned} &\boldsymbol{v = r\dot{\theta}} \\ &\boldsymbol{a_n = v^2/r = r\dot{\theta}^2 = v\dot{\theta}} \\ &\boldsymbol{a_t = \dot{v} = r\ddot{\theta}} \end{aligned}} \qquad (21)$$

Notes:

1- If the particle moves along a straight line, then $\rho \rightarrow \infty$
$\boldsymbol{a_n = 0}$, $\boldsymbol{a = a_t = \dot{v}}$

2- If the particle move along curve with constant speed then;
$\boldsymbol{a = a_t = \dot{v} = 0}$ and $\boldsymbol{a = a_n = \frac{v^2}{\rho}}$

3- The a_t acts in the positive direction of s if the particle's speed is increasing or in the opposite direction if the particle's speed is decreasing.

4- The relations between a_t , v, t and s are the same as for rectilinear motion, namely,
$\boldsymbol{a_t = \dot{v}}$, $\boldsymbol{a_t ds = v dv}$

5- If a_t is *constant*, $a_t = (a_t)_c$, the above equations, when integrated, give;
$\boldsymbol{v = v_0 + (a_t)_c\, t}$, $\boldsymbol{v^2 = v_0^2 + 2(a_t)_c(s - s_0)}$ and $\boldsymbol{s = s_0 + v_0 t + \frac{1}{2}(a_t)_c\, t^2}$

6- The a_n component is always directed toward the center of curvature of the path (along the positive *n* axis).

7- If the path is expressed as $\boldsymbol{y = f(x)}$, the radius of curvature ρ at any point on the path is determined from the equation

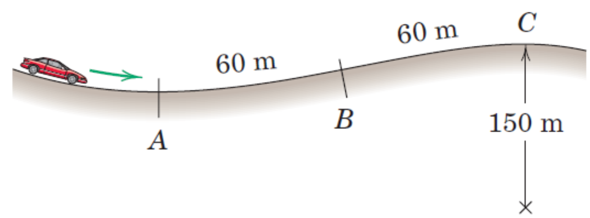
$$\boldsymbol{\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|d^2y/dx^2\right|}}$$

Ex. (11): To anticipate the dip and hump in the road, the driver of a car applies her brakes to produce a uniform deceleration. Her speed is 100 km/h at the bottom *A* of the dip and 50 km/h at the top *C* of the hump, which is 120 m along the road from *A*. If the passengers experience a total acceleration of $3\ \mathrm{m/s^2}$ at *A* and if the radius of curvature of the hump at *C* is 150 m, calculate:

(*a*) The radius of curvature at *A*.

(*b*) The total acceleration at the inflection point *B*.

(*c*) The total acceleration at *C*.



Sol.:

(a). $v_A = 100\frac{km}{h} = 100\left(\frac{1000}{360}\right) = 27.8\ \mathrm{m}/s$

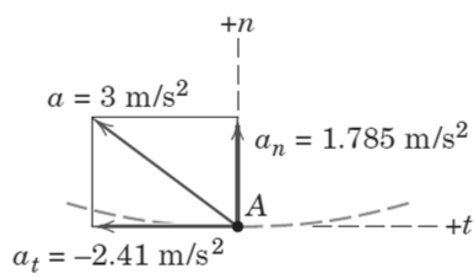
$v_C = 50\frac{\mathrm{km}}{\mathrm{h}} = 50(1000/3600) = 13.89\ \mathrm{m}/s$

For constant deceleration, we can use the following formulae

$v_C^2 = v_A^2 + 2as$

$(13.89)^2 = (27.8)^2 + 2a_t(120)$

$\therefore\ a_t = -2.14\ \mathrm{m}/s^2$



Total acceleration at *A* is given as: $a = 3\ \mathrm{m}/s^2$

$a = \sqrt{a_n^2 + a_t^2}$

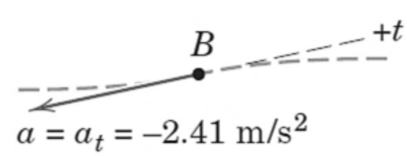
$3 = \sqrt{a_n^2 + (-2.14)^2}$ $\therefore\ a_n = 1.785\ \mathrm{m}/s^2$

$a_n = \frac{v_A^2}{\rho_A}$, $\rho_A = \frac{(27.8)^2}{1.785} = 432\ \mathrm{m}$ **Ans.**

