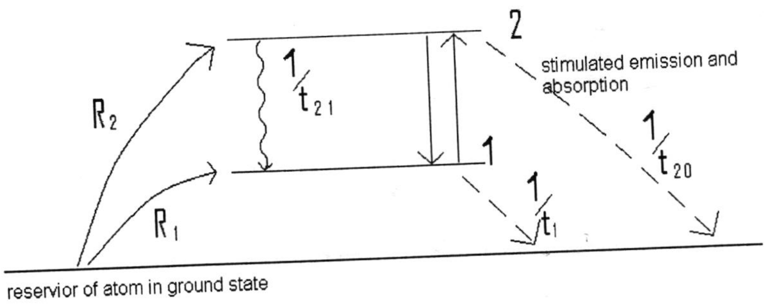
- Assume that the center wavelength is 5000Å
    - $\nu_o = 6 \times 10^{14}$ Hz , $\Delta\nu \sim 1$ Ghz
      Volume of the medium is 10 cm^3 , n=1
    - There are $(*\pi\nu^2\Delta\nu/C^3)V = 3.35\times10^{10}$ different modes in whch the atom can give up its internal energy to this electromagnetic field.
    - Few are toward the mirror, for instance, during the initial stage, those resonant field close to line center were excited spontaneously. Then it will be amplified by stimulated emission process. After a few round-trips through the cavity
        - Sufficient to make the field quite large.

    Example

    TEM $_{00q}$ (0,0,q) mode has initial intensity of 1 $\mu w/cm^2$ and let say the net gain $(R_1R_2)^{1/2}$ $\exp(\gamma_o(\nu_q)l)=4$. After five round trips, the intensity will grow, $4^{10} \sim 1.05\times10^6$ times (assuming the gain weff is constant. The another modes, maybe gain $(R_1R_2)^{1/2}$ $\exp(\gamma_o(\nu_q)l)=2$ with initial intensity $=0.5\mu W/cm^2$, after 5 round trip = 0. $5mW/cm^2$.

    But the intensity cannot keep growing indefinitely through more and more cavity transit times.

    - When the stimulating field is so large as to cause the atom to give up their energy as fast as they are being pumped --->equilibrium.
        - Gain of the system must change to a lower value until the rate of production of the excess inverted population is balance by the destruction rate by stimulation emission →gain saturation.
        - The laser gain coefficient has decreased(or saturated) to the loss coefficient at the frequency of laser oscillation.
        - Eventually only the centre mode will oscillate
            - Consequence of homogeneous broadening.
- Gain saturation in a homogeneous Broadened transition
    - Need mathematical description of gain saturation using generalized 2-level model.



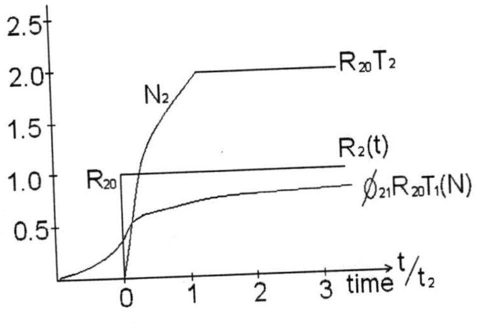
- R₁ includes direct excitation from the ground state to state 1 (include indirect routes)such excitation to higher states followed by spontaneous emission to 1. R_2 pumping directly from ground state to state 2 (include indirect rountes).
- State 2 can radiate photon of energy $h\nu_{12}$ spontaneously
- State 1 →similar
- Assume that the population in 2 and 1 are very small compared to that in 0.
- Rates equation.

$$\frac{dN_2}{dt} = R_2 - \frac{N_2}{t_2} - \frac{\sigma I_\nu}{h\nu}(N_2 - N_1) \ldots\ldots (10.1)$$

$$\frac{dN_1}{dt} = R_1 + \frac{N_2}{t_{21}} - \frac{N_1}{t_1} + \frac{\sigma I_\nu}{h\nu}(N_2 - N_1) \ldots\ldots (10.2)$$

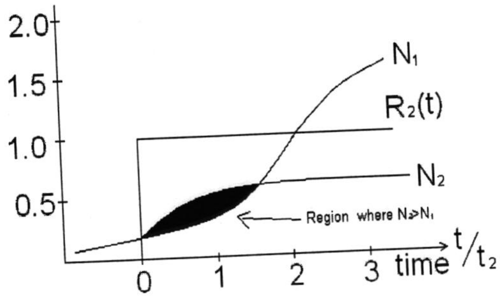
- $\frac{1}{t_{21}}$ = spontaneous decay 2→1
- $\frac{1}{t_{20}}$ = loss os state 2 to ground state
- Total decay rate of state 2 $\frac{1}{t_2} = \frac{1}{t_{21}} + \frac{1}{t_{20}}$
- Decay rate of state 1 due to all causes $\frac{1}{t_1}$

