**Method of Solving Homogeneous Equations with Variable Coefficients;**

The homogeneous diff. eq. with form second order ;

….(1)

Where the coefficients  **;(*j***=0,1,2) are functions for ***x*** ,can be written as;

Where for all ***x*** in the interval ***I .***

Thus  **….(2)**

Where the functions are continuous of ***x*** in the interval ***I***

If is a solution for eq.(2),then we shall find a second solution such that are linearly independent on ***I*** .

Let

Where is an unknown function to be determined.

Thus

And

Are put in eq.(2) then

Since is solution of eq.(2) i.e satisfy the equation

By integrating ;

Thus

The solutions are linearly independent since the wronskian determinant is ;

Which is not zero in any interval .

Then the general solution for eq.(2) is ;

To find values for  either by trial and error or by these rules;

**Rule 1: If for some real number**

**Then**

**Rule 2: If for some real number**

**Then**

**Note ; If the sum of coefficients of *y'', y', y* is zero in the equation , Then the solution of**

**is**

Example 1; Solve the equation in any interval ***I*** not containing the origin .

The equation can formed as;

Since in ***I***

Here

And

Hence

And

Then the general solution is ;

Example 2; Solve the differential equation in the interval ***I=***(-1,1) .

Can rewritten as;

Then

And

For

So by Rule 1 ; is solution

Thus

Then the general solution is ;

Example 3; Solve the differential equation in the interval ***I=***(0,+∞) .

Can rewritten as;

So will be

here

Then

If

Thus is solution, and

Hence the general solution is;

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