(b). Since the radius of curvature is infinite at the inflection point *B* ($\rho_B = \infty$)

$a_n = 0$ and $= a_t - 2.41\ \mathrm{m}/s^2$

Total acceleration at *B* is $a = at = -2.41\ \mathrm{m}/s^2$

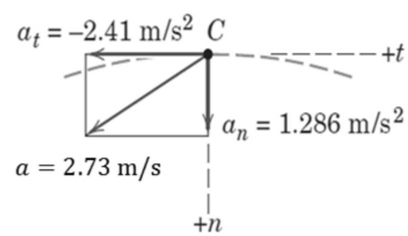


(c). $\mathrm{a_t} = -2.14\ \mathrm{m/s^2}$

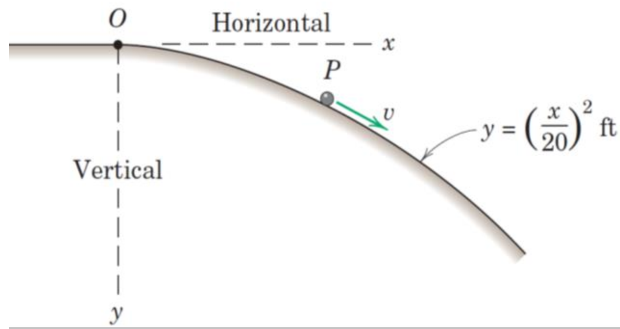
$a_n = \frac{v_C^2}{\rho_C} = \frac{(13.89)^2}{150} = 1.286\ \mathrm{m}/s^2$

Total acceleration at C is

$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(1.286)^2 + (-2.14)^2} = 2.73\ \mathrm{m}/s^2$ **Ans.**



Ex. (12): A small particle P starts from point O with a negligible speed and increases its speed to a value $v = \sqrt{2gy}$, where y is the vertical drop from O. When $x = 50$ ft, determine the n-component of acceleration of the particle.



Sol.:

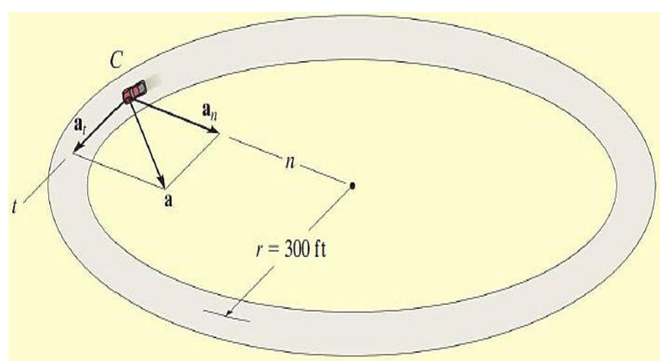
$$y = \left(\frac{x}{20}\right)^2 = \left(\frac{50}{20}\right)^2 = 6.25 \text{ ft}$$

$$v = \sqrt{2gy} = \sqrt{2 * 32.2 * 6.25} = 20 \text{ ft/s}$$

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{|d^2y/dx^2|} = \frac{\left[1+\left(\frac{x}{200}\right)^2\right]^{3/2}}{|1/200|} = \frac{\left[1+\left(\frac{50}{200}\right)^2\right]^{3/2}}{|1/200|} = 219 \text{ ft}$$

$$\therefore \quad a_n = \frac{v^2}{\rho} = \frac{20^2}{219} = 1.838 \text{ ft/s}^2$$ ***Ans.***

Ex. (13): A race car C travels around the horizontal circular track that has a radius of 300 ft. If the car increases its speed at a constant rate of 7 ft/s^2, starting from rest, determine the time needed for it to reach an acceleration of 8 ft/s^2. What is its speed at this instant?



Sol.:

The magnitude of acceleration is

$$a = \sqrt{a_t^2 + a_n^2}$$

$$a_t = 7 \text{ ft}/s^2$$

$$v = v_0 + a_t t$$

$$v = 0 + 7t = 7t$$

$$a_n = \frac{v^2}{\rho} = \frac{(7t)^2}{300} = 0.163t^2$$

The time needed for the acceleration to reach 8 ft/s^2 is therefore

$$a = \sqrt{a_t^2 + a_n^2}$$

$$8 = \sqrt{7^2 + (0.163t^2)^2}$$

Solving for the positive value of t gives

$$0.163t^2 = \sqrt{8^2 + 7^2} \quad \Longrightarrow \quad t = 4.87\text{s}$$ ***Ans.***

