COMPRESSIBLE AND INCOMPRESSIBLE FLUIDS

Compressible fluids: are the fluids with variable density.

 Incompressible fluid: are the fluids with constant density. They could be liquids and gases.

Although there is no such thing in reality as an incompressible fluid, we use this term where the change in density with pressure is so small as to be negligible. This is usually the case with liquids. We may also consider gases to be incompressible when the pressure variation is small compared with the absolute pressure.

In problems involving water hammer we must consider the compressibility of the liquid. The flow of air in a ventilating system is a case where we may treat a gas as incompressible, for the pressure variation is so small that the change in density is of no importance. But for a gas or steam flowing at high velocity through a long pipeline, the drop in pressure may be so great that we cannot ignore the change in density. For an airplane flying at speeds *below 250 mph (100 m/s)*, we may consider the air to be of constant density. But as an object moving through the air approaches the velocity of sound, which is of the order of *760 mph (1200 km/h)* depending on temperature, the pressure and density of the air adjacent to the body become materially different from those of the air at some distance away, and we must then treat the air as a compressible fluid.

COMPRESSIBILITY OF LIQUIDS

The compressibility (change in volume due to change in pressure) of a liquid is inversely proportional to its volume modulus of elasticity, also known as the bulk modulus. This modulus is defined as

$$E_v = -v\frac{dp}{dv} = -\left(\frac{v}{dv}\right)dp \qquad\qquad (2.1)$$

Where:

v= specific volume, dv= change in volume due to change in pressure.

p= pressure, dp= change in pressure.

 As v/dv = a dimensionless ratio, the units of Ev = uits of p.

The bulk modulus is similar to the modulus of elasticity for solids; however, for fluids it is defined on a volume basis rather than in terms of the familiar one-dimensional stress–strain relation for solid bodies.

 By rearranging the definition of Ev, as an approximation we may use for the case of a fixed mass of liquid at constant temperature:

$$\frac{\Delta v}{v} = -\frac{dp}{E_v} \quad \textbf{or} \quad \frac{v_2 - v_1}{v} = -\frac{p_2 - p_1}{E_v}$$

where E_v = the mean value of the modulus for the pressure range,
and the subscripts 1 and 2 refer to the before and after conditions.

Example 1:
At a depth of 8 km in the ocean the pressure is 81.8 MPa. Assume that the specific weight of seawater at the surface is 10.05 kN/m3 and that the average volume modulus is 2.34 109 N/m2 for that pressure range. (a) What will be the change in specific volume between that at the surface and at that depth? (b) What will be the specific volume at that depth? (c) What will be the specific weight at that depth?

Sol:

a. $v_1 = \frac{1}{\rho_1} = \frac{g}{\gamma_1} = \frac{9.81}{10050} = 0.000976$ m³/kg

$\Delta v = -0.000976(81.8 * 10^{-6} - 0)/(2.34 * 10^9) = -34.1\ 10^{-6}$ m³ /kg

b. $v_2 = v_1 + \Delta v = 0.00942$ m³ /kg

c. $\gamma_2 = \frac{g}{v_2} = 9.81/0.000942 = 10410$ N/m³

Example 2:
Calculate the modulus of liquid elasticity that reduced 0.035 per cent of its volume by applying a pressure of 5[Bar].
Sol:

$$E_v = -v\frac{dp}{dv} = \frac{v}{dv}\ dp = \frac{5}{0.00035} = 14285.714\ bar$$

SURFACE TENSION :

Liquids have cohesion and adhesion, both of which are forms of molecular attraction.

Cohesion enables a liquid to resist tensile stress, while adhesion enables it to adhere to another body.

At the interface between a liquid and a gas, i.e., at the liquid surface, and at the interface between two immiscible (not mixable) liquids, the out-of-balance attraction force between molecules forms an imaginary surface film which exerts a tension force in the surface.

This liquid property is known as *surface tension*. Because this tension acts in a surface, we compare such forces by measuring the tension force per unit length of surface. When a second fluid is not specified at the interface, it is understood

that the liquid surface is in contact with air. The surface tensions of various liquids decrease slightly with increasing temperature.

Capillarity: is the property of exerting forces on fluids by fine tubes or porous media; it is due to both cohesion and adhesion.

When the cohesion is of less effect than the adhesion, the liquid will wet a solid surface it touches and rise at the point of contact; if cohesion predominates, the liquid surface will depress at the point of contact. For example, capillarity makes water rise in a glass tube, while mercury depresses below the true level.

We call the curved liquid surface that develops in a tube a *meniscus*. From free-body considerations, equating the lifting force created by surface tension to the gravity force,

$$2\pi r \sigma cos\theta = \pi r^2 h\gamma$$

$$h = \frac{2\sigma cos\theta}{\gamma r}$$

where σ=surface tension (sigma) in units of force per unit length

θ= wetting angle (theta)

γ= specific weight of liquid

r= radius of tube

h= capillary rise

Note: For tube diameters larger than in (12 mm), capillary effects are negligible.

Surface tension effects are generally negligible in most engineering situations. However, they can be important in problems involving capillary rise, such as in the soil water zone.

When we use small tubes to measure fluid properties, such as pressures, we must take the readings while aware of the surface tension effects; a true reading would occur if surface tension effects were zero. These effects are also important in hydraulic model studies when the model is small, in the breakup of liquid jets, and in the formation of drops and bubbles. The formation of drops is extremely complex to analyze, but is, for example, of critical concern in the design of inkjet printers,

Example1 :

Water at 10°C stands in a clean glass tube of 2-mm diameter at a height of 35 mm. What is the true static height?

at 10°C: γ= 9804 N/m^3 , σ= 0.0742 N/m. for clean glass tube: θ=-0°.

$$h = \frac{2\sigma cos\theta}{\gamma r} = 0.015\ 14\ m = 15.14\ mm$$

True static height= 35.00 - 15.14 =19.86 mm

Example2:
Derive an expression for the change in height h in a circular tube of a liquid with surface tension σ and contact angle θ, as in the Fig.

Solution

The vertical component of the ring surface-tension force at the interface in the tube must balance the weight of the column of fluid of height h

$$2\pi R\sigma \cos \theta = \gamma\pi R^2 h$$

Solving for h, we have the desired result

$$h = \frac{2\sigma \cos \theta}{\gamma R} \qquad\qquad Ans.$$

Thus the capillary height increases inversely with tube radius R and is positive if $\theta < 90°$ (wetting liquid) and negative (capillary depression) if $\theta > 90°$.

Suppose that $R = 1$ mm. Then the capillary rise for a water-air-glass interface, $\theta \approx 0°$, $\sigma = 0.073$ N/m, and $\rho = 1000$ kg/m³ is

$$h = \frac{2(0.073 \text{ N/m})(\cos 0°)}{(1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.001 \text{ m})} = 0.015 \text{ (N} \cdot \text{s}^2)/\text{kg} = 0.015 \text{ m} = 1.5 \text{ cm}$$

For a mercury-air-glass interface, with $\theta = 130°$, $\sigma = 0.48$ N/m, and $\rho = 13{,}600$ kg/m³, the capillary rise is

$$h = \frac{2(0.48)(\cos 130°)}{13{,}600(9.81)(0.001)} = -0.46 \text{ cm}$$