***Vectors:***

The vector represented by the directed line segment has **initial point**and **terminal point** and its **length**is denoted by.

Two vectors are equal if they have the same length and direction.

***Component Form***

* If is a two-dimensional vector in the plane equal to the vector with initial point at the origin and terminal point, then the **component form** of is

The numbers and are the components of **v**.

* If is a three-dimensional vector equal to the vector with initial point at the origin and terminal point , then the **component form** of **v** is

The numbers and are the components of **v**.

* The **magnitude** or **length** of the vector is the nonnegative number

Example: Find the (a) component form and (b) length of the vector with initial point and terminal point.

Solution:

(a) The standard position vector representing has components

Then the component form of is

(b)The length or magnitude of is

***Vector Algebra Operations***

If and are vectors, and is scalar

1) Vector addition

2) Scalar multiplication

***Properties of Vector Operations***

Let be vectors and be scalars

Example: let and.Find the components of

Solution:

b)

c)

Homework: let, and.Find the components of

***Unit Vectors***

* A vector of length **1**is called a **unit vector**.

The standard unit vectors are

* Any vector can be written as a linear combination of the standard unit vectors as follows:
* If , then:

Example: Find a unit vector **u** in the direction of the vector from to

Solution:

Then

Therefore, the unit vector **u** in the direction of is:

Example: Find a vector of magnitude 7 in the direction

Solution:

The unit vector **u** in the direction of is:

Then the vector of magnitude 7 in the direction is:

Homework: Find a vector of magnitude 3 in the direction

Example: A force of 6newtons is applied in the direction of the vector. Express the force **F** as a product of its magnitude and direction.

Solution:

The magnitude of force vector = 6

Therefore, we can express the force **F** as a product of its magnitude and direction as:

***The Dot Product***

The dot product of vectors and is

Example:

***Angle between Two Vectors***

The angle between two nonzero vectors and is given by

* Vectors and are **orthogonal** or **perpendicular**) if and only if.

Example: Find the angle betweenand

Solution:

Homework: Find the angle between and

Example: using vector methods, find the angle in the triangle ABC shown in the figure below.

Solution:

From figure we have: and

The angle is the angle between the vectorsand

The component forms of these vectors are

Then

Example: Determine if the following two vectors are orthogonal (perpendicular) or not:

and

and

and

Solution:

To determine if two vectors are orthogonal, we calculate their dot product

and

, then the two vectors are orthogonal.

and

, then the two vectors are orthogonal.

and

, then the two vectors are orthogonal.

***The Cross Product***

* Unlike the dot product, the cross product is a vector.
* The cross product (“**u** cross **v**”) is the vector defined as follows:

Where is a unit vector perpendicular to the plane determine by and.

* The vector is orthogonal to both and because it is a scalar multiple of.
* Nonzero vectors and are parallel if and only if.

***Calculating the Cross Product as a Determinant***

Ifandthen:

Example: Find and if and.

Solution:

Example: Find a unit vector perpendicular to the plane of, and as shown in figure below.

Solution:

The vector is perpendicular to the plane because it is perpendicular to both vectors.

In terms of components,

So,

Then a unit vector perpendicular to the plane is

***Lines and Planes in Space***

* In space, a line is determined by a point and a vector that giving the direction of the line.



Suppose that is a line in space passing through a point parallel to a vector.

Then is the set of all points for which is parallel to.

Thus, for some scalar parameter. The value ofdepends on the location of the point along the line.

The expanded form of the equationis:

which can be rewritten as:

Therefore, a vector equation for the line through parallel to a vector is

And the parametric (scalar) equations

, , ,

Example: Find parametric equations for the line through parallel to.

Solution:

With equal to and

equal to

The parametric equations are

Example: Find parametric equations for the line through and.

Solution:

The vector

is parallel to the line

With equal to, then the parametric equations are

Homework:

1) Find parametric equations for the line through parallel to .

2) Find parametric equations for the line through and.

3) Ifand, Find parametric equations for the line through parallel to the vector.

***The Distance from a Point to a Line in Space***

The distance from a point to a line through parallel to,is given as

Example: Find the distance from the point to the line

Solution:

From equations:

Then the line passes through parallel to

Then the distance is

Homework:

1) Find the distance from the point to the line through parallel to.

2) Find the distance from the point to the line through and.

3) Find the distance from the point to the line

***An Equation for a Plane in Space***

* In space, a plane is determined by knowing a point on the plane and its “tilt” or orientation. This “tilt” is defined by specifying a vector that is perpendicular or normal to the plane.

Suppose that plane passes through a point and is normal to the nonzero vector. Then is the set of all points for which is orthogonal to.

Accordingly, the dot product. This equation is equivalent to:

Or:

Which can be rewritten as:

Therefore, the plane through normal to has:

* Vector equation:
* Component equation:
* Component equation simplified:

where

Example:Find an equation for the plane through perpendicular to**.**

Solution:

From simplified component equation:

Then the equation for the plane:

Homework:

1) Find an equation for the plane through perpendicular to**.**

2) Find an equation for the plane through perpendicular to the line:

Example:Find an equation for the plane through and .