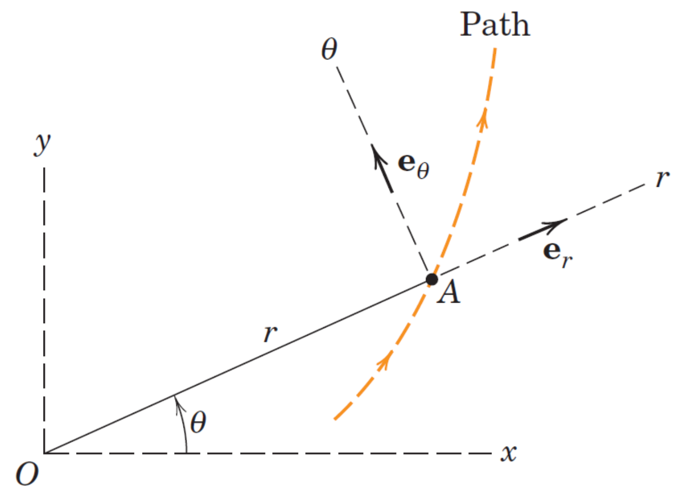
The speed at time $t = 4.87\text{s}$ is

$$v = 7t = 7(4.87) = 34.1\ \text{ft/s}$$ ***Ans.***

2.3. Polar Coordinates ($r - \theta$):

The third description of plane curvilinear motion is the *polar* coordinates. Where the particle is located by the radial distance (r) from a fixed point (O) and by an angle (θ) measured from the radial line.

The position vector is



$\text{r} = re_r$

$\tan d\theta = d\theta = \frac{de_r}{e_\theta}$

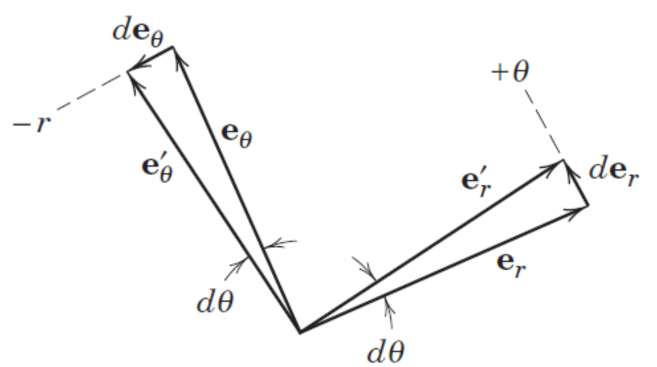
$\therefore \quad de_r = e_\theta d_\theta$

$de_\theta = -e_r d_\theta$

e_r, e_θ : are unit vectors in r and θ directions

$\frac{de_r}{dt} = \left(\frac{d_\theta}{dt}\right) e_\theta$

And $\frac{de_\theta}{dt} = -\left(\frac{d_\theta}{dt}\right) e_r$



$\therefore \quad \dot{e}_r = \dot{\theta} e_\theta \qquad \text{and} \qquad \dot{e}_\theta = -\dot{\theta} e_r$

- ***Velocity***

$$\text{v} = \dot{\text{r}} = \dot{r} e_r + r \dot{e}_r$$

$$\boldsymbol{v} = \dot{\boldsymbol{r}} \boldsymbol{e}_r + \boldsymbol{r} \dot{\boldsymbol{\theta}} \boldsymbol{e}_\theta = \boldsymbol{v}_r \boldsymbol{e}_r + \boldsymbol{v}_\theta \boldsymbol{e}_\theta$$

where:

$$\boxed{\begin{aligned} \boldsymbol{v_r} &= \dot{\boldsymbol{r}} \\ \boldsymbol{v_\theta} &= \boldsymbol{r}\dot{\boldsymbol{\theta}} \end{aligned}} \qquad (22)$$

The magnitude of velocity v is: $\boxed{\boldsymbol{v} = \sqrt{\boldsymbol{v}_r^2 + \boldsymbol{v}_\theta^2}}$ (23)

