Lecture 14

Linear programming: sensitivity analysis

1. Change Affecting optimality:

The current solution will be not optimal only if the coefficients of the objective equation violate the optimality condition.

* 1. Changes in the objective function:

1. If the changes in the objective function involve the coefficients of a current basic variable, determine the new dual values and then use them to recompute the new Z- equation coefficients.
2. If the changes involve non basic variables only, use the current dual values (directly from the current tableau) and recomputed the Z-equation coefficients of the involved non basic variables only. No other changes will occur in the tableau.

**Example(3):** Suppose that the objective function in example (1) is changed from Z=3x1 +2x2 to Z=5x1 +4x2

the primal model:

Max Z=5x1+4x2

s.t x1+2x26

2x1+2x28

-x1+x21

x22

x1,x2 0

The dual model:

Min W=6y1+8y2+y3+2y4

s.t y1+2y2 –y35

2y1+y2+y3+y44

y1 ,y2 ,y3 ,y40

1

The changes involve both x1 and x2 which happen to be basic in the current solution. Thus we must determine the new dual values. Notice that the order of the basic variables in the current tableau is(x2,x1,S3,S4)

(y1 ,y2 ,y3,y4)=(4,5,0,0)\* =(1,2,0,0)

|  |  |  |  |
| --- | --- | --- | --- |
| 2/3 | -1/3 | 0 | 0 |
| -1/3 | 2/3 | 0 | 0 |
| -1 | 1 | 1 | 0 |
| -2/3 | 1/3 | 0 | 1 |

The next step is to recompute the Z-equation coefficients by taking the difference between the left and right sides of the dual constraints.

x1 – coefficient =y1+2y2–y3-5

=1(1) +2(2)-0-5=0

x2 – coefficient =2y1+y2+y3+y4-4

=2(1) +2+0+0-4=0

s1 – coefficient =y1- 0 =1-0 =1

s2 – coefficient =y2-0 =2-0 =2

s3 – coefficient =y3-0 =0-0 =0

s4 – coefficient =y4-0 =0-0 =0

Z-Cj >= 0, the changes indicated in the objective function will not change the optimum variables or their values. The only change is the value of Z=5\*(10/3)+4\*(4/3)=22 ,where(x1=10/3,x2=4/3 in the primal optimal tableau in example (1) ).

2

**Example(4):** Suppose that the objective function change from: Z=3x1+2x2 to Z=4x1+x2

|  |  |  |  |
| --- | --- | --- | --- |
| 2/3 | -1/3 | 0 | 0 |
| -1/3 | 2/3 | 0 | 0 |
| -1 | 1 | 1 | 0 |
| -2/3 | 1/3 | 0 | 1 |

( y1 ,y2 ,y3,y4)=(1,4,0,0) \*

=(-2/3,7/3,0,0)

Since (S1 ) has a negative coefficient ,S1 must enter the solution and optimality is recovered by applying the regular simplex method.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **B.V** | **X1** | **X2** | **S1** | **S2** | **S3** | **S4** | **R.H.S** |
| **Z** | 0 | 0 | -2/3 | 7/3 | 0 | 0 | 44/3 |
| **X2** | 0 | 1 | 2/3 | -1/3 | 0 | 0 | 4/3 |
| **X1** | 1 | 0 | -1/3 | 2/3 | 0 | 0 | 10/3 |
| **S3** | 0 | 0 | -1 | 1 | 1 | 0 | 3 |
| **S4** | 0 | 0 | -2/3 | 1/3 | 0 | 1 | 2/3 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **B.V** | **X1** | **X2** | **S1** | **S2** | **S3** | **S4** | **R.H.S** |
| **Z** | 0 | 1 | 0 | 2 | 0 | 0 | 16 |
| **S1** | 0 | 3/2 | 1 | -1/2 | 0 | 0 | 2 |
| **X1** | 1 | 1/2 | 0 | 1/2 | 0 | 0 | 4 |
| **S3** | 0 | 3/2 | 0 | 1/2 | 1 | 0 | 5 |
| **S4** | 0 | 1 | 0 | 0 | 0 | 1 | 2 |

Since Z-Cj>=0, (R.H.S +v), the optimal solution is : Z=16, x1=4, x2=0 And the dual (y1 ,y2 ,y3,y4)=(0,2,0,0) (row of objective function)

3

* 1. Changes in activity’s usage of Resources (aij):

A change in an activity’s usage of resources (aij) can affect only the optimality of the solution; since it affects the left side of its dual constraint .However we must restrict this statement to activities that are currently non basic. A change in the constraint coefficients of the basic activities will affect the inverse and could lead to complications in the computations