- Stimulated emission
  - $B_{21}g(\nu)\Delta\nu = (c'^3 A_{21}/8\pi h\nu^3 n^3)g(\nu)\,\Delta\nu$
  - Note: we have written c'=c/n velocity of light in medium
- From $\frac{A_{21}}{B_{21}} = \frac{h\,8\pi\nu^3}{c'^3} \rightarrow \frac{h\,8\pi\nu^3 n^3}{c^3}$ and $\Delta\nu = I/c$ therefore $B_{21}g(\nu) = (\lambda_0^2/8\pi h\nu n^2)A_{21}\,g(\nu)\,I\nu$
- 10.1 and 10.2 are so fundamental lead to so many important consequences.
- Case 1:
  - Assume $I_\nu=0$ (no stimulated emission) pumping to state 2 only (i.e. $R_1=0$) and R_2 is in the form of a step function, $R_2(t)=R_{20}u(t)$ where u(t)=Heaviside step function (=0 for t<0 and = 1 for t>0) therefore equation 10.1 becomes $\frac{dN_2}{dt} + N_2/t_2 = R_{20}$ which has a solution, homogeneous, $A\exp(-t/t_2)$ particular, $R_{20}t_2$ using the initial condition, $N_2(0)=0$ lead to $N_2(t)= R_{20}t_2(1- \exp(-t/t_2))$ 10.3
  - Dynamic of state 1 is complicated, due to source term N_2/t_{21} and finite population of state 1 (no direct pumping)
  - From eqn 10.2
  $$\frac{dN_1}{dt} + \frac{N_1}{t_1} = \frac{N_2(t)}{t_{21}} = \Phi_{21}R_{20}(1 - \exp(-t/t_2))\ldots\ldots (10.2)$$
  Where $\Phi_{21}=t_2/t_{21}$=branching ratio $(1/t_2=1/t_{21}+1/t_{20})$
  - Using integrating factor approach multiply both sides by a factor $\exp(t/t_2)$
  $\frac{d}{dt}\left\{N_1(t)\ \exp\left(\frac{t}{t_1}\right)\right\} = (N_2(t)\ \exp\left(\frac{t}{t_2}\right))/t_{21}$ prefect differential
  - Thus $N_1(t) = \left\{\exp\left(\frac{t}{t_1}\right)\right\}/t_{21}\int_0^t N_2(t)\ \exp\left(\frac{t}{t_1}\right)dt$ 10.5
  - For times $t<<t_1$ the exponential term =1 and thus $N_1(t)\ \alpha \int N_2(t)dt$, for $t>>t_1$, must evaluate the integral with $N_2(t)$ as given by 10.3
  $N_1(t) = \Phi_{21}R_{20}t_1\{1 + \frac{\frac{t_1}{t_2}}{1-\frac{t_1}{t_2}}\exp(-{}^{t}/{}_{t_1}) - \frac{1}{1-\frac{t_1}{t_2}}\exp(-{}^{t}/{}_{t_2})$ 10.6
  As $t\rightarrow\infty$ steady state is reached with $N_2(\infty)=R_{21}t_2$ and $N_1(\infty)=\Phi_{21}R_{20}t_1$



(a) for $t_2/t_1>1$
notes: the density in state 2 is always greater than that of state 1 and the system exhibits gain for all values of t =>Favorable lifetime ratio

(b) for $t_2/t_1 \leq 1$ =>unfavorable lifetime ratio then gain is possible for only yhr short initial interval of roughly t_2



Excitation must be in the form of pulse. Example N_2 laser with wavelength 337nm (uv) unfavorable lifetime t_1~10µs ,t_2`10ns

But YAG→ favorable lifetime ratio with t_2=255µs and t_1=30ns

- case 2 :modified case 1 for a finite intensity but assume that state 1 decay at infinite fast rate (t_1=0)
  - ideal laser system
  - to decouple 10.2 and 10.1
  - if t_1=0 implies N_1=0 hence 10.1 becomes

$$\frac{dN_2}{dt}+\frac{1}{t_2}(1+\frac{\sigma t_2}{h\nu}I_\nu)N_2 = R_{20}u(t).....(11.1)$$

  - simplifying

$$\frac{dN_2}{dt}+\frac{1}{t_2}(1+\frac{I_\nu}{I_s})N_2 = R_{20}u(t).....(11.1)$$

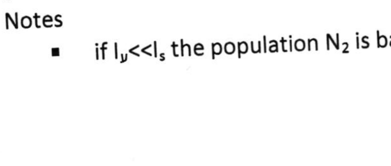
where I_s=saturation intensity

  - it has a solution in the form $N_2(t) = \frac{R_{20}t_2}{1+\frac{I_\nu}{I_s}}\{1-\exp\left((-t/t_2)(1+\frac{I_\nu}{I_s})\right)$.............11.2

equation 11.2 is similar to 10.3

  - stimulated emission by the intemsity $I_\nu$affect both these quantities.

Example for $\frac{I_\nu}{I_s}$=1.5



The quantity $h\nu/[\sigma(\nu)t_2]$ is a important collection of atomic constants with dimension of watts/area (i.e.$J.T^{-1}L^{-2}$) =>saturation intensity

Notes

- if $I_\nu \ll I_s$ the population N_2 is barely affected by the stimulating wave

- if $I_y = I_s$, the steady state peak amplitude of N_2 is one-half of the value given by $N_2(t)=R_{21}t_2(1-\exp[-t/t_2])$
  - notes: the time constant controlling the approach to equilibrium is reduces. i.e. $t_2' = t_2/(1+I_y/I_s)$
- if $I_y >> I_s$, the peak population in N_2 is greatly reduces compare to 1

- case 3 d/dt =0
  - assuming all time derivatives equal to zero reduces the differential equation to simple form

$$\frac{dN_2}{dt} = 0 = R_2 - \frac{N_2}{t_2} - \frac{\sigma I_y}{h y} N_2 - N_1 \ldots\ldots (11.3)$$

$$\frac{dN_1}{dt} = 0 = R_1 + \frac{N_2}{t_{21}} - \frac{N_1}{t_1} + \frac{\sigma I_y}{h y} N_2 - N_1 \ldots\ldots (11.4)$$