- ***Acceleration.***

$$\mathbf{a} = \dot{\mathbf{v}} = (\ddot{r}\mathbf{e}_r + \dot{r}\dot{\mathbf{e}}_r) + (\dot{r}\,\dot{\theta}\,\mathbf{e}_\theta + r\,\ddot{\theta}\,\mathbf{e}_\theta + r\,\dot{\theta}\,\dot{\mathbf{e}}_\theta)$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$$

$$\mathbf{a} = \boldsymbol{a_r e_r} + \boldsymbol{a_\theta e_\theta}$$

Where:

$$\boxed{\begin{aligned} a_r &= \ddot{r} - r\dot{\theta}^2 \\ a_\theta &= r\ddot{\theta} + 2\dot{r}\dot{\theta} \end{aligned}} \qquad (24)$$

The magnitude of acceleration **a** is: $$\boxed{a = \sqrt{a_r^2 + a_\theta^2}} \qquad (25)$$

Where:

$\boldsymbol{v_r}$: radial component of velocity

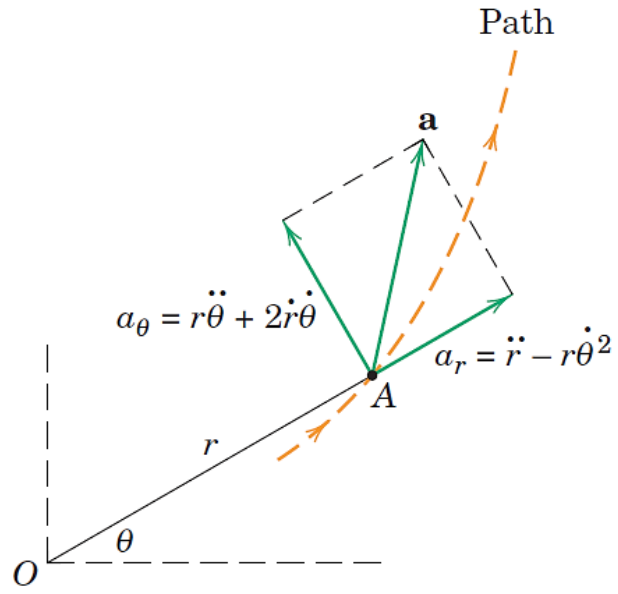
$\boldsymbol{v_\theta}$: transverse component of velocity

$\dot{\theta} = \frac{d\theta}{dt}$: angular velocity (rad/s)

$\ddot{\theta} = \frac{d_\theta^2}{dt^2}$: angular acceleration (rad/s^2)

$\boldsymbol{a_r}$: radial component of acceleration

$\boldsymbol{a_\theta}$: transverse component of acceleration

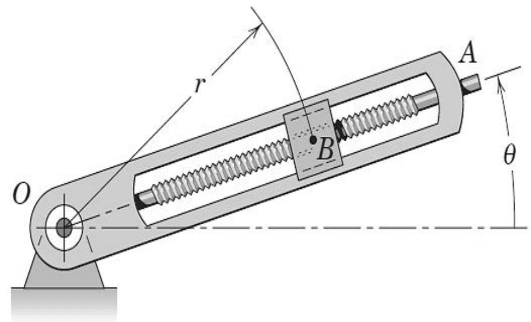


Circular Motion:

For motion in a circular path with r constant, the components of velocity and acceleration are:

$$\boxed{\begin{aligned} v_r &= 0 & v_\theta &= r\dot{\theta} \\ a_r &= -r\dot{\theta}^2 & a_\theta &= r\ddot{\theta} \end{aligned}}$$

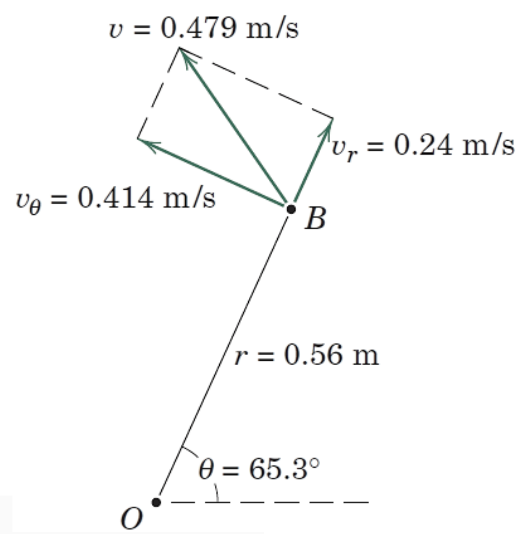
Ex. (14): Rotation of the radially slotted arm is governed by $\theta = 0.2t + 0.02t^3$, where θ is in radians and *t* is in seconds. Simultaneously, the power screw in the arm engages the slider *B* and controls its distance from *O* according to $r = 0.2 + 0.04t^2$, where r is in meters and t is in seconds. Calculate the magnitudes of the velocity and acceleration of the slider for the instant when $t = 3\ s$.



