***Chapter Six***

***DIMENSIONAL ANALYSIS AND***

***PERFORMANCE OF TURBOMACHINES***

***6-1.Introduction:***

In the previous studies we have discussed and analyzed the ideal performance of turbomachines based largely on one-dimensional frictionless flow assumptions. The actual performance of a turbomachines will differs from the ideal situation because of the occurrence of losses due to fluid dynamic drag, leakage and mechanical friction, for example, the actual performance curve of a centrifugal pump running at constant speed is given in ***Figure(6-1)***.



***Figure (6-1): Performance Characteristics of Centrifugal***

***Pump.***

Where the head developed by the pump is shown as a function of delivered volume flow rate. This performance curve differs from the ideal curve shown in ***Figure (6-2)*** by the effects of non-uniform velocity distribution (Circulatory flow), the frictional losses in the stationary and moving vanes of the pump, and the separation losses (Turbulence losses) that occur at off-design conditions. As shown in ***Figure (6-2),*** at off-design conditions the flow through the pump, and hence the radial velocity differs from that at design, and hence the angle of relative rotor inflow velocity will be different from the blade angle ***β1***. The ensuing flow separation causes a loss that is proportional to the square of the difference in flow rates, ***(Q-Qdesign)2***. The frictional losses will be proportional to the square of the flow rate ***Q2***, while the flow non-uniformity losses are proportional to ***ΔH***, the actual performance of the pump is reduced from the ideal by the amount indicated in ***Figure (6-2).***



***\***

***Figure (6-2): Performance Characteristics of Centrifugal Pump***

***Indicating Various Losses.***

In addition to the head losses and reductions, pumps and blowers have torque losses due to bearing and packing friction and disk friction losses from the fluid between the moving impeller and housing. Internal leakage is also an important power loss, in that fluid which has passed through the impeller, with its energy increased, escape through clearances and flows back to the suction side of the impeller.

In order to avoid testing each pump for performance characteristics, we can be made of similitude considerations to establish such characteristics for classes of turbomachines having geometric similarity (having similar velocity vector diagram, and similar streamlines).

***6-2.Turbomachine Affinity Laws:***

The similarity grouping can be used to compare or estimate the performance of a whole family of geometrically similar turbomachine in terms of the performance of one of the machines of the family. Consider two geometrically similar turbomachines with dynamically similar flow. If we neglect small flow variations due to Reynolds number effects, dynamic similarity of the two flows requires:

|  |  |
| --- | --- |
|  | …(6-1)  …(6-2) |

It is customary to write the head change simply as ***H*** rather than ***(ΔHi)1→2***

And, since

|  |  |
| --- | --- |
|  | …(6-3) |

Then ***equation (6-2)*** becomes

|  |  |
| --- | --- |
|  | …(6-4) |

The volume flow rate through an axial flow turbomachine is

|  |  |
| --- | --- |
|  | …(6-5) |

Where ***D*** is the blade tip diameter and ***d*** is the hub diameter, for geometrically similar machines the hub tip ratio ***d/D*** is the same. Then



Matching capacity coefficients to give:

|  |  |  |
| --- | --- | --- |
|  | | …(6-6) |
|  | …(6-7) | |

|  |  |
| --- | --- |
|  | …(6-8) |

We can obtain a dynamic similarity relationship involving only the power by noting that



Then

|  |  |
| --- | --- |
|  | …(6-9) |

If we use ***equation (6-2)*** to eliminate the heads ratio and ***equation (6-7)*** to eliminate the flow rates ratio, then ***equation (6-9)*** becomes:

|  |  |
| --- | --- |
|  | …(6-10) |

In two dynamically similar flows all corresponding velocities must be in the same proportion, in two velocity triangles are similar, and

|  |  |
| --- | --- |
|  | …(6-11) |

Using ***equation (6-11)*** in ***equation (6-10)*** and remembering that ***U*** is proportional to ***ND***, to obtain:

|  |  |
| --- | --- |
|  | …(6-12) |

***Equations (6-4),(6-8), and (6-12)*** are called the ***turbomachine Affinity Laws***. They are valid for incompressible flow and compare dynamically similar flow in geometrically similar turbomachines.