**Example:** Suppose that the activity (x2) of the first and second constraints is

changed from 2

1

to 4

3

f

in the following problem:

Primal

Max Z=4x1+x2

s.t x1+2x26

2x1+x28

-x1+x21

x22

x1,x2 0

New primal Max Z=4x1+x2

s.t x1+4x26

2x1+3x28

-x1+x21

x22

x1,x2 0

Dual

Min W=6y1+8y2+y3+2y4

s.t y1+2y2 –y34

4y1+3y2+y3+y41

y1 ,y2 ,y3 ,y40

The associated dual constraint is 4y1 +3y2 +y3 +y41 since the objective function remains unchanged, the dual values in the primal optimal tableau in example(1) is:

(0, 2,0,0)= (y1 ,y2 ,y3 ,y4)

New x2 –coefficient=4(0)+3(2)+1(0)+1(0)-1=5 0

Hence the proposed change does not affect the optimum solution. 4

* 1. Addition of a new Activity (xi):

We can think of adding a new activity as a non basic activity that started originally in the model with all zero coefficients in the objective and constraints.

**Example**: suppose the original problem:

Max Z=3x1+2x2

s.t x1+2x26 2x1+x28

-x1+x21

x22 x1,x2 0

And the primal optimal tableau is:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **B.V** | **X1** | **X2** | **S1** | **S2** | **S3** | **S4** | **R.H.S** |
| **Z** | 0 | 0 | 1/3 | 4/3 | 0 | 0 | 38/3 |
| **X2** | 0 | 1 | 2/3 | -1/3 | 0 | 0 | 4/3 |
| **X1** | 1 | 0 | -1/3 | 2/3 | 0 | 0 | 10/3 |
| **S3** | 0 | 0 | -1 | 1 | 1 | 0 | 3 |
| **S4** | 0 | 0 | -2/3 | 1/3 | 0 | 1 | 2/3 |

Let we add the new activity x3 to the objective function and the constraints as follows:

Max Z=3x1+2x2+(3/2)x3

s.t

x1+2x2+(3/4)x36

2x1+x2+(3/4)x38

-x1+x2-x31

x22

x1,x2 x30

5

The addition of a new activity is equivalent to combining the analysis of making changes in the objective and the resource usages the first thing to do is to check the corresponding dual constraint:

(3/4)y1 + (3/4)y2 –y3 3/2

Since x3 is regarded as a non basic variable in the original tableau, the dual values remain unchanged, thus the coefficients of x3 in the current optimal tableau is:

(3/4)(1/3)+(3/4)(4/3)-1(0)-3/2= -1/4 coefficient of x3 in Z-equation

Where (y1 ,y2 ,y3)=(1/3 ,4/3 ,0) in the optimal tableau in example (1)

Then we find the coefficients of x3 in the rows of tableau (in constraints) as follows:

3/2 -1/3 0 0

-1/3 2/3 0 0

-1 1 1 0

-2/3 1/3 0 1

3/4

\* =

3/4

-1

0

61/4

1/4

-1

-1/4

Since the coefficient of x3 is negative (-1/4) ,we use simplex tableau to find the optimal solution as follows:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B.V.** | **X1** | **X2** | **X3** | **S1** | **S2** | **S3** | **S4** | **R.H.S** |
| **Z** | 0 | 0 | -1/4 | 1/3 | 4/3 | 0 | 0 | 38/3 |
| **X2** | 0 | 1 | 1/4 | 2/3 | -1/3 | 0 | 0 | 4/3 |
| **X1** | 1 | 0 | 1/4 | -1/3 | 2/3 | 0 | 0 | 10/3 |
| **S3** | 0 | 0 | -1 | -1 | 1 | 1 | 0 | 3 |
| **S4** | 0 | 0 | -1/4 | -2/3 | 1/3 | 0 | 1 | 2/3 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **B.V.** | **X1** | **X2** | **X3** | **S1** | **S2** | **S3** | **S4** | **R.H.S** |
| **Z** | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 14 |
| **X3** | 0 | 4 | 1 | 8/3 | -4/3 | 0 | 0 | 16/3 |
| **X1** | 1 | -1 | 0 | -1 | 1 | 0 | 0 | 2 |
| **S3** | 0 | 4 | 0 | 5/3 | -1/3 | 1 | 0 | 25/3 |
| **S4** | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 2 |

The new optimal solution is: Z =14, x1=2, x2=0, x3=16/3

7