Solution:

So, the vector normal to the plane is:

Taking

Then from simplified component equation:

Then the equation for the plane:

***Planes Intersection***

* Two planes are parallel if and only if their normal vectors are parallel.
* If two planes are not parallel then they intersect in a line.
* is a vector parallel to the line of intersection of the planes.
* The angle between two intersecting planes is defined to be the angle between their normal vectors.

Example: For the planesand

a) Find a vector parallel to the line of intersection of the planes.

b) Find the angle between the planes.

Solution:

The vectors:

arenormals to the planes.

a) The vector parallel to the line of intersection of the planes is:

b)

The angle between the planes is:

Homework:

1) For the planesand, find a unit vector parallel to the line of intersection of the planes.

2) if plane is normal to the vector and plane is normal to the vector, Find the angle between the planes.

3) If plane pass through point and perpendicular to the line:

and plane equation given as, then:

a) Find an equation for the plane.

b)Find a vector parallel to the line of intersection of plane with plane.

c) Find the angle between plane and plane.

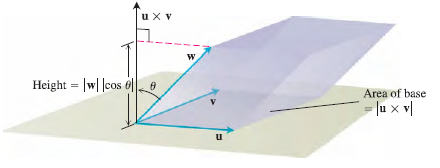
***TRIPLE PRODUCTS***

Dot and cross multiplication of three vectors may produce important products of the form.

Example:If and , find:

Solution:

***Triple Scalar or Box Product***

* The product is called the triple scalar productor the box Product of.
* The absolute value of this product is the volume of the parallelepiped (parallelogram-sided box)determined by.

The triple scalar product can be evaluated as a determinant:

If**,**, Then:

So, from matrices properties:

Example:Find the volume of the box (parallelepiped) determined by:

and**.**

Solution:

Homework:

If and , then find:

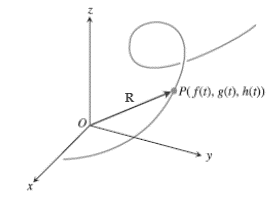
1)

2)

3)

4)

***Vector-Valued Functions and Motion in Space***

* When a particle moves through space during a time, it make a curve (particle’s path) in space.
* Particle’s position at any time is the points which can describe by the three functions:
* The vector from the origin to the particle's position at time is the particle's position vector and given as:

where functions and are the component functions (scalar functions) of the position vector.

* The velocity vector is the derivative of the position vectorwith respect to
* At any time, the direction of motion (*tangent to the path curve****)*** is also called the unit tangent vector and given as:
* Speed is the magnitude of velocity
* We can express the velocity of a moving particle as the product of its speed and direction:
* The accelerationvector is the derivative of velocity

Example:Ifthe vector is the position of a particle in space at time t, then find:

a) The particle's velocity vector, acceleration vector and speed at any time.

b) The particle's speed and direction of motion at.

Solution:

a) The velocity vector

The acceleration vector

The particle's speed

b) The particle's speed at:

The direction of motion at:

Homework:

Ifthe vector is the position of a particle in space at time, find the unit tangent vector.

***Curvature***

If is the unit tangent vector of a smooth curvethat specified by the position vector as a function of some parameter, then the curvature (of the curve) is:

where is the unit tangent vector.

Example:Find the curvature of a circlewith radius

Solution:

Therefore,for any value of the parameter t,the curvature of the circle is:

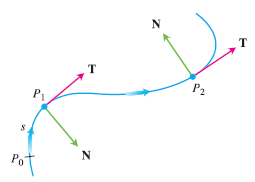
Homework:

Find the unit tangent vector, and the curvature for the space curves:

a)

b)

***The principal unit normal vector***

* The **principal unit normal** vector has a particular importance because it points in the direction in which the curve is turning.
* The principal unit normal vector is orthogonal to the unit tangent vector (i.e. ).
* If is a smooth curve, then the principal unit normal is:

where is the unit tangent vector.

Example:Find and for the circular motion

Solution:

Homework:

Find the unit tangent vector, the curvature and the principal unit normal vector for the space curve given as:

***Circle of Curvature***

* The circle of curvature (or osculating circle) at a point on a plane curve whereis the circle in the plane of the curve that:

1. is tangent to the curve at (has the same tangent line the curve has).
2. has the same curvature the curve has at .
3. lies toward the inner side (or concave) of the curve.

* The **radius of curvature** of the curve at is the radius of the circle of curvature
* The **center of curvature**of the curve at P is the center of the circle of curvature.

Example:Find the radius of curvature and the center of curvature of the parabola at the origin.

Solution:

By parameterize the parabola using the parameter, then:

At the origin, :

2**j**

Therefore, the curvature is

Also at the origin