***PI theorem*** is used as another method to obtain the applicable non-dimensional parameters, the physical variables considered significant in this case must be listed. Specifically, we expect that the change in total head ***(gΔH)*** should be a function of the size of the machine such as, its diameter ***D***, the rotational speed of the rotor ***ω***, the flow rate through the machine ***Q***, density of the flow ***ρ***, and the dynamic viscosity of the working fluid. This statement takes the form:

|  |  |
| --- | --- |
|  | …(6-13) |

|  |  |  |
| --- | --- | --- |
| Quantity | Symbol | Dimensions |
| Discharge | Q | L3T-1 |
| Density | ρ | ML-3 |
| Speed of Rotation | ω | T-1 |
| Diameter | D | L |
| Viscosity | μ | ML-1T-1 |
| (gΔH) | gH | L2T-2 |

No. of dimensionless parameter=n-m

n-no.of quantity=6

m-no. of dimensions=3

∴no. of dimensionless parameter=6-3=3

|  |  |
| --- | --- |
| ∴ | …(6-14)  …(6-15)  …(6-16) |



***at π1***

***For M***

***x1=0***

***for L***

***-3x1+z1+3=0 → z1=-3***

***For T***

***-y1-1=0 → y1***

***at π2***

***for M***

***x2=0***

***for L***

***-3x2+z2+2=0 → z2=-2***

***for T***

***-y2-2=0 → y2=-2***

***at π3***

***for M***

***x3=1***

***for L***

***-3x3+z3+1=0 → z3=2***

***y3=1***

Substituting the above variables into ***equations (6-14), (6-15),*** and ***(6-16),*** then gets:

|  |  |
| --- | --- |
|  | …(6-17)  …(6-18)  …(6-19) |

This can be written as

|  |  |
| --- | --- |
|  | …(6-20) |

The dimensionless parameter in this equation is the Flow Coefficient. The dimensionless parameter is a form of Reynolds number for sufficiently large Reynolds numbers, the effect of Reynolds number on the performance of the machine is less important than the other parameters, consequently, we can write with sufficient accuracy:

|  |  |
| --- | --- |
|  | …(6-21) |

The same procedure can be used to find the power coefficient as:

|  |  |
| --- | --- |
|  | …(6-22) |

***6-3.Efficiency:***

The efficiency of family of geometrically similar turbomachines depends on the capacity coefficient; ***Figure (6-3)*** shows a representative plot of efficiency for family of geometrically similar turbomachines exhibiting the characteristic behavior of having efficiency of zero at zero flow, which increases to a maximum near the design flow range.



***Figure (6-3): Efficiency of Dynamically Similar***

***Turbomachine.***

Dynamically similar flows in geometrically similar turbomachines have the same efficiency. As previously mentioned, the effect of changes in Reynolds number on the performance of a turbomachine is very small compared with changes in the other similarity groupings.

However, to obtain a more accurate prediction of the efficiency, various correlation formulas have been developed for the dependence of efficiency on Reynolds number, of which ***equation (6-23)*** is representative. The exponent n for most turbomachines is between ***0.1*** and ***0.25:***

|  |  |
| --- | --- |
|  | …(6-23) |

The formula shows that operation at higher Reynolds numbers by either increased size or speed results in improved efficiency.

***Moody, L.F., 1942***, suggested an empirical equation that may be used to estimate the maximum efficiency of a prototype pump based on test data from a geometrically similar model of the prototype pump. His equation is written:

|  |  |
| --- | --- |
|  | …(6-24) |

***Moody*** assumed that only surface resistance changes with model scale so that losses in passages of the same roughness vary as ***1/D5***. Unfortunately, it is difficult to maintain the same relative roughness between model and prototype pumps. Further the ***Moody*** model does not account for any difference in mechanical losses between model and prototype, nor does it allow determination of off-peak efficiencies. Nevertheless, scaling of the maximum efficiency point is useful to obtain a general estimate of the efficiency curve for the prototype pump.

Turning to hydrodynamic problems ***Nixon and Cairney***, ***1972***, ***Osterwalder, 1978*** and ***Osterwalder and Ettig, 1977*** suggest the following relation:

 Here, ***δM***and ***δI***are unaffected by the Reynolds number and ***δI*** is usually assumed to remain the same. ***δM*** is considered to vary as speed, in contrast to the other hydrodynamic losses which tend to follow ***N3*** law, and reduces with reducing speed at a lesser rate, thus being proportionally more important at low speeds. ***Nixon and Cairney, 1972*** present a method of finding ***δM***, and suggest that prediction from low speed tests be limited to differential head readings.

