Electrostatics

Coulomb's Law

Due to a French physicist "Coulomb", the force between two charges is :-

a) Directly proportional to the product of their charges,

b) Inversely proportional to the square of the distance between them,

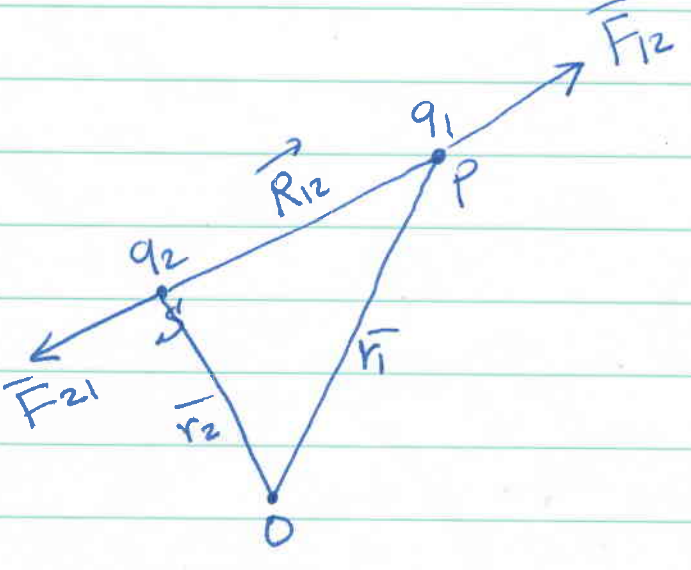
c) Directed along the line joining them, and

d) Repulsive (attractive) for like (unlike) charges.

If q_1 and q_2 are two charged particles situated at points $P(x, y, z)$ and $S(\acute{x}, \acute{y}, \acute{z})$ as shown in figure.

The electric force acting on q_1 due to q_2 is:-

$$\bar{F}_{12} = K \frac{q_1 q_2}{R_{12}^2} \bar{a}_{12}$$ (Force $\bar{F}_{12}$ in Newtons (N))



②

Where :-

ⓐ $\overline{F_{12}}$ is the force experienced by q_1 dueto q_2

ⓑ K is the constant of Proportionality, which depends upon the system of unit used,

ⓒ R_{12} is the distance between P and S,

ⓓ $\overline{a_{12}}$ is the unit vector pointing in the direction from point S to point P

S ↑ q_2 location, P ↑ q_1 location

The distance vector from S to P is :

$$\overline{R_{12}} = R_{12}\,\overline{a_{12}} = \overline{r_1} - \overline{r_2}$$

Where :- $\overline{r_1}$ and $\overline{r_2}$ are the Position vectors of points P and S, respectively.

$$K = \frac{1}{4\pi\epsilon_0}, \quad \overline{a_{12}} = \frac{\overline{R_{12}}}{R_{12}}$$

where : $\epsilon_0 = 8.85 \times 10^{-12} \simeq \frac{10^{-9}}{36\pi}$ farad/meter is the Permitivity of free space.

Thus,

$$\overline{F_{12}} = \frac{q_1 q_2}{4\pi\epsilon_0 R_{12}^2}\,\overline{a_{12}}$$

or

$$\boxed{\overline{F_{12}} = \frac{q_1 q_2 (\overline{r_1} - \overline{r_2})}{4\pi\epsilon_0 |\overline{r_1} - \overline{r_2}|^3}}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

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* It should be noted that the force exerted by q_1 on q_2 is equal in magnitude but opposite in direction to the force that q_2 exerts on q_1, That is

$$\bar{F}_{21} = -\bar{F}_{12}$$

__Example:__ Two point charges of 0.7 mC and 4.9 μC are situated in free space at (2,3,6) and (0,0,0). Calculate the force acting on the 0.7 mC charge.

