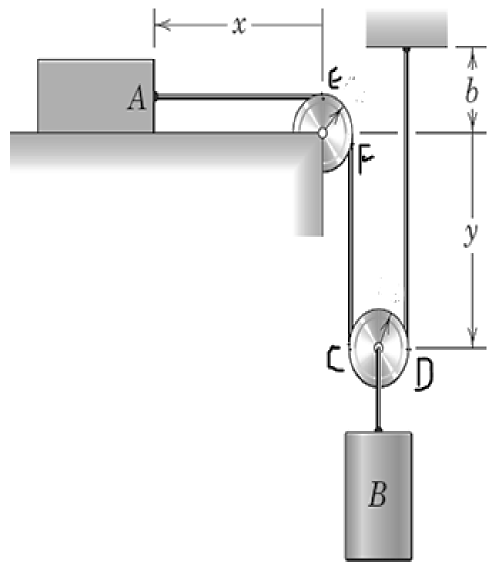
4- Constrained Motion (Absolute dependent motion) of Connected Particles:

A- One cord (cable) case:-

The coordinates *x* and *y* are measured from fixed point (O) or fixed datum, to the block.



The lengths L_{CD} and L_{EF} covered the pulleys remain constant during the motion.

The total length of the cable is

$$L_t = x + L_{EF} + 2y + L_{CD} + b \qquad \text{……(a)}$$

Where ; L_{EF}, L_{CD} and b all constant.

Tike the first time derivative for eq.(1)

$$0 = \frac{dx}{dt} + 0 + 2\frac{dy}{dt} + 0 + 0$$

$$\therefore \quad \dot{x} + 2\dot{y} = 0$$

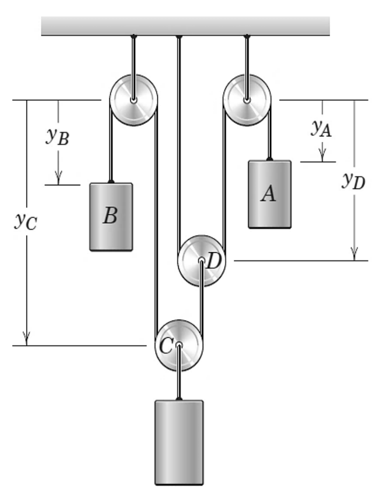
$$\boldsymbol{v_A + 2v_B = 0}$$

And the **acceleration** is:

$$\ddot{x} + 2\ddot{y} = 0 \qquad \boldsymbol{a_A + 2a_B = 0}$$

B- Two or more cables case:

The lengths of the cables attached to cylinders *A* and *B* can be written, respectively, as



$$L_A = y_A + 2y_D + \text{constant}$$

$$L_B = y_B + y_C + (y_C - y_D) + \text{constant}$$

and their time derivatives are:

$$0 = \dot{y}_A + 2\dot{y}_D \qquad \text{and} \qquad 0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D$$

$$0 = \ddot{y}_A + 2\ddot{y}_D \qquad \text{and} \qquad 0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

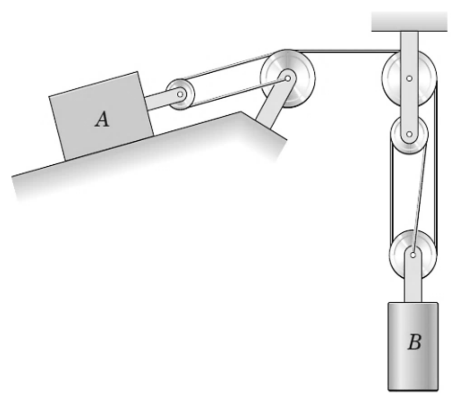
Eliminating the terms in $\dot{y}_D$ and $\ddot{y}_D$ gives

$$\dot{y}_A + 2\dot{y}_B + 4\dot{y}_C = 0 \qquad \text{or} \qquad v_A + 2v_B + 4v_C = 0$$

$$\ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C = 0 \qquad \text{or} \qquad a_A + 2a_B + 4a_C = 0$$

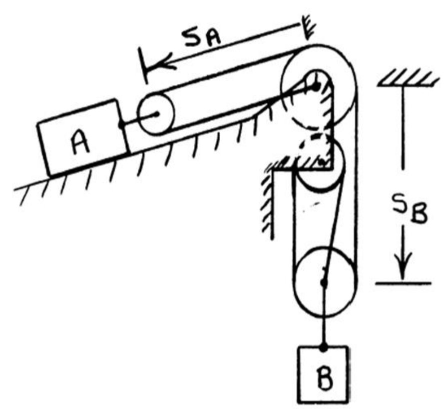
If both *A* and *B* have downward (positive) velocities, then *C* will have an upward (negative) velocity.