The estimation of disc friction loss has been a subject for argument, as the classical work was done on plain thin discs rotating in a close fitting closed casing. ***Nixon*** used work by ***Necce and Daily, 1960*** and ***Watabe, 1958*** for 'smooth' and 'rough' discs, and showed an error from measured data of about ***10%***. ***Sutton, 1968*** studied this problem, particularly the effect of leakage flow through wear rings and its relation to disc friction. ***Osterwalder, 1978*** commented that there is little current data of general applicability. But ***Kurokawa and Toyokura***, ***1976*** and ***Wilson and Goulburn, 1976*** extended the database. ***Table (6-1)*** shows a selection of model scale formula (as quoted for example by ***Nixon, 1965***).

Table (6-1): A Selection of Model Formula, ***Turton, R.K., 1995***

|  |  |
| --- | --- |
| ***Moody:*** |  |
| ***Anderson:*** |  |
| ***Pfleiderer:*** | ***valid between 1/12<Rm/R<20*** |
| ***Hutton:*** |  |
| ***Ackeret*** |  |

***6-4.Specific Speed***

We have seen from previous sections how the dynamic similitude allows us to predict the performance of whole family of geometrically similar turbomachines from the performance of a single machine.

If we suppose we want a pump to turn at a certain speed and deliver a certain flow rate at a specified head; ***what type of pump will give the most efficient performance for these operating condition?***

Similarly; ***what type of turbine will give the most efficient operation at a certain speed, power output, and head change?***

Question like this type are best answered by developing a similarity grouping called the ***Specific Speed***.

The appropriate specific speed for a pump is consequently developed by eliminating the diameter between Affinity laws ***(6-4)*** and ***(6-8)*** in such away that the speed appears to the first power. This is done most simply by dividing the square root of ***equation (6-8)*** by the three fourths power of ***equation (6-4)*** to give:

|  |  |
| --- | --- |
|  | …(6-25) |

The quantity called the Specific Speed ***Ns*** which is the form used for pumps.

An appropriate specific speed for turbines is developed by eliminating the diameter between Affinity laws ***(6-12)*** and ***(6-4)*** and having the speed appear to the first power. This is accomplished by dividing the square root of ***(6-12)*** by the five fourths power of ***(6-4)*** resulting in:

|  |  |
| --- | --- |
|  | …(6-26) |

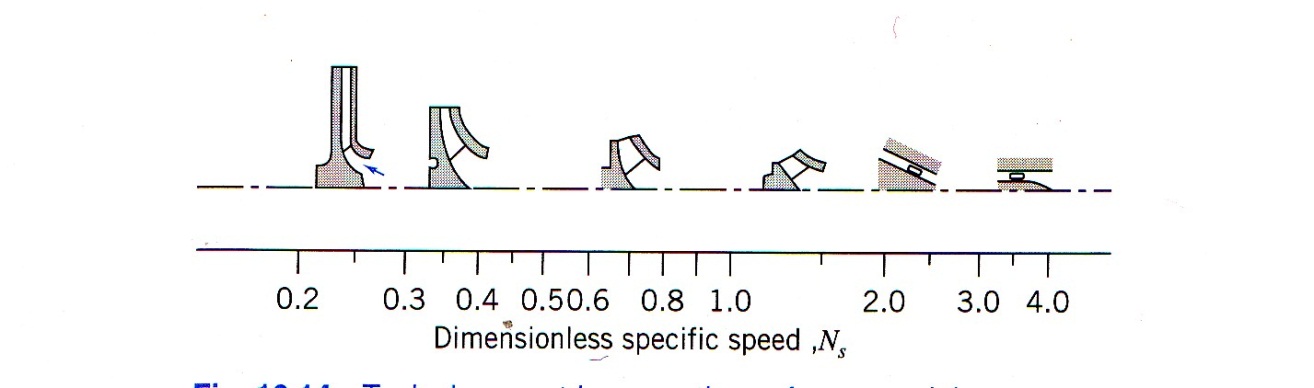
Since most incompressible flow turbines use water as the working fluid, the density is usually omitted. The quantity is the specific speed ***Ns*** used to characterize different types of turbines. When head is expressed as energy per unit mass (i.e., with dimensions equivalent to ***L2/t2***, or ***g*** times head in height of liquid), and ***N*** is expressed in radians per second, the specific speed for pump is dimensionless.

Although specific speed is a dimensionless parameter, it is common practice to use a convenient but inconsistent set of units to specify the variables, ***N*** and ***Q*** and to use energy per unit weight, in place of energy per unit mass, when this is done, the specific speed is not a unitless parameter and the magnitude of the specific speed depends on the units used to calculate it. Customary unit used in ***U.S.*** engineering practice for pump are ***r.p.m.*** for ***N***, ***gpm*** for ***Q*** and ***feet*** (energy per unit weight) for ***h***. customary units used in ***U.S***. engineering practice for hydraulic turbine are ***r.p.m.*** for ***N***, horsepower for ***P*** and ***feet*** for ***h***.

