**Solving of Nonhomogeneous Linear Differential Equations ;**

The nonhomogeneous diff. eq. with form second order ;

….(1)

Where and and are continuous functions for ***x*** on an interval ***I .***

To find the general solution for nonhomogeneous eq.(1) ,it is necessary to;

1. Find the general solution of the associated homogeneous equation

….(2)

as a complementary function

1. Add to the particular solution of eq.(1) .

Thus there are several methods to find of eq.(1)

**Variation of Parameters Method ;**

This method is powerful more than any other method ,it can be applied to all linear differential equations regardless of the nature of coefficients and the function (if zero or function ).

Since in ***I***

Or …..(3)

and …..(4)

Where  **,**

Let the complementary function for eq.(3) be;

Where are arbitrary constants and are two linearly independent solutions of

eq.(4)

Let the particular solution for eq.(3) be ;

Where are two twice-differentiable functions to determined.

This all since in are replaced by variables to get so it called **variation of parameters ,**

Since two functions are to be determined. There are two conditions

First : put value of in eq.(3) .

Second : Assume …(5)

Then

and

Put in eq.(3) ,then ;

…. (6)

Since are solutions for eq.(4) ,then

and

Consequently eq.(6) becomes ;

… (7)

Thus the two conditions (5), (7) to determine ,solve equations by **Cramer's rule** ,we get ;

….. (8)

…… (9)

By integration eqs.(8),(9) ,we obtain ,(integration constants are disregarded) .

Thus the general solution of eq.(3) is formed as;

… (10)

Example1: Solve

The associated homogeneous eq. is

The characteristic eq. is with the roots  **.**

The complementary function

So let ;

Putting

Then

Now

Substitute the values ;

Thus the two eqs. to determine are ;

Now by integration ;

Hence the general solution is

Example2: Solve the differential equation in (-1,1)

The complementary function

By integration

Thus the general solution is

**Exercise:**

1. Solve

sol;

1. Solve

sol;