__Solution:__

The distance vector from the 4.9 μC charge to the 0.7 mC charge is

$$\bar{R}_{12} = \bar{r}_1 - \bar{r}_2 = 2\bar{a}_x + 3\bar{a}_y + 6\bar{a}_z$$

Thus, $|R_{12}| = \sqrt{2^2 + 3^2 + 6^2} = 7 \text{ m}$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

$$\bar{F}_{0.7mC} = \frac{9 \times 10^9 \times 0.7 \times 10^{-3} \times 4.9 \times 10^{-6}}{7^3}\left[2\bar{a}_x + 3\bar{a}_y + 6\bar{a}_z\right]$$

$$= 0.18\,\bar{a}_x + 0.27\,\bar{a}_y + 0.54\,\bar{a}_z \quad N$$

The magnitude of the force experienced by either charge is 0.63 N

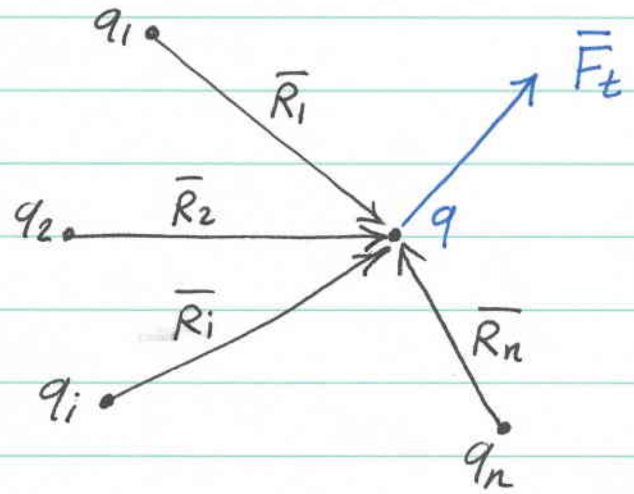
④

Principle of Superposition

The total force $\bar{F}_t$ acting on a point charge q due to a system of n point charges is the vector sum of the forces exerted individually by each charge on q, as shown in figure.

$$\bar{F}_t = \sum_{i=1}^{n} q \frac{q_i(\bar{r} - \bar{r}_i)}{4\pi\epsilon_0 |\bar{r} - \bar{r}_i|^3}$$

Where $\bar{r}$ and $\bar{r}_i$ are the position vectors of point charges q and q_i.



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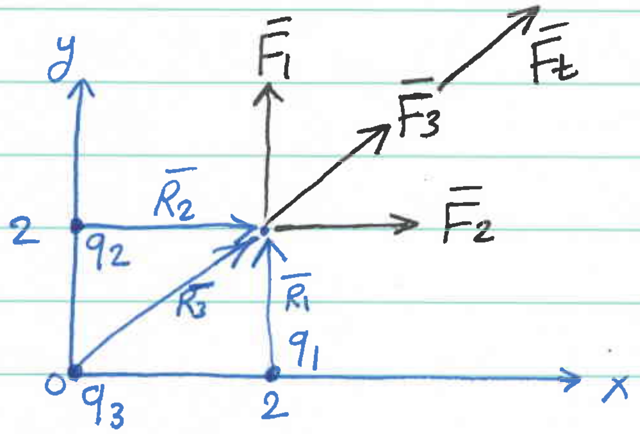
Example: Three equal charges of 200 nC are placed in free space at (0,0,0), (2,0,0), and (0,2,0). Determine the total force acting on a charge of 500 nC at (2,2,0).

Solution:

The distance vectors

$$\bar{R}_1 = \bar{r} - \bar{r}_1 = 2\bar{a}_y \Rightarrow |R_1| = 2\,m$$
$$\bar{R}_2 = \bar{r} - \bar{r}_2 = 2\bar{a}_x \Rightarrow |R_2| = 2\,m$$
$$\bar{R}_3 = \bar{r} - \bar{r}_3 = 2\bar{a}_x + 2\bar{a}_y \Rightarrow |R_3| = 2.828\,m$$