Specific speed may be thought of as the operating speed in which a machine produces unit head at unit volume flow rate (or unit power at unit head). Holding specific speed constant describes all the operating conditions of geometrically similar machines with similar flow conditions.

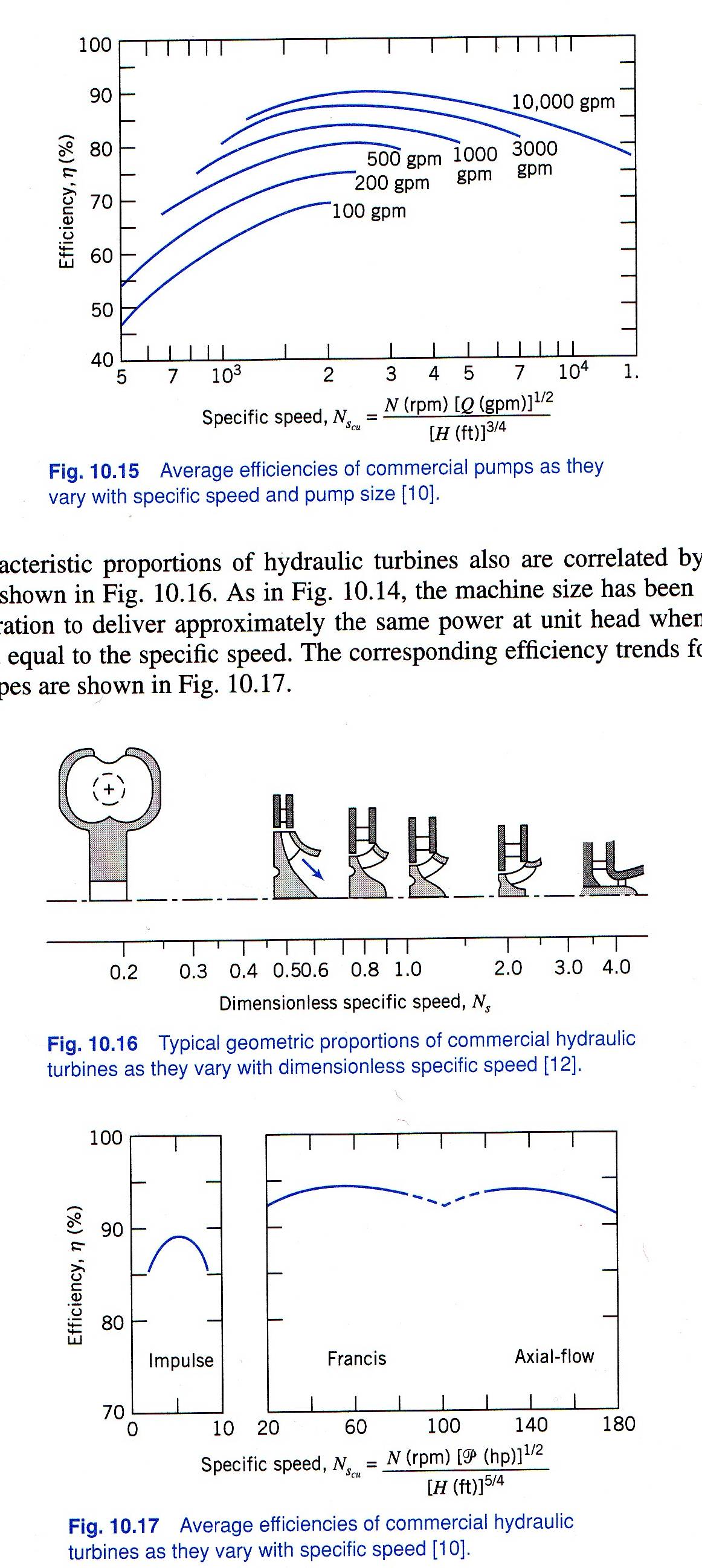
It is customary to characterize a machine by its specific speed at the design point. This specific speed has been found to characterize the hydraulic design features of a machine. Low specific speeds correspond to efficient operation of radial flow machines. High specific speeds correspond to efficient operation of axial flow machines. For a specified head and flow rate, one can choose either a low specific speed machine (which operates at low speed) or a high specific speed machine (which operates at higher speed).

