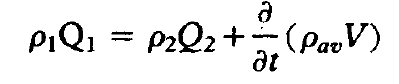
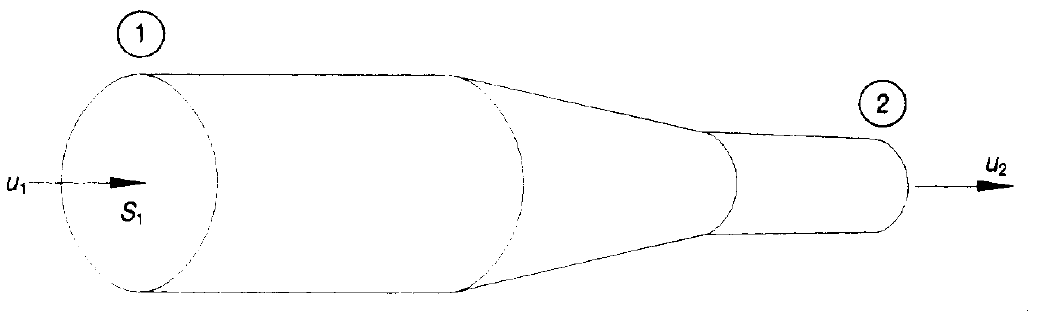
## Conservation of mass

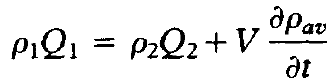
Consider flow through the pipe-work shown in Figure 1, in which the fluid occupies the whole cross section of the pipe. A mass balance can be written for the fixed section between planes 1 and 2, which are normal to the axis of the pipe. The mass flow rate across plane 1 into the section is equal to ρl Qland the mass flow rate across plane ***2*** out of the section is equal to ρ2 Q2***,*** where ***ρ*** denotes the density of the fluid and Qthe volumetric flow rate.

Thus, a mass balance can be written as

**Mass flow rate in = mass flow rate out + rate of accumulation within section**

 --------------------------------- 1.3

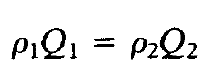


 -------------------------------- 1.4

Where Vis the constant volume of the section between planes 1 and 2, and **ρav** is the density of the fluid averaged over the volume V. This equation represents the conservation of mass of the flowing fluid: it is frequently called the ‘continuity equation’ and the concept of ‘continuity’ is synonymous with the principle of conservation of mass.

In the case of unsteady compressible flow, the density of the fluid in the section will change and consequently the accumulation term will be non-zero. However, for steady compressible flow the time derivative must be zero by definition. In the case of incompressible flow, the density is constant so the time derivative is zero even if the flow is unsteady.

Thus, for incompressible flow or steady compressible flow, there is no accumulation within the section and consequently equation 1.4 reduces to

---------------------------------- 1.5

This simply states that the mass flow rate into the section is equal to the mass flow rate out of the section. In general, the velocity of the fluid varies across the diameter of the pipe but an average velocity can be defined.

If the cross-sectional area of the pipe at a particular location is S, then the volumetric flow rate Q is given by

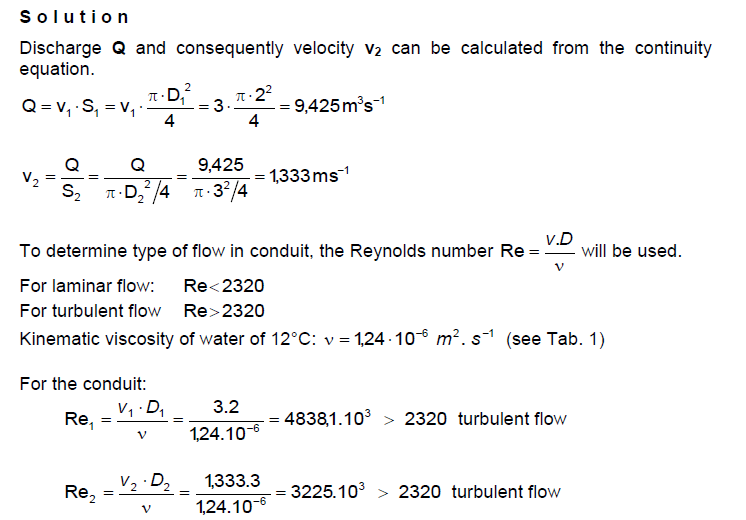
 ------------------ 1.6

Equation **1.6** defines the volumetric average velocity **u:** it is the uniform velocity required to give the volumetric flow rate Q through the flow area S. Substituting for **Q** in equation 1.5, the zero accumulation mass balance becomes

---------------------- 1.7

This is the form of the Continuity Equation that will be used most frequently but it is valid only when there is no accumulation. Although Figure 1.3 shows a pipe of circular cross section, equations 1.4 to 1.7 are valid for a cross section of any shape

## 



## Energy relationships and the Bernoulli equation

The total energy of a fluid in motion consists of the following components: internal, potential, pressure and kinetic energies. Each of these energies may be considered with reference to an arbitrary base level. It is also convenient to make calculations on unit mass of fluid.