Sol.:

For $t = 3\ s$

$r = 0.2 + 0.04t^2$	$r_3 = 0.2 + 0.04(3^2) = 0.56$ m
$\dot{r} = 0.08t$	$\dot{r}_3 = 0.08(3) = 0.24$ m/s
$\ddot{r} = 0.08$	$\ddot{r}_3 = 0.08$ m/s^2
$\theta = 0.2t + 0.02t^3$	$\theta_3 = 0.2(3) + 0.02(3^3) = 1.14$ rad
	or $\theta_3 = 1.14(180/\pi) = 65.3°$
$\dot{\theta} = 0.2 + 0.06t^2$	$\dot{\theta}_3 = 0.2 + 0.06(3^2) = 0.74$ rad/s
$\ddot{\theta} = 0.12t$	$\ddot{\theta}_3 = 0.12(3) = 0.36$ rad/s^2



The velocity components are ;

$[v_r = \dot{r}]$ $\qquad v_r = 0.24$ m/s

$[v_\theta = r\dot{\theta}]$ $\qquad v_\theta = 0.56(0.74) = 0.414$ m/s

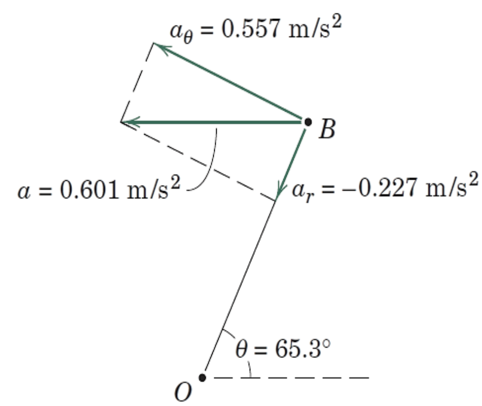
$[v = \sqrt{{v_r}^2 + {v_\theta}^2}]$ $\qquad v = \sqrt{(0.24)^2 + (0.414)^2} = 0.479$ m/s $\qquad$ *Ans.*

The acceleration components are

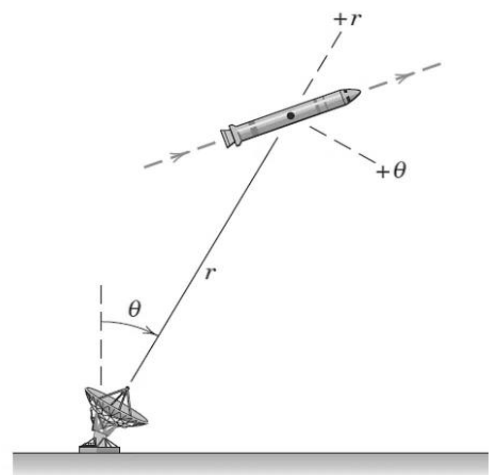
$[a_r = \ddot{r} - r\dot{\theta}^2]$ $\qquad a_r = 0.08 - 0.56(0.74)^2 = -0.227$ m/s^2

$[a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}]$ $\qquad a_\theta = 0.56(0.36) + 2(0.24)(0.74) = 0.557$ m/s^2

$[a = \sqrt{{a_r}^2 + {a_\theta}^2}]$ $\qquad a = \sqrt{(-0.227)^2 + (0.557)^2} = 0.601$ m/s^2 $\qquad$ *Ans.*

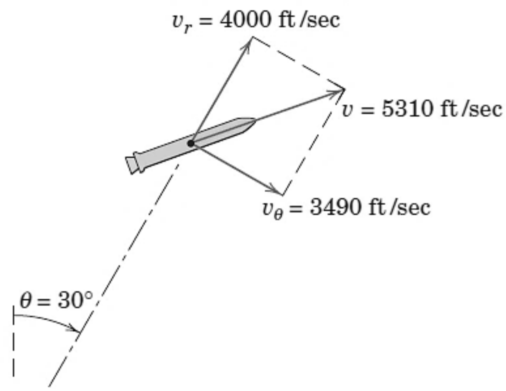


Ex. (15): A tracking radar lies in the vertical plane of the path of a rocket which is coasting in unpowered flight above the atmosphere. For the instant when $\theta = 30^o$, the tracking data give $r = 25(10^4)$ ft, $\dot{r} = 4000$ ft/s, and $\dot{\theta} = 0.80$ deg/s. The acceleration of the rocket is due only to gravitational attraction and for its particular altitude is $31.4\ \text{ft/s}^2$ vertically down. For these conditions determine the velocity v of the rocket and the values of $\ddot{r}$ and $\ddot{\theta}$.