The force on q due to q_1 is

$$\bar{F}_1 = \frac{9\times10^{9}\times200\times10^{-9}\times500\times10^{-9}}{2^3}[2\bar{a}_y]$$
$$= 225\,\bar{a}_y\ \mu N$$

similarly;

$\bar{F}_2 = 225\bar{a}_x\ \mu N$ and $\bar{F}_3 = 79.6[\bar{a}_x + \bar{a}_y]\ \mu N$

Thus, the total force experienced by q, is

$$\boxed{\bar{F}_t = \bar{F}_1 + \bar{F}_2 + \bar{F}_3} = 304.6[\bar{a}_x + \bar{a}_y]\ \mu N.$$

(6)

Electric Field Intensity ($\bar{E}$)

It is quite useful to define the force acting on a charge in the presence of another charge in terms of a field.
We say that there exists an electric field or electric field intensity everywhere in space sorrounding the charge. When another charge q_t is brought into this electric field, it experiences a force acting on it. given by

$$\bar{F}_t = \frac{q_1 q_t}{4\pi\epsilon_0 \; R_{1t}^2} \bar{a}_{1t}$$

The electric field intensity is then defined as the force per unit charge $\left(\frac{\bar{F}_t}{q_t}\right)$

$$\therefore \; \bar{E} = \frac{\bar{F}_t}{q_t} = \frac{q_1}{4\pi\epsilon_0 R_{1t}^2} \bar{a}_{1t} \qquad (V/m)$$

In general:

effect charge

$$\bar{E} = \frac{q}{4\pi\epsilon_0} \frac{(\bar{r}_1 - \bar{r}_2)}{|\bar{r}_1 - \bar{r}_2|^3} = \frac{q'}{4\pi\epsilon_0 \; R^2} \bar{a}_R$$

Where: $\bar{R}$:- is the distance between the point at which the point charge (q) is Located and the Point at which ($\bar{E}$) is desired.

$\bar{a}_R$:- is the Unit Vector in the direction of $\bar{R}$

* The electric field intensity due to n point charges, is

$$\bar{E} = \sum_{i=1}^{n} \frac{q_i}{4\pi\varepsilon_0} \frac{(\bar{r}-\bar{r}_i)}{|\bar{r}-\bar{r}_i|^3}$$

Where: $\bar{r}_i$ is the distance vector directed from the Location of the charge q_i toward the point of measurement of $\bar{E}$.

Examples. Two point charges of 20 nC and -20 nC are situated at (1,0,0) and (0,1,0) in free space. Determine the electric field intensity at (0,0,1).

Solutions The two distance vectors are:

$$\bar{R}_1 = \bar{r}-\bar{r}_1 = -\bar{a}_x + \bar{a}_z$$

$$|R_1| = |\bar{r}-\bar{r}_1| = 1.414 \text{ m}$$

and

$$\bar{R}_2 = \bar{r}-\bar{r}_2 = -\bar{a}_y + \bar{a}_z$$

$$|R_2| = |\bar{r}-\bar{r}_2| = 1.414 \text{ m}$$

$$\therefore \bar{E} = 9\times10^{9}\left[\frac{20\times10^{-9}}{(1.414)^3}(-\bar{a}_x+\bar{a}_z) - \frac{20\times10^{-9}}{(1.414)^3}(-\bar{a}_y+\bar{a}_z)\right]$$

$$= 63.67\,[-\bar{a}_x+\bar{a}_y] \text{ V/m}.$$

charge distributions

Line charge density:

When the charge is distributed over a linear element, we define the line charge density, the charge per unit length, as

$$\rho_\ell = \lim_{\Delta \ell \to 0} \frac{\Delta q}{\Delta \ell}$$

Where Δq is the charge on a linear element $\Delta \ell$.

Surface charge density:

When the charge is distributed over a surface, we define the surface charge density, the charge per unit area, as

$$\rho_s = \lim_{\Delta S \to 0} \frac{\Delta q}{\Delta S}$$

where Δq is the charge on a surface element ΔS.