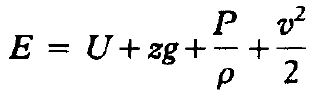
**Internal energy This** is the energy associated with the physical state of the fluid, ie, the energy of the atoms and molecules resulting from their motion and configuration**.** Internal energy is a function of temperature. The internal energy per unit mass of fluid is denoted by ***U.***

**Potential energy** this is the energy that a fluid has by virtue of its position in the Earth’s field of gravity. The work required to raise a unit mass of fluid to a height ***z*** above an arbitrarily chosen datum is *zg,* where g is the acceleration due to gravity. This work is equal to the potential energy of unit mass of fluid above the datum.

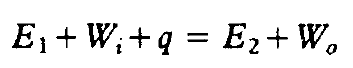
**Pressure energy** this is the energy or work required to introduce the fluid into the system without a change of volume. If *P* is the pressure and V is the volume of mass ***m*** of fluid, then ***PV/***mis the pressure energy per unit mass of fluid. The ratio m /Vis the fluid density ***ρ.*** Thus the pressure energy per unit mass of fluid is equal to ***P/ρ.***

**Kinetic energy** this is the energy of fluid motion. The kinetic energy of unit mass of the fluid is ***v***2***/2,*** where ***v*** is the velocity of the fluid relative to some fixed body.

**Total energy** Summing these components, the total energy E per unit mass of fluid is given by the equation

 --------------------------------- 1.8

Consider fluid flowing from point 1 to point 2 as shown in Figure **below** Between these two points, let the following amounts of heat transfer and work is done per unit mass of fluid: heat transfer **q**to the fluid, work **W**idone on the fluid and work ***W*º** done by the fluid on its surroundings. ***W*i**and **Wᵒ**may be thought of as work input and output. Assuming the conditions to be steady, so that there is no accumulation of energy within the fluid between points 1 and 2, an energy balance can be written per unit mass of fluid as

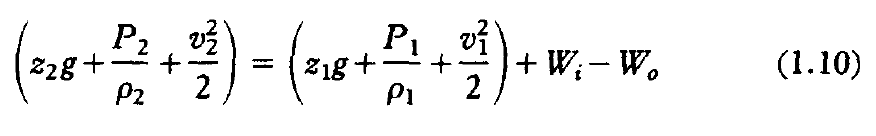


Or, after rearranging

----------------------------------------- 1.9

A flowing fluid is required to do work to overcome viscous frictional forces so that in practice the quantity **Wᵒ**is always positive. It is zero only for the theoretical case of an inviscid fluid or ideal fluid having zero viscosity. The work **Wi** may be done on the fluid by a pump situated between points 1 and 2.

If the fluid has a constant density or behaves **as** an ideal gas, then the internal energy remains constant if the temperature is constant. If no heat transfer to the fluid takes place, ***q*** ***=***0***.*** For these conditions, equations 1.8and 1.9 may be combined and written as



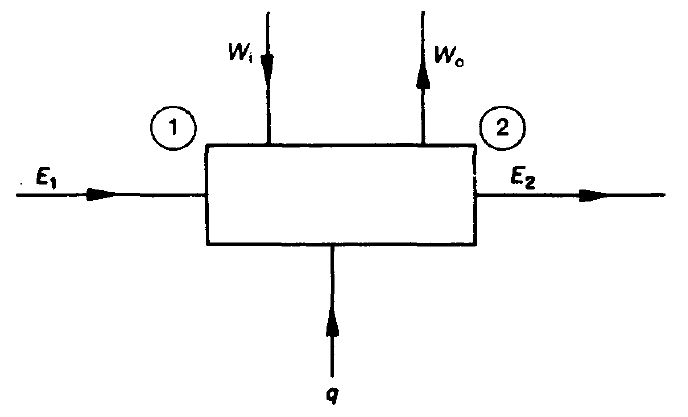
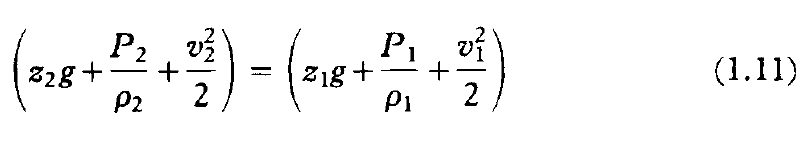
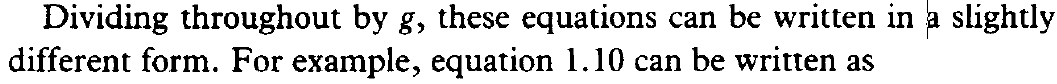


