**Power set**

The power set of some set S, denoted P(S), is the set of all subsets of S (including S itself and the empty set)

Example 1: Let A = { 1,2 3}

Power set of set A = P(A) = [{1},{2},{3},{1,2},{1,3},{2,3},{},A]

Example 2: P({0,1})={{},{0},{1},{0,1}}

**Classes of sets:** Collection of subset of a set with some properties

Example: Suppose A = { 1,2 3} , let X be the class of subsets of A which contain exactly two elements of A. Then

class X = [{1,2},{1,3},{2,3}]

### Cardinality

The cardinality of a set S, denoted **|S|**, is simply the number of elements a set has. So

**|**{a,b,c,d**}| =** 4, and so on. The cardinality of a set need not be finite: some sets have infinite cardinality.

### The cardinality of the power set

Theorem: If |A| = n then |P(A)| = 2n (Every set with n elements has 2n subsets)

**Problem set**

write the answers to the following questions. 1. |{1,2,3,4,5,6,7,8,9,0}|

2. |P({1,2,3})|

3. P({0,1,2})

4. P({1})

**Answers**

1. 10

2. 23=8

3. {{},{0},{1},{2},{0,1},{0,1,2},{0,2},{1,2}}

4. {{},{1}}

**The Cartesian product**

The Cartesian Product of two sets is the set of all tuples made from elements of two sets. We write the Cartesian Product of two sets A and B as A × B. It is defined as:

A \times B = \{(a,b)|a\in A\ \mbox{and}\ b\in B\}

It may be clearer to understand from examples;



Example: If A = {1, 2, 3} and B = {x, y} then

A . B = {(1, x), (1, y), (2, x), (2, y), (3, x), (3, y)}

B . A = {(x, 1), (x, 2), (x, 3), (y, 1), (y, 2), (y, 3)}

It is clear that, the cardinality of the Cartesian product of two sets A and B is:

|A\times B|=|A||B|

A Cartesian Product of two sets A and B can be produced by making tuples of each element of A with each element of B; this can be visualized as a grid (which *Cartesian* implies) or table: if, *e.g.*, A = { 0, 1 } and B = { 2, 3 }, the grid is

|  |  |  |  |
| --- | --- | --- | --- |
| **×** | | **A** | |
| **0** | **1** |
| **B** | **2** | (0,2) | (1,2) |
| **3** | (0,3) | (1,3) |

**Problem set**

Answer the following questions: 1. {2,3,4}×{1,3,4}

2. {0,1}×{0,1}

3. |{1,2,3}×{0}|

4. |{1,1}×{2,3,4}|

### Answers

1. {(2,1),(2,3),(2,4),(3,1),(3,3),(3,4),(4,1),(4,3),(4,4)}

2. {(0,0),(0,1),(1,0),(1,1)}

3. 3

4. 6

### Partitions of set:

Let S be any nonempty set. A partition (  ) of S is a subdivision of S into nonoverlapping, nonempty subsets. A partition of S is a collection {Ai} of non-empty subsets of S such that:

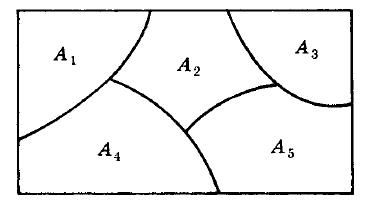
1) Ai , where i=1,2,3,……

2) The sets of {Ai } are mutually disjoint

or Ai ∩ Aj =  where i  j.

3) UAi = S, where A1  A2  ..................  Ai = S

The partition of a set into five cells, A1, A2,A3,A4,A5, can be represented by Venn diagram



Example 1: let A = {1,2,3,n}

A1 = {1}, A2 = {3,n}, A3 = {2}

∏ = {A1, A2, A3} is a partition on A because it satisfy the three above conditions.

**Example 2 :** Consider the following collections of subsets of S = {1,2,3,4,5,6,7,8,9} (i) [{1,3,5},{2,6},{4,8,9}]

(ii) [{1,3,5},{2,4,6,8},{5,7,9}]

(iii) [{1,3,5},{2,4,6,8},{7,9}]

Then

1. is not a partition of S since 7 in S does not belong to any of the subsets.
2. is not a partition of S since {1,3,5} and {5,7,9} are not disjoint.
3. is a partition of S.