Ex. (19): At a certain instant, the velocity of cylinder *B* is 2 ft/s down and its acceleration is 0.5 ft/s^2 up. Determine the corresponding velocity and acceleration of block *A*.



Sol.:

Cable length $\quad L = 2s_A + 3s_B + constants$



Velocity; $\quad \dot{L} = 0 = 2v_A + 3v_B$

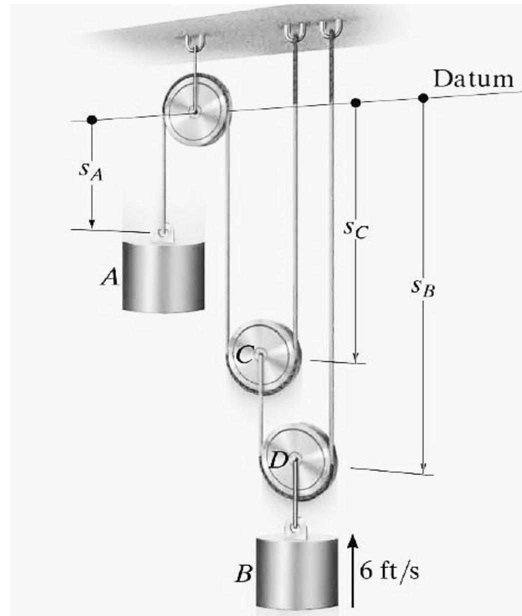
$$v_A = -\frac{3}{2}v_B = -\frac{3}{2}(2) = -3 \text{ ft/s}$$

$$v_A = 3 \text{ ft/s} \quad \text{up the incline} \qquad \textbf{\textit{Ans}}.$$

Acceleration; $\quad \ddot{L} = 0 = 2a_A + 3a_B$

$$a_A = -\frac{3}{2}a_B = -\frac{3}{2}(-0.5) = 0.75 \text{ ft}/s^2 \quad \text{down the incline} \qquad \textbf{\textit{Ans.}}$$

Ex. (20): Determine the speed of A in Fig. shown if B has an upward speed of 6 ft/s.



Sol.:

$l_1 = y_A + 2y_C$ (1)

$l_2 = y_B + (y_B - y_C) = 2y_B - y_C$ (2)

$\therefore \quad \dot{l}_1 = 0 = v_A + 2v_C$ (3)

$\dot{l}_2 = 0 = 2v_B - v_C$

$v_C = 2v_B$ sub. in (3)

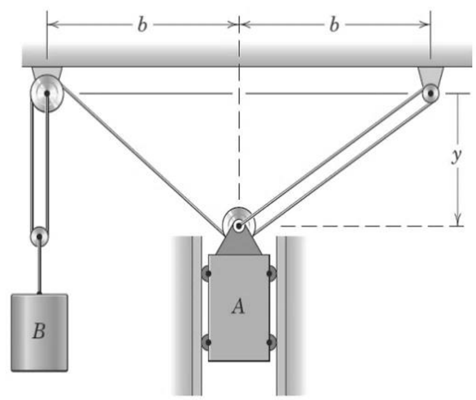
$v_A + 4v_B = 0$

$v_B = -6$ ft/s upward (the position y_B decrease)

$\therefore \quad v_A + 4(-6) = 0$

$v_A = 24 \text{ ft}/s^2$ ***Ans.***

Ex. (21): Neglect the diameters of the small pulleys and establish the relationship between the velocity of *A* and the velocity of *B* for a given value of *y*



Sol:.

The length of the cable is

$L = 2x + 3\sqrt{y^2 + b^2}$

Differentiate to obtain

$\dot{L} = 0 = 2\dot{x} + 3\frac{y\dot{y}}{\sqrt{y^2+b^2}}$

$\dot{x} = v_B$ and $\dot{y} = v_A$

$\therefore \quad v_B = -\frac{3yv_A}{2\sqrt{y^2+b^2}}$