Context- Free Grammar

A Context- Free Grammar (CFG) G is a 4-tuple (N,Σ,R,S), where :

1- N is a finite set of *nonterminals* or variables.

2- Σ is a finite set of *terminals*.

3- R is a finite set of rules, each consisting of a variables (the left- hand side) and a sentential form (the right- hand side).

4- S is the start symbol.

The binary relation $\rightarrow$ on sentential forms is defined as follows:

Let u, v, and w be sentential forms then $uAw\rightarrow uvw$ iff $A\rightarrow v$ is a rule in R

$\rightarrow$ captures a single derivation step.

$\rightarrow^*$ is the reflexive transitive closure of $\rightarrow$,

and $L(G)=\{s\in\Sigma^* \,/\, S\rightarrow^* s\}$

Derivations

A sequence of rewritings that transforms the start variable S of a grammar G to a sentence s is called a derivation of s from

G. A derivation in which every derivation step uses the left most variable in the sentential form is called a left most derivation.

A grammar G is called ambiguous if there exist a string s with tow different left most derivations from G.

For example: the arithmetic expression grammar

$E \rightarrow 0\backslash 1 \ldots..\backslash 9\backslash (E)\backslash E*E\backslash E+E$

Is ambiguous because the sentence

2+3*4 Has tow different left most derivations

$E \rightarrow E*E \rightarrow E+E*E \rightarrow 2+E*E \rightarrow 2+3*E \rightarrow 2+3*4$

$E \rightarrow E+E \rightarrow 2+E \rightarrow 2+E*E \rightarrow 2+3*E \rightarrow 2+3*4$

Pars trees

Here is another grammar for arithmetic expressions:

$E \rightarrow T|T+E$

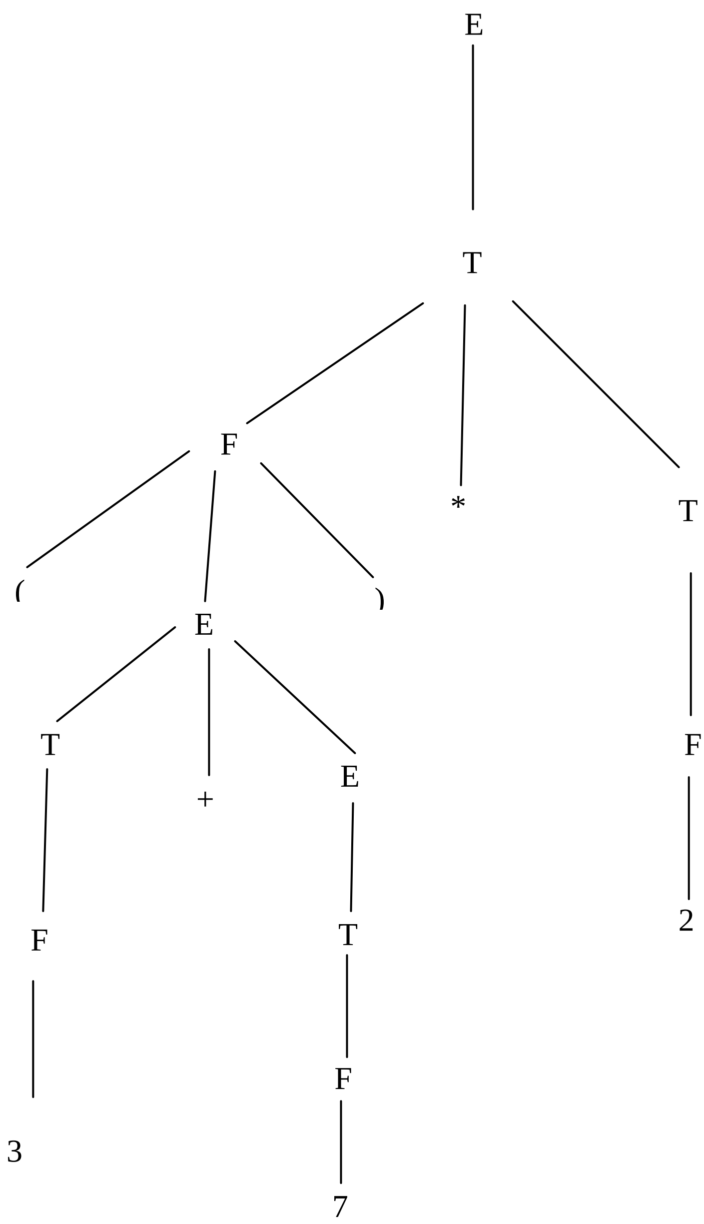
$T \rightarrow F|F*T$

$F \rightarrow 0|1|\ldots|9|(E)$

When the start variable is unspecified, it is assumed to be the variable of the first rule in this case E.

