**Linear Differential Equations in n th-order ;**

An *n*th-order linear differential equation has the form ;

…. (1)

Where the coefficients  **;(*j***=0,1,2,…,n-1) ,and depends solely on the variable (not on or any derivative of )

If then Eq.(1) is **homogenous** ,if not then is **nonhomogeneous** .

……(2)

A linear differential equation **has constant coefficients** if all the coefficients are constants ,if one or more is not constant then has **variable coefficients** .

Examples on linear differential equations ;

**first order nonhomogeneous**

**second order homogeneous**

**third order nonhomogeneous**

**fifth order homogeneous**

**second order nonhomogeneous**

**Theorem 1:** Consider the initial value problem given by the linear differential equation(1)and the n initial

Conditions;

…(3)

Define the differential operator ***L(y)*** by

…. (4)

Where  **;(*j***=0,1,2,…,n-1) is continuous on some interval of interest then

***L(y)*** =

and a linear homogeneous differential equation written as;

***L(y)*** =0

**Definition: Linearly Independent Solution**; A set of functions is linearly dependent on if there exists constants not all zero ,such that

…. (5)

on

A set of functions is linearly dependent if there exists another set of constants not all zero, that is satisfy eq.(5).

If the only solution to eq.(5) is ,then the set of functions is linearly independent on

The functions are linearly independent

**Theorem 2:** If the functions ***y1 ,y2, …, yn*** are solutions for differential equation and are arbitrary constants

then is a solution for homogeneous eq.(2)

i.e; The *n* th-order linear homogeneous differential equation always has n linearly independent solutions

**The Wronskian for differentiable functions :**

The Wronskian of a set of functions on the interval where each function has n-1 derivatives on this interval ,is the determinant;

**Theorem 3:** If the Wronskian of a set of n functions defined on the interval is non zero for at least one point in this interval ,then the set of functions is linearly independent .

i.e let  **;(*j***=0,1,2,…,n-1) is continuous on some interval then the solutions ***y1 ,y2, …, yn*** for the homogeneous differential equation are linearly independent on that interval iff ;

For all ***x*** in the interval

If the Wronskian is zero on this interval and if each of the functions is a solution to the same linear differential equation ,then the set of functions is linearly dependent.

The general solution for homogenous equation is a complementary function for nonhomogeneous equation .

**Theorem 4:** If ***u(x)*** is a particular solution for nonhomogeneous equation and ***v(x)*** is a general solution for homogeneous equation then

***y= u+v***

is a general solution for nonhomogeneous equation .

**Example** :Determine whether the set {***1-x , 1+x , 1-3x***} of the solutions to the differential equation is linearly dependent on (-∞,∞)

The Wronskian of this set as;

As in the theorem .Consider

Or as

This linear equation can be satisfied for all only if both coefficients are zero ;

Thus and by solution

Then , with arbitrary . If we chose (non zero number) ,then ;

, ,and

Thus the given set of functions is linearly dependent

**Example:** Find the general solution of if it known that two solutions are;

the Wronskian of the set as;

Which is non zero then the two solutions are linearly independent and then the general solution is ;

Example : Find on

Since

Then for on one of its variables:

For

Thus ;   
 on [-1,1]

Wronskian is in general a nonconstant function as in this example ;

**Example:** Find the Wronskian of the set {}

**Exercise :**

Find the Wronskian determinate ,and show if it are linear independent or not ;

**1.  *cosh ax , sinh ax***

**2. 1 ,**



**5.** 

**6. **

**7.  *x , 1 , 2x-7***