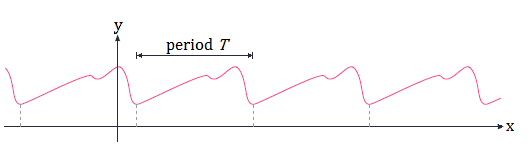
***Periodic Functions***

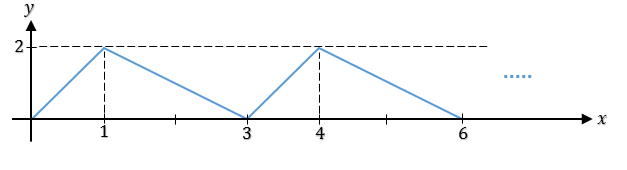
* A function is periodic if there is a positive number such that for every value of:

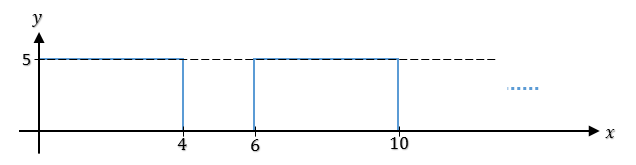
The smallest such value of is called the fundamental period or simply the period.

* If we know what the graph looks like in an interval of length, then we can use replication to sketch the entire graph.



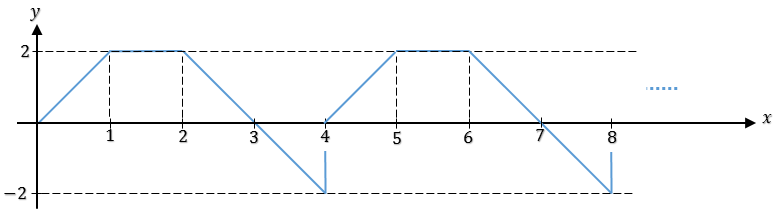
Example 1: What is the fundamental period of the following function?





(b)

(a)



(c)

Solution:

a) The period is 3

b) The period is 6

c) The period is 4

***Period of Trigonometric Functions***

**Period 𝜋:**

**Period 2𝜋:**

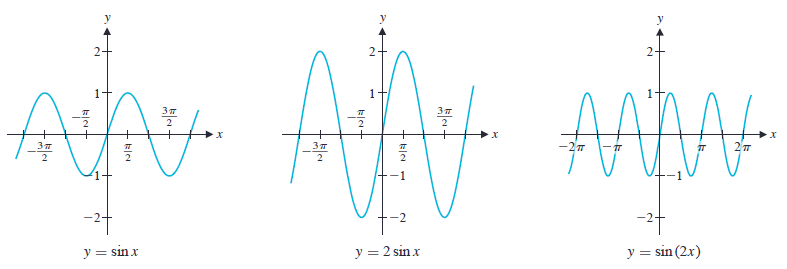


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Example 2: Sketcha),and*,*

What is the period of each function?

Solution:

a) The period is 2π b) The period is 2π c) The period is π

***Fourier Series***

Let be defined in the interval and outside of this interval by

(i.e assume that has a period ). Then the trigonometric Fourier series corresponding to is given by:

Where are constants and usually referred to as Fourier coefficients.

The Fourier coefficients is given by Euler formulas as:

* represent the DC component of
* With , we can rewrite Fourier series as:
* is the first harmonic of

is the second harmonic of

is the *n*th harmonic of



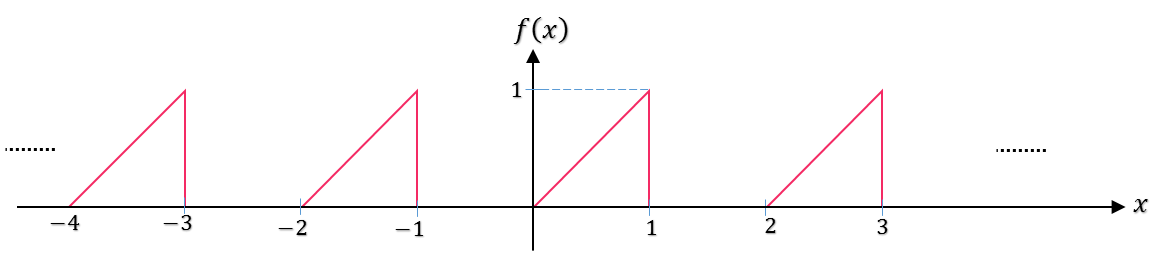
Example 3: Find the Fourier series corresponding to the square-wave function:

where is assumed to be periodic outside of the interval

Solution:The period

* we can write the even- indexed and odd-indexed coefficients separately as , for and , for

Example 4: Find the Fourier series corresponding to the following periodic function of period that defined by:

Solution:The period

* we can write the even- indexed and odd-indexed coefficients separately as , for and , for

***Even Functions and Odd Functions:***

A function is an:

**Evenfunction** of if,

**Odd function** of if,

for every in the function's domain.

The graphs of evenand oddfunctions have characteristic symmetry properties.

* + The graph of an **even function** is **symmetric about the -axis**.
  + The graph of an **odd function** is **symmetric about the origin**.

Example 4:

is an even function, since for all, (symmetry about y-axis).



Example 5:

is an odd function, since for all, (symmetry about the origin).

Example 6:

is an odd function, since for all , (symmetry about the origin).

Example 7:

is neither even nor odd:

Not even function since,where.

Not odd function since, where, but.

**Even**

**Odd**

Example 8:

is: an even function: for all, (symmetry about y-axis).

H.W: Which of the functions below is even, odd, or neither.

Give reasons for your answer.

a)b)

c)d)

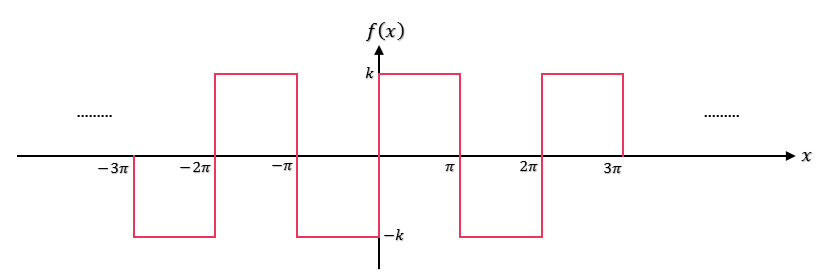
e)f)

g)h)

* In the Fourier series corresponding to an **even function**, **only cosine terms** (and possibly a **DC component**, which we shall consider a cosine term) can be present.
* In the Fourier series corresponding to an **odd function**, **onlysine terms** can be present.

Example 9: Find the Fourier series corresponding to the following periodic function:

where

Solution:The period

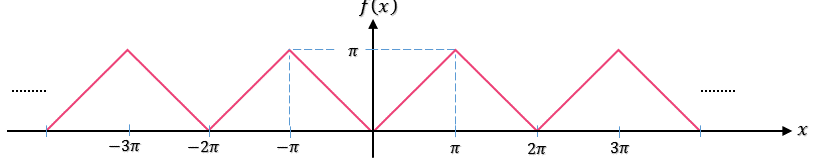
* we can write the even- indexed and odd-indexed coefficients separately as , for and , for

H.W:Find the Fourier series corresponding to, for, where is assumed to be periodic, of period 2π, outside of the interval [−π, π].

Answer:

,

* is the triangular-wave functionshown in the figure below.



***Half Range Fourier series***

A half range Fourier sine or cosine series is a series in which only sine terms or only cosine terms are present, respectively. When a half range series corresponding to a

given function is desired, the function is generally defined in the interval , which is half of the interval , and then the function is specified as odd or even, so that it is clearly defined in the other half of the interval, .

