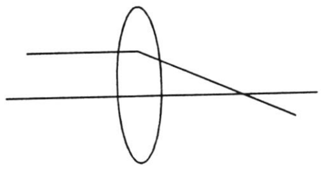
In the case of ray β, with input slope $\frac{r_1}{f}$ and exits parallel to the axis, $r_2^{'} = 0$

$$r_{2\beta}^{'} = 0 = \frac{1}{f}(r_{1\beta}) + Dr_{1\beta}^{'} \ ; \ r_{1\beta}^{'} = r_{1\beta} / f$$

Therefore D = 1

The other ray,



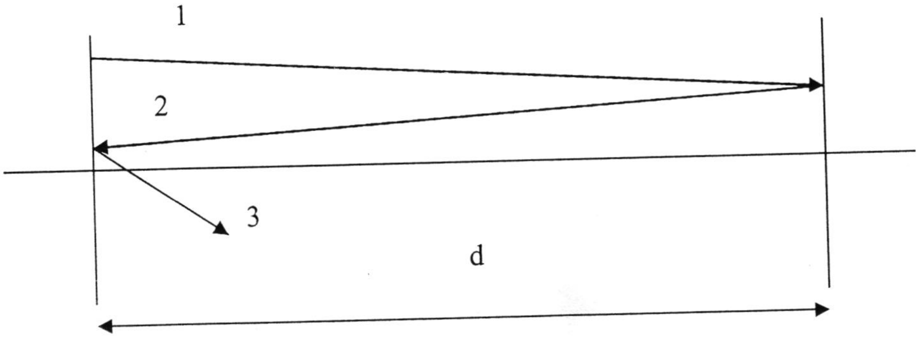
$[1,0]$

The ray matrix of a lens,

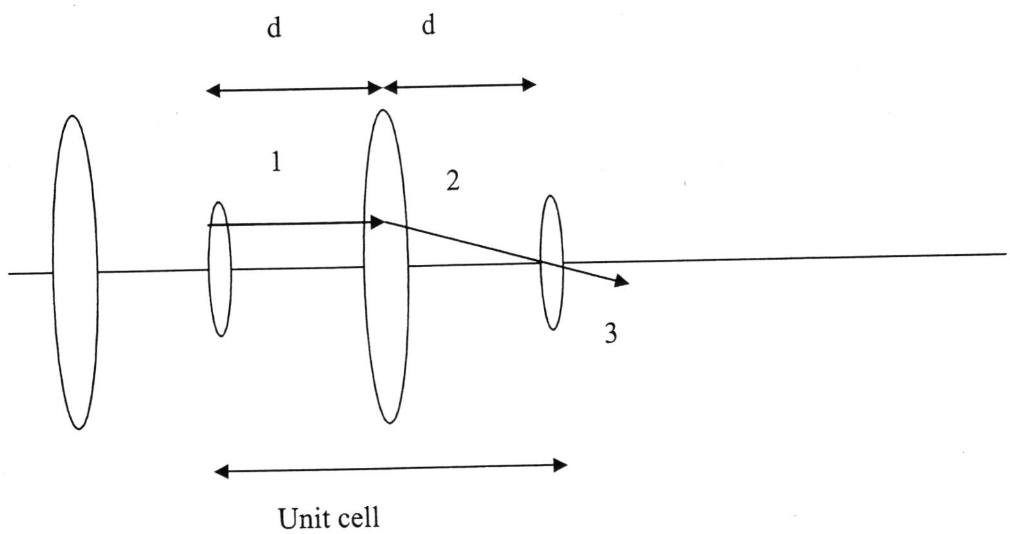
$$T = \begin{pmatrix} 1 & 0 \\ -\frac{1}{f} & 1 \end{pmatrix}$$

Note: The determinant T is unity.

<u>Optical cavities</u>



This can be represented by an equivalent lens waveguide system.