Typical proportions for commercial pump designs and their variation with dimensionless specific speed are shown in ***Figure (6-4).*** In this figure, the size of each machine has been adjusted to give the same head and flow rate for rotation at a speed corresponding to the specific speed. Thus it can be seen that if the machine's size and weight are critical, one should choose a higher specific speed. ***Figure (6-4)*** shows the trend from radial (purely centrifugal pumps), through mixed flow, to axial flow geometries as specific speed increases.



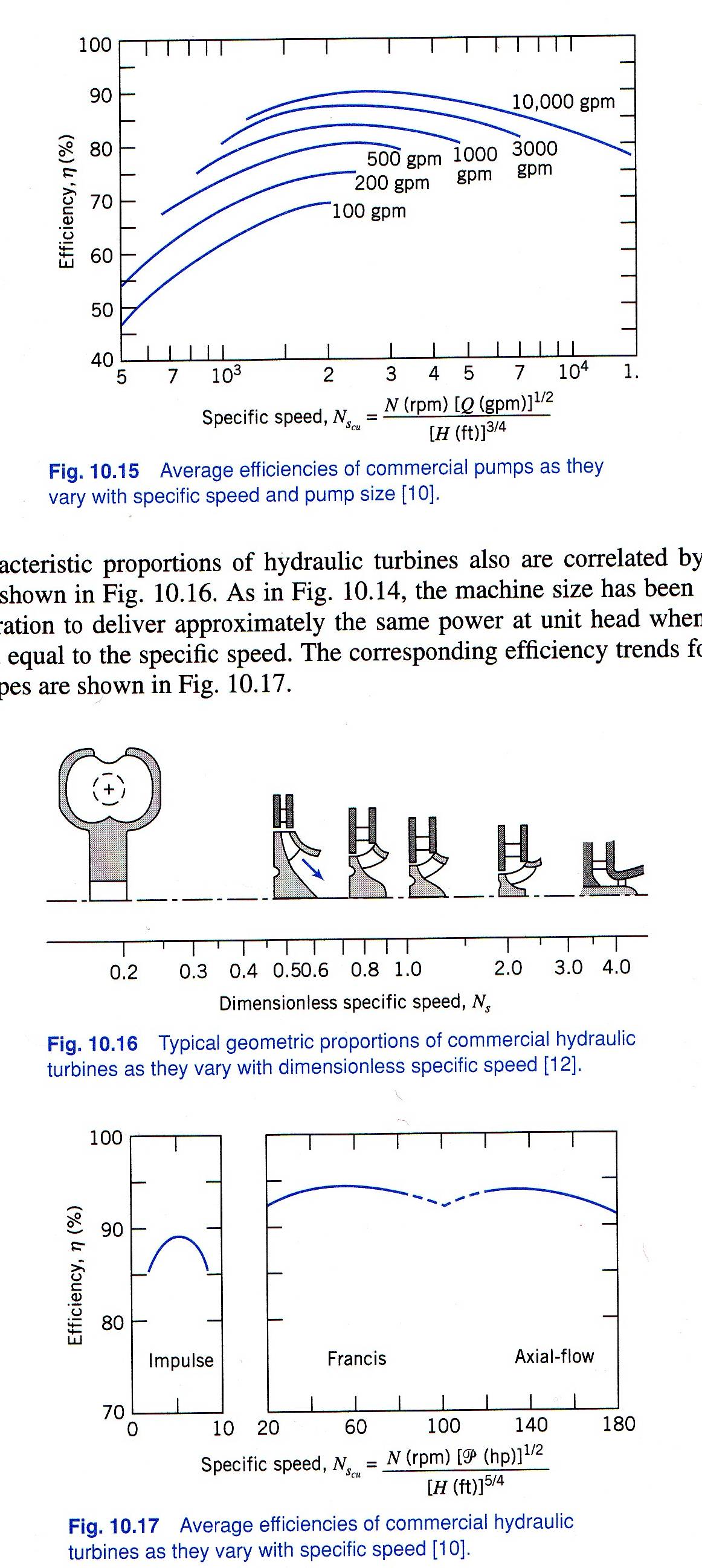
***Figure (6-4): Typical Geometric Proportions of Commercial Pumps as they Vary with Dimensionless Specific Speed (Fox, Robert W., et. al., 1998).***

The corresponding efficiency trends for typical pumps are shown in ***Figure (6-5)***. ***Figure (6-5)*** shows that pump capacity generally increases as specific speed increases; the figure also shows that at any given specific speed, efficiency is higher for large pumps than for small ones. Physically this scale effect means that viscous losses become less important as the pump size is increased.