Sol.:

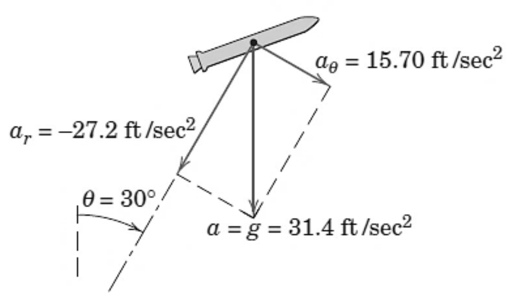
The components of velocity are



$v_r = \dot{r} = 4000$ ft/s

$v_r = r\dot{\theta} = 25(10^4) * \left(0.80\left(\frac{\pi}{180}\right)\right) = 3490$ ft/s

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{4000^2 + 3490^2} = 5310$ ft/s



The total acceleration of the rocket is $g = 31.4\ \text{ft/s}^2$ down, the components of acceleration are:

$a_r = -31.4\cos 30 = -27.2\ \text{ft/s}^2$

$a_\theta = -31.4\sin 30 = 15.7\ \text{ft/s}^2$

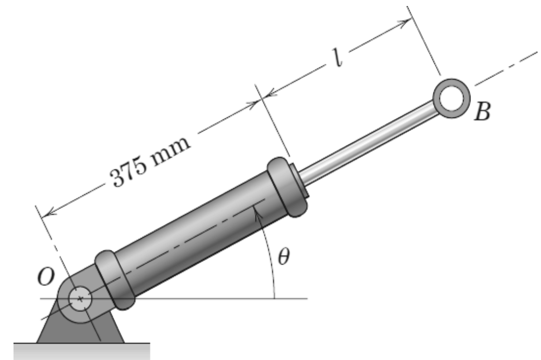
$\therefore \quad a_r = \ddot{r} - r\dot{\theta}^2$

$-27.2 = \ddot{r} - 25(10^4) * \left(0.80\left(\frac{\pi}{180}\right)\right)^2 \Longrightarrow \ddot{r} = 21.5\ \text{ft/s}^2$ ***Ans.***

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$15.7 = 25(10^4)\ddot{\theta} + 2(4000)\left(0.80\left(\frac{\pi}{180}\right)\right) \Longrightarrow \ddot{\theta} = -3.84(10^{-4})\ \text{ft/s}^2$ ***Ans.***

Ex. (16): As the hydraulic cylinder rotates around *O*, the exposed length *l* of the piston rod *P* is controlled by the action of oil pressure in the cylinder. If the cylinder rotates at the constant rate $\dot{\theta} = 60\,\text{deg}/s$ and l is decreasing at the constant rate of $150\,\text{mm/s}$, calculate the magnitudes of the velocity $\boldsymbol{v}$ and acceleration $\boldsymbol{a}$ of end *B* when $l = 125\,\text{mm}/s$.



Sol.:

$r = 375 + 125 = 500\ mm \ , \ \dot{r} = \dot{l} = -150\ \frac{mm}{s}$

$\ddot{r} = 0 \ , \quad \dot{\theta} = 60\left(\frac{\pi}{180}\right) = \frac{\pi}{3}\ rad/s \ , \ \ddot{\theta} = 0$

$v_r = \dot{r} = -150\ \frac{mm}{s} \ , \ v_\theta = r\dot{\theta} = 500\left(\frac{\pi}{3}\right) = 524\ \frac{mm}{s}$

$v = \sqrt{v_r^2 + v_\theta^2} = \sqrt{(-150)^2 + (524)^2} = \underline{545\ \frac{mm}{s}}$

$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 500\left(\frac{\pi}{3}\right)^2 = -548\ mm/s^2$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-150)\left(\pi/3\right) = -314\ mm/s^2$

$a = \sqrt{a_r^2 + a_\theta^2} = \sqrt{(-548)^2 + (-314)^2} = 632\ mm/s^2$