***For half range Fourier sine series (odd functions):***

***For half range Fourier cosine series (even functions):***

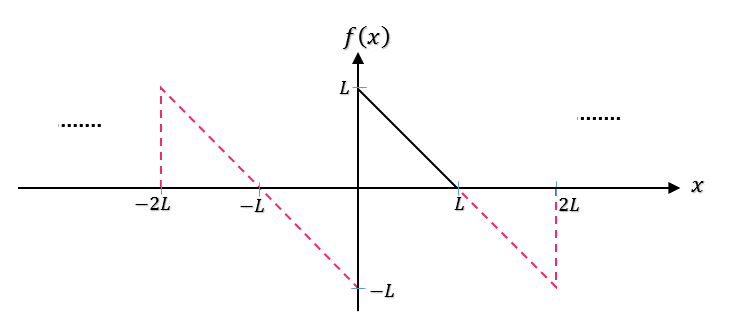
Example 10: Determine the half range Fourier sine series corresponding to:

.

Solution:

* A Fourier series consisting of sine terms alone is obtained only for an odd function.

Hence, we extend the definition of so that it becomes odd.



Taking the period

Since the function is odd then: and

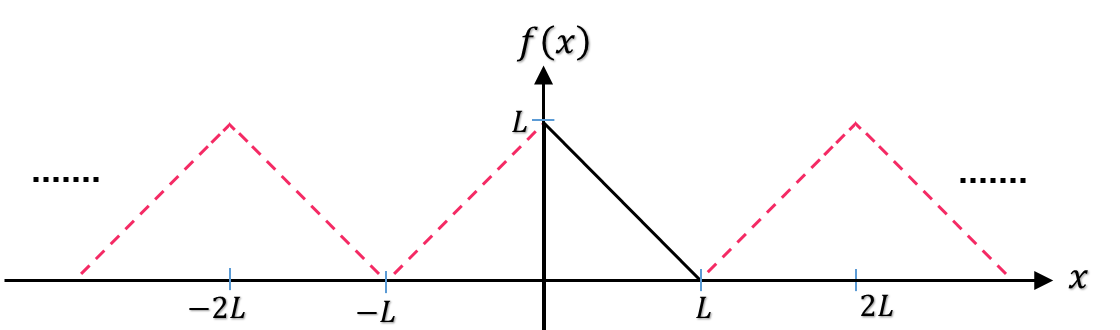
Example 11: Determine the half range Fourier cosine series corresponding to:

.

Solution:

* A Fourier series consisting of cosine terms alone is obtained only for an even function.

Hence, we extend the definition of so that it becomes even.



Taking the period

Since the function is even then:

H.W: Sketch each of the following functions then determine the corresponding half range Fourier cosine series:

1)

2), for

3)

H.W: Sketch each of the given functions then find the correspondinghalf range Fourier sine series:

1) , .

2),

3)

4)

5)

6)

***Complex representation for Fourier series***

Using Euler’s identities:

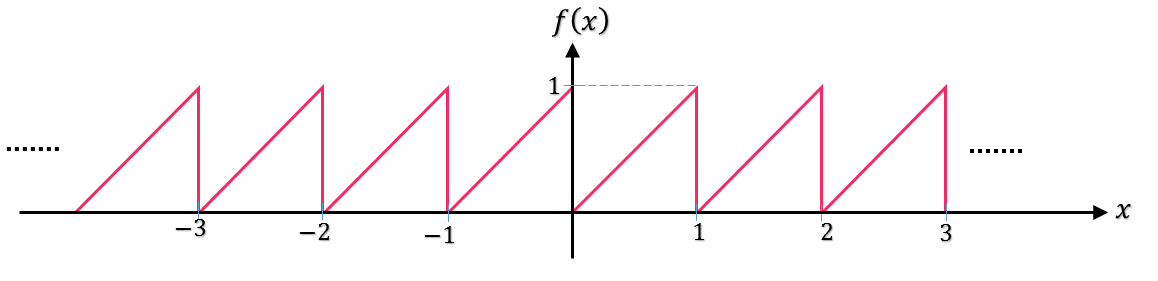
The trigonometric Fourier series corresponding to given by:

Can be written in complex form as:

Where:

Example 12: Determine the complex Fourier series corresponding to the following periodic function of period that given by:

.

Solution:The period

* From Euler’s identities:

So,