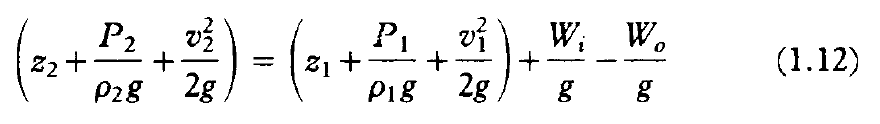
Figure 1

For an inviscid fluid, ie frictionless flow, and no pump, equation (1.10) becomes

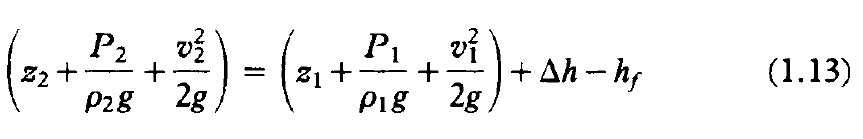


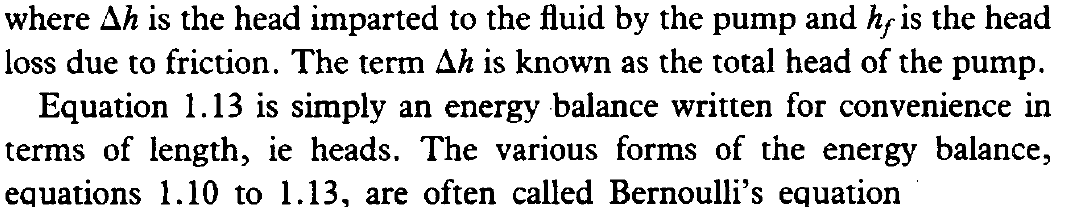
Equation 1.11 is known as Bernoulli’s equation.





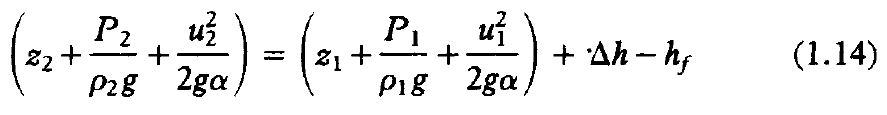
In this form, each term has the dimensions of length. The terms *z*,P/(ρg) and υ2/(2g) are known as the potential, pressure and velocity heads, respectively. Denoting the work terms as heads, equation 1.12 can also be written as

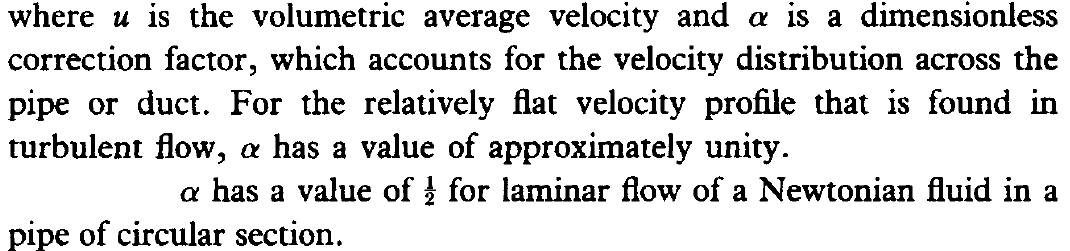




The various forms of energy are interchangeable and the equation enables these changes to be calculated in a given system. In deriving the form of Bernoulli’s equation without the work terms, it was assumed that the internal energy of the fluid remains constant. This is not the case when frictional dissipation occurs, ie. there is a head loss ***hf.*** In this case ***hf*** represents the conversion of mechanical energy into internal energy and, while internal energy can be recovered by heat transfer to a cooler medium, it cannot be converted into mechanical energy.