Transmission matrix of a unit cell
-Product of 2 matrices
Writing from right to left,

$$T = \begin{pmatrix} 1 & d \\ -\frac{1}{f_1} & (1-\frac{d}{f_1}) \end{pmatrix} \begin{pmatrix} 1 & d \\ -\frac{1}{f_2} & (1-\frac{d}{f_2}) \end{pmatrix} \quad (8.3)$$

Generally, it is better to find the second-order difference equation for the ray as it passes the various planes of the succeeding unit cell as it makes successive round-trips through the cavity.

$$r_{s+1} = Ar_s + Br_s' \quad \text{or} \quad r_s' = \frac{1}{B}(r_{s+1} - Ar_s)$$

Slope,

$r'_{s+1} = Cr_s + Dr'_s$ and given $r'_{s+1} = \frac{1}{B}(r_{s+2} - Ar_{s+1})$

Substituting yields

$$r'_{s+1} = Cr_s + \frac{D}{B}(r_{s+1} - Ar_s)$$

Noting that AD-BC = 1 for a complete roundtrip in any cavity leads to:

$$r_{s+2} - 2\left(\frac{A+D}{2}\right)r_{s+1} + r_s = 0 \qquad (8.4)$$

(Does 8.4 have solutions in which the magnitude of r is less than some max value?)
If yes, the ray is bounded along the lens waveguide.
If the ray is unbounded, it will eventually become big that it will miss one of the components (mirrors) and thus walk off the mirror.

Note:The ABCD matrix relates the 2 planes s+1 to s but tells you nothing about the trajectory of the ray.

Solution of (8.4)

Assume $r_s = r_0(e^{j\theta})^s$

$$= r_0 e^{js\theta} \qquad (8.5)$$

(r must be real.Two solutions, whose combination yields a real answer)
Substitute (8.5) into (8.4)

$$r_0 e^{js\theta}[e^{2j\theta} - 2\left(\frac{A+D}{2}\right)e^{j\theta} + 1] = 0$$

r_0 is set by the initial conditions – cannot be set equal to 0
Therefore quantity in [...] must be zero
i.e quadratic equation in $e^{j\theta}$

Two solutions are,

$$e^{j\theta} = \frac{A+D}{2} \pm j[1-(\frac{A+D}{2})^2]^{\frac{1}{2}}$$

If all the quantities in (8.7) are real, then the solutions are complex conjugates.The solutions are,

$r_s = r_0 e^{js\theta} + r_0^* e^{-js\theta}$

OR $r_s = r_{\max} \sin(s\theta + \alpha)$ (8.8)

8.3 Stability

It is sufficient for $\frac{A+D}{2}$ to be less than 1, so that θ be real and have a bounded solution (as in 8.8)

General condition for stability

$$-1 \le (\cos\theta = \frac{A+D}{2}) \le 1 \qquad (8.9)$$

By adding 1 to this equation

$$0 \le (\frac{A+D+2}{4}) \le 1 \qquad \text{(stable)} \qquad (8.10)$$

For the case of spherical cavity,

$$\frac{A+D+2}{4} = \frac{1}{4}[1 - \frac{d}{f_2} - \frac{d}{f_1} + (1-\frac{d}{f_2})(1-\frac{d}{f_1}) + 2]$$

$$= 1 - \frac{d}{2f_1} - \frac{d}{2f_2} + \frac{d^2}{4f_1f_2}$$

$$= (1 - \frac{d}{2f_1})(1 - \frac{d}{2f_2})$$

$$(g_1) \quad (g_2)$$

The stability condition can be written as

$$0 \le g_1 g_2 \le 1 \qquad (8.11)$$

Whereby $g_{1,2} = 1 - \frac{d}{R_{1,2}}$ (8.12)

8.5 Unstable Resonator

The unstable region is described by the condition:

$$(\frac{A+D}{2})^2 > 1 \qquad \underline{\text{unstable}} \qquad (8.13)$$

-normally we avoid operating in this regime

However,

Resonators operating in the unstable have become useful for high-gain laser systems (i.e the rays that walk-off the mirrors -> as output)