Volume charge density:

If the charge is confined within a volume, we define the volume charge distribution, the charge per unit volume, as

$$\rho_v = \lim_{\Delta v \to 0} \frac{\Delta q}{\Delta v}$$

where Δq is the charge contained in a volume element Δv.

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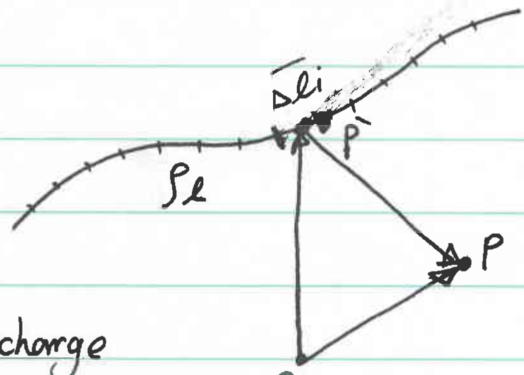
Electric field intensity due to charge distributions:

Suppose we are given a line charge distribution and our aim is to determine the electric field intensity at some point $P(x,y,z)$. We divide the line into n small sections, each of which contributes to the electric field intensity.



Line charge distribution.

$$\therefore \quad \Delta q_i = \rho_l \, \Delta l_i \Rightarrow dq_i = \rho_l \, dl_i$$

$\therefore$ The net electric field intensity is then given as:-

$$E = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{\Delta q_i}{4\pi\epsilon_0} \frac{(\bar{r} - \bar{r_i}')}{|\bar{r} - \bar{r_i}'|^3}$$

where:-

$\bar{r}$ is the position vector of the point P.

and

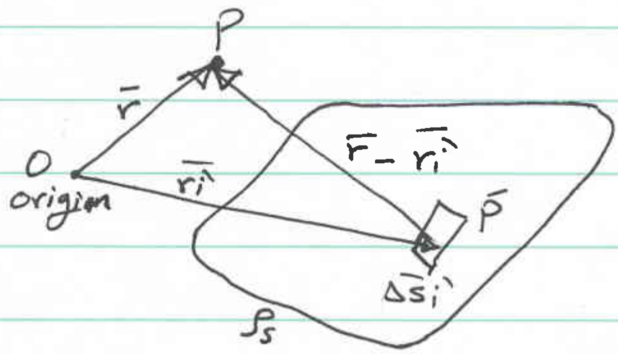
$\bar{r_i}'$ is the position vector of the point P' of the charge element $\overline{\Delta l_i}$

Note:- We will generally use primed letters for the coordinates of the source point and unprimed letters for points at which the desired quantity is to be determined in order to avoid confusion.

We can express the above equation as:

$$\boxed{E = \frac{1}{4\pi\epsilon_0}\int_c \frac{\rho_\ell\,(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}\,d\ell'}$$

Likewise, due to a surface charge distribution.



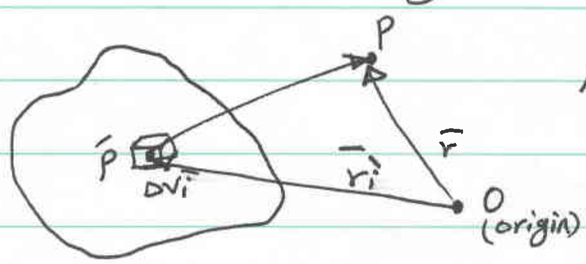
$\Delta q_i = \rho_s\,\Delta s_i'$

or

$dq_i = \rho_s\,ds_i'$

$$\boxed{\bar{E} = \frac{1}{4\pi\epsilon_0}\int_s \frac{\rho_s\,(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}\,ds'}$$

Finally, due to a volume charge distribution.



$\Delta q_i = \rho_v\,\Delta v_i'$

or

$dq_i = \rho_v\,dv_i'$

$$\boxed{\bar{E} = \frac{1}{4\pi\epsilon_0}\int_v \frac{\rho_v\,(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}\,dv'}$$