To enable Bernoulli’s equation to be used for the fluid flowing through the whole cross section of a pipe or duct, equation 1.13 can be modified as follows:





Consider the case of steady, fully developed flow of a liquid (incompressible) through an inclined pipe of constant diameter with no pump in the section considered. Bernoulli’s equation for the section between planes 1 and 2 shown in Figure 1.5 can be written as

## 

For the conditions specified, ***u1=u2,*** and ***α*** has the same value because the flow is fully developed. The terms in equation 1.15 are shown schematicalk ly in Figure 1.5. The total energy ***E2*** is less than ***El*** by the frictional losses ***hf***

## 

Figure 2

The velocity head remains constant as indicated and the potential head increases owing to the increase in elevation. As a result the pressure energy, and therefore the pressure, must decrease. It is important to note that **this** upward flow occurs because the upstream pressure P1is sufficiently high (compare the two pressure heads in Figure 2). This high pressure would normally be provided by a pump upstream of the section considered; however, as the pump is not in the section there must be no pump head term Ah in the equation. The effect of the pump is already manifest in the high pressure P1that it has generated.

The pressure drop P1 - *P2* experienced by the fluid in flowing from location 1 to location 2 is given by

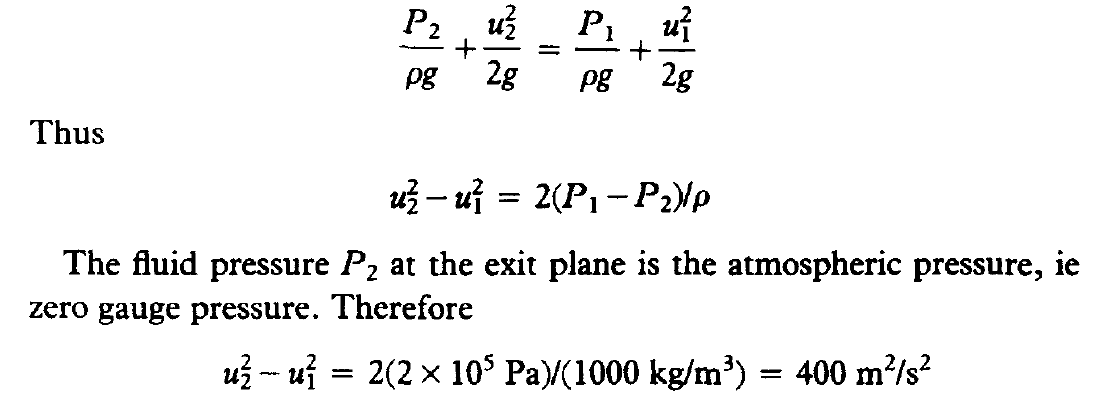


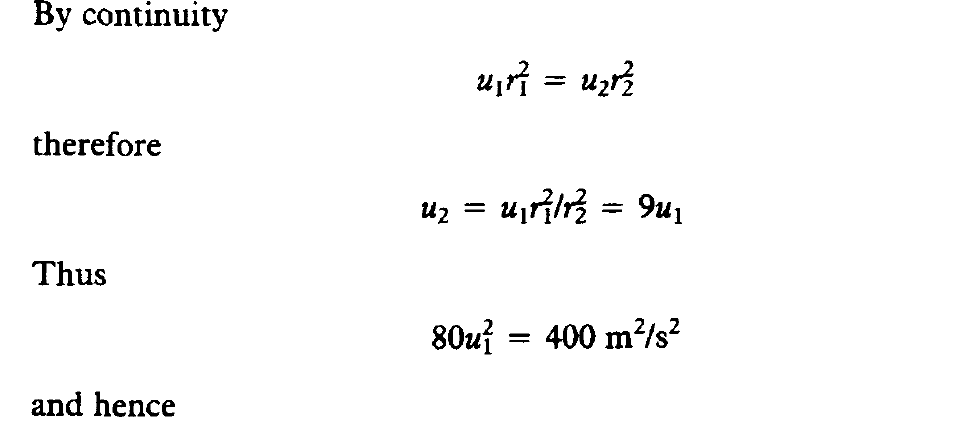
## Example 1.2

Water issues from the nozzle of a horizontal hose-pipe. The hose has an internal diameter of 60 mm and the nozzle tapers to an exit diameter of 20 mm. If the gauge pressure at the connection between the nozzle and the pipe is 200 Way what is the flow rate? The density of water is 1000 kg/m3.

## Solution

The nozzle is horizontal so z1 = z2 and for turbulent flow **α** = **1.0.** With these simplifications and the fact that there is no pump in the section, Bernoulli's equation reduces to







The volumetric discharge rate can be calculated from either velocity and the corresponding diameter. **Using** the values for the pipe