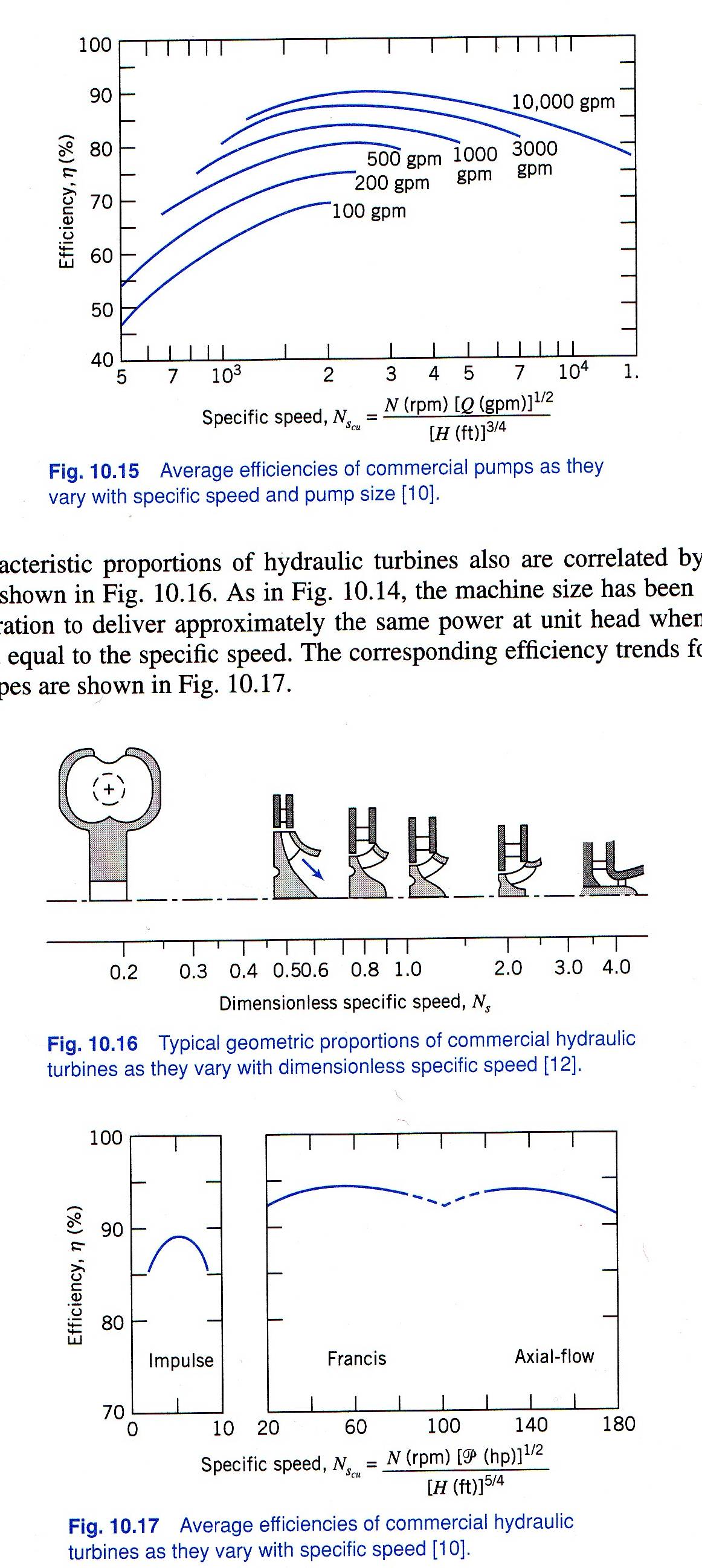


***Figure(6-5): Average Efficiencies of Commercial Pumps as they vary with Specific Speed and Pump Size (Fox, Robert W., et. al., 1998).***

Characteristic proportions of hydraulic turbines also are correlated by specific speed, as shown in ***Figure (6-6)***, as in ***Figure (6-4)***, the machine size has been scaled in this illustration to deliver approximately the same power at unit head when rotating at a speed equal to the specific speed. The corresponding efficiency trends for typical turbine types are shown in ***Figure (6-7)***.