##### FINITE SETS, COUNTING PRINCPLE:

A set is said to be finite if it contains exactly m distinct elements where m denotes some nonnegative integer. Otherwise, a set is said to be infinite. For example, the empty set  and the set of letters of English alphabet are finite sets, whereas the set of even positive integers, {2,4,6,…..}, is infinite.

If a set A is finite, we let *n*(A) or #(A) denote the number of elements of A.

Example: If A ={1,2,a,w} then

*n*(A) = #(A) = |A| = 4

Lemma: If A and B are finite sets and disjoint Then A   is finite set and:

##### *n*(A  B) = *n*(A) + *n*(B)

**Theorem (Inclusion–Exclusion Principle):** Suppose A and B are finite sets. Then A  B and A ∩ B are finite and

|A  B| = |A| + |B| - | A  B|

That is, we find the number of elements in A or B (or both) by first adding n(A) and n(B) (inclusion) and then subtracting n(A ∩ B) (exclusion) since its elements were counted twice.

We can apply this result to obtain a similar formula for three sets:

**Corollary:**

If A,B,C are finite sets then

|A  B  C | = |A| + |B| + |C| - | A  B| - |A  C| - |B  C| + |A  B  C|

##### Example (1) :

A= {1,2,3}

B= {3,4}

C= {5,6}

A  B  C = {1,2,3,4,5,6}

|A  B  C| = 6

|A| =3 , |B| = 2 , |C| = 2

  B = {3} , |   B | = 1

  C    , |   C | = 0

B  C = { } , |   C | = 0

  B  C = { } , |   B  C | = 0

|A  B  C | = |A| + |B| + |C| - | A  B| - |A  C| - |B  C| + |A  B  C|

|A  B  C | = 3 + 2 +2 -1 – 0 – 0 + 0 = 6

##### Example (2):

Suppose a list A contains the 30 students in a mathematics class, and a list B contains the 35 students in an English class, and suppose there are 20 names on both lists. Find the number of students:

* 1. only on list A
  2. only on list B
  3. on list A  B

Solution:

1. List A has 30 names and 20 are on list B; hence 30 − 20 = 10 names are only on list A.
2. Similarly, 35 − 20 = 15 are only on list B.
3. We seek n(A  B). By inclusion–exclusion,

n(A  B) = n(A) + n(B) − n(A ∩ B) = 30 + 35 − 20 = 45.

##### Example (3):

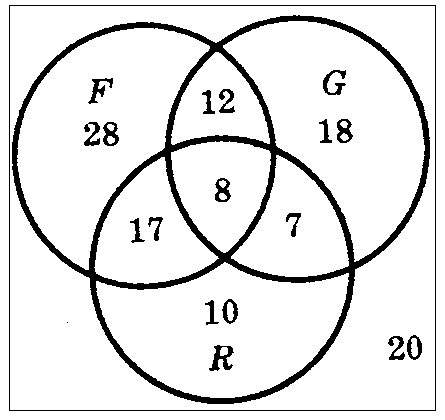
Suppose that 100 of 120 computer science students at a college take at least one of languages: French, German, and Russian and:

65 study French (F). 45 study German (G). 42 study Russian (R).

20 study French & German F  G. 25 study French & Russian F  R. 15 study German & Russian G  R.

Find the number of students who study:

1. All three languages ( F  G  R)
2. The number of students in each of the eight regions of the Venn diagram



Solution:

|F  G  R| = |F| + |G| + |R| - |F  G| - |F  R| - |G  R| + |F  G  R| 100 = 65 + 45 + 42 - 20 - 25 - 15 + |F  G  R|

100 = 92 + |F  G  R|

****|F  G  R| = 8 students study the 3 languages

20 – 8 = 12 (F  G) - R

25 – 8 = 17 (F  R) - G

15 – 8 = 7 (G  R) - F

65 – 12 – 8 – 17 = 28 students study French only

45 – 12 – 8 7 = 18 students study German only

42 – 17 – 8 7 = 10 students study Russian only

120 – 100 = 20 students do not study any language