This grammar is unambiguous (convince yourself of this fact)

Here is the parse tree for (3+7)*2

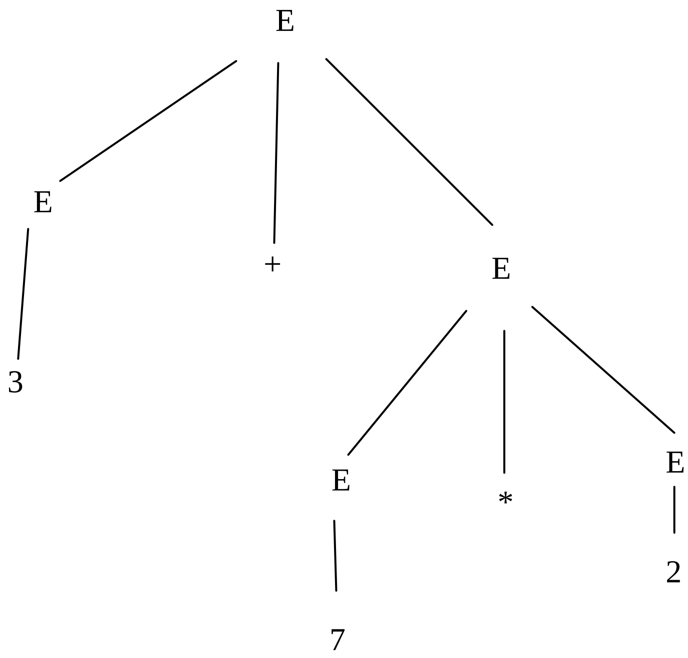


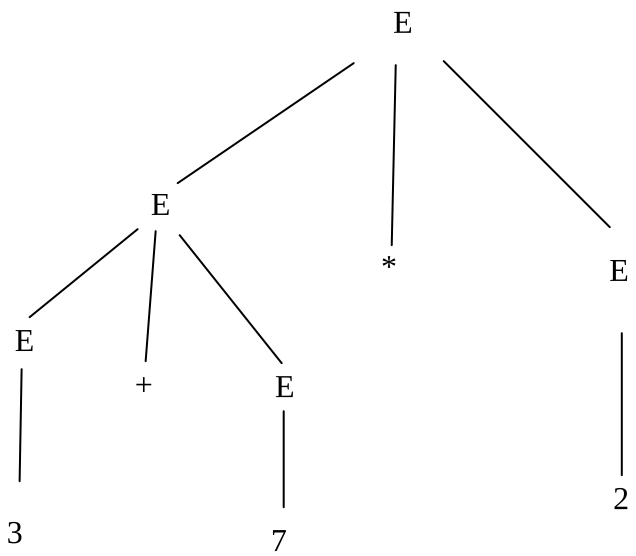
This is only parse tree for this sentence (using this grammar).

In contrast, consider the previous grammar

$E \rightarrow 0|1|\ldots|9|(E)|E*E|E+E$

This grammar has two different parse trees for the sentence 3+7*2.





Previously, we said a grammar was ambiguous if there exists some sentence with two different leftmost derivations. Equivalently, a grammar is ambiguous if there exists some sentence with two different parse tree.

Examples:

1- $L(G)=\{0^n1^n, n>=0\}$

$S \rightarrow 0S1 \backslash \lambda$

the derivation of (0011) is

$S \rightarrow 0S1 \rightarrow 00S11 \rightarrow 0011$

2- $L(G)=\{w \in \Sigma^*, w=w^R\}$ where $\Sigma=(a,b)$

$S \rightarrow aSa \backslash bSb \backslash a \backslash b \backslash \lambda$

The derivation of (abaaba) is

$S \rightarrow aSa \rightarrow abSba \rightarrow abaSaba \rightarrow abaaba$

H.W 1 : $L(G)=\{ 0^n1^n, n>=1\}$

H.W 2 : $L(G)=\{a^ib^j , i>=j\}$