***Figure (6-6): Typical Geometric Proportions of Commercial Hydraulic Turbines as they Vary with Dimensionless Specific Speed (Fox, Robert W., et. al., 1998).***

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***Figure (6-7): Average Efficiencies of Commercial Hydraulic Turbines as they vary with Specific Speed (Fox, Robert W., et. al., 1998).***

***Example (6-1):***

At the best efficiency point a centrifugal pump, with impeller diameter ***D=8 in***, produces ***H= 21.9 ft*** at ***Q=300 gpm*** with ***N=1170 rpm***. Compute the corresponding specific speed using a) ***U.S.*** customary units. B) ***SI units*** (rad/sec, m3/sec, m2/sec2).c) ***European units*** (rev/sec, m3/sec, m2/sec2). Develop conversion factors to relate the specific speeds.

***Solution:***

Since 

From the given information the specific speed in ***U.S. customary*** units is:



To get the specific speed in ***S.I. unit*** we must convert the information to S.I.:



The dimensionless specific speed is



Convert the operating speed to hertz to get the specific speed in European units as:



Finally the specific speed in European units is:



To relate the specific speed, from ratios:



***6-5.Specific Speed for Pelton Wheels:***

The equation may be developed for a Pelton Wheel as follows:

|  |  |
| --- | --- |
|  | …(6-27) |

Where

***D***-mean wheel diameter.

***U***-bucket velocity

|  |  |
| --- | --- |
|  | …(6-28) |

Since

|  |  |
| --- | --- |
|  | …(6-29) |

Where

***H***-head behind the nozzle.

***V***j-nozzle velocity.

For fixed speed wheel

|  |  |
| --- | --- |
|  | …(6-30) |

Hence,

|  |  |
| --- | --- |
|  | …(6-31) |

|  |  |
| --- | --- |
|  | …(6-32) |

Where

***d***-nozzle diameter.

|  |  |
| --- | --- |
|  | …(6-33) |

If the specific speed defined in terms ***Q, H,*** and ***N*** as in ***equation (6-25)***:



Substituting all in the formula for ***Ns*** above, to get

|  |  |
| --- | --- |
|  | …(6-34) |

The value of ***k*** has to be deduced from the data of the wheel and nozzle.

***Note:***

***Sometimes Ns is defined in terms of water powers as in equation (6-26)***

******

***This is just an alternative formula and the same result can be easily obtained other way. You will need the substitution.***

******

***6-6.Performances of Turbines:***

Sometimes, we have to compare the performances of turbine, of different outputs and speeds, working under different heads. This comparison will be much convenient, if we study the following three characteristic of turbine (power, speed, discharge, head) under fixed diameter we get:

From the Affinity law:



Since the diameter remain fixed

|  |  |
| --- | --- |
|  | …(6-35) |

|  |  |
| --- | --- |
|  | …(6-36) |

|  |  |
| --- | --- |
|  | …(6-37) |

If we substitute ***equation (6-37)*** into ***(6-35)*** we get:

|  |  |
| --- | --- |
|  | …(6-38) |

And the power for the same procedure

|  |  |
| --- | --- |
|  | …(6-39) |

Sub. ***equation (6-37)*** into ***(6-39)*** to get:

|  |  |
| --- | --- |
|  | …(6-40) |

***6-7.Performances of Pumps:***

Sometimes there is a minor change in the requirement of the head of water or discharge of a pump from its designed head of water or discharge. In such a case a slight adjustment in the pump is made to suit the new set of conditions. This is done either:

* 1. Varying speed of the pump impeller, or
  2. Changing the diameter of the pump impeller.

Now, we shall study the effect of the two variations on the discharge, head of water and the power required to drive the pump.

1. Effect of variations in speed:

The same equations used in turbine in the previous section.

1. Effect of variations in diameter:

As discussed in previous section that minor change in the requirement of the head of water or discharge of a pump is made either by varying speed of the pump impeller or by changing the diameter of the pump impeller. It has been experienced that the former (i.e. varying the speed of the pump impeller) is not possible because the pump impeller is driven by motor, whose speed is fixed. It is thus obvious, that in majority of the cases, the diameter of the pump impeller is enlarged or reduced, whenever the head of water or discharged is to be increased or decreased. It is done either by changing the blades of the impeller or fixing rings to its outside diameter.

From Affinity laws:

 with fixed speed ***(ω or N)***

|  |  |
| --- | --- |
|  | …(6-41) |



|  |  |
| --- | --- |
|  | …(6-42) |

And



|  |  |
| --- | --- |
|  | …(6-43) |

It is impractical to manufacture and tests a series of pump models that differ in size by only a scale ratio. Instead it is common practice to test a given pump casing at fixed speed with several impellers of different diameter. Because pump casing width is the same for each test, impeller width also must be the same: only impeller diameter ***D*** is changed. As a result, volume flow rate scales in proportion to ***D2***, no to ***D3***. Pump power input at fixed speed scales as the product of flow rate and head, so it becomes proportional to ***D4***.

***Example (6-2):***

Estimate the size, flow rate, and rotational speed of a Kaplan hydraulic turbine which is dynamically similar to the one have (***P=7500 kw, Q=60 m3/sec, N=140 r.p.m.***) and delivers ***18000 KW*** at the same total head. Estimate the possible change in efficiency if the turbine has an efficiency of ***90%***.

***Solution:***

From the similarity grouping, ***equation (6-4)***, Since H1=H2,



Using the similarity grouping, ***equation (6-12)***



All linear dimensions of the proposed turbine are increased by factor ***1.55*** over the linear dimensions of the prototype. The speed ratio is then:



We use similarity grouping of equation (6-8) to estimate the flow rate of the proposed turbine and get:



Finally we use ***equation (6-23)*** to estimate the possible change in efficiency. The Reynold-number ratio becomes:



For dynamically similar turbomachines, the velocity triangles must be similar



Then



For the possible range of the exponent ***n*** (0.1 to 0.25) in ***equation (6-23)***, an increase in efficiency of ***0.5*** to **1** percent may be expected for the proposed turbine, i.e., efficiency between ***90.5*** and ***91%***.

***Problems***

***Q6-1:***

The turbines in a river barrage hydroelectric plant are designed to give ***44 MW*** each when the level difference is ***25 m*** and they are running at ***94.7 r.p.m.***, the designed overall efficiency is ***93 %*** and the runner diameter is ***6 m***. a model with a runner diameter of ***300 mm*** is to be tested under the same level difference. Suggest the probable rotational speed, flow rate, efficiency and power produced when the model is operating in dynamically similar conditions.

***Q6-2:***

Catalog data for a centrifugal water pump at design conditions are ***250 gpm*** and ***Δp 18.6 psi*** at ***1750 r.p.m.***, a laboratory flume requires ***200 gpm*** at ***32 ft*** of head. The only motor available develops ***3 hp*** at ***1750 r.p.m.***, is this motor suitable for the laboratory flume? How might the pump/ motor match be improved?

***Q6-3:***

A Centrifugal pump rotates at ***185 rad/sec*** and at best efficiency has a pressure rise of ***4.5\*105 N/m2*** when pumping water at the rate of ***0.28 m3/sec***, predict the corresponding best efficiency, flow rate and pressure rise when rotating at ***80%*** of the design speed. If the efficiency is ***85%*** in both cases estimate the input power required.

***Q6-4:***

One of the Kaplan turbines is rated at ***34000 hp*** when working under ***30 m*** head at ***166.7 r.p.m.***, find the diameter of the runner, overall efficiency of the turbine is ***0.91***, assume a flow ratio of ***0.65*** and diameter of runner hub equal to ***0.3*** times the external diameter of runner, also find the specific speed of turbine.

***Q6-5:***

In a hydroelectric station, the water is available under a head of ***5 m*** at the rate of ***300 m3/sec***. Calculate the number of Kaplan turbines with a speed of ***50 r.p.m.*** and ***82%*** efficiency. The specific speed of the turbines is not to exceed ***500(r.p.m., hp,m)***, also calculate the power produced by each turbine.

***Q6-6:***

A Centrifugal water pump operates at ***1750 r.p.m***.; the impeller has backward curved vanes with ***β2=60o*** and ***b2=1.27*** cm. at a flow rate of ***0.0220 m3/sec***, the radial outlet velocity is ***νr2=3.5685 m/sec***. Estimate the head at this pump could deliver at ***1150***.

***Q6-7***

Explain the significance and use of "specific Speed" ******

Explain why in the case of a Pelton wheel with several nozzles, P is the power per nozzle.

Calculate the specific speed of a Pelton wheel given the following:

***d***-nozzle diameter.

***D***-wheel diameter.

***U***-optimum blade speed=***0.46 Vjet***

η-**88%**. ***Cv***-coefficient of velocity=***0.98***.

***Q6-8:***

A reaction turbine is designed to produce ***25000 hp*** at ***90 rpm*** under ***150 ft*** of head laboratory facilities are available to provide ***25 ft*** of head and to absorb ***50 hp*** from the model turbine. Assume comparable efficiencies for the model and prototype turbines. Determine the appropriate model test speed, scale ratio, and volume flow rate.