Lecture 16

# Transportation problem :

*Methods for Initial Basic Feasible Solution*

*( North - West corner rule and matrix minimum method )*

## 16.1 Methods for Initial Basic Feasible Solution

Some simple methods to obtain the initial basic feasible solution are

* + 1. North-West Corner Rule
    2. Lowest Cost Entry Method (Matrix Minima Method)
    3. Vogel’s Approximation Method (Unit Cost Penalty Method)

## North-West Corner Rule

### Step 1

* + The first assignment is made in the cell occupying the upper left-hand (north-west) corner of the table.
  + The maximum possible amount is allocated here i.e. x11 = min (a1, b1). This value of x11 is then entered in the cell (1,1) of the transportation table.

### Step 2

1. If b1 > a1, move vertically downwards to the second row and make the second allocation of amount x21 = min (a2, b1 - x11) in the cell (2, 1).
2. If b1 < a1, move horizontally right side to the second column and make the second allocation of amount x12 = min (a1 - x11, b2) in the cell (1, 2).
3. If b1 = a1, there is tie for the second allocation. One can make a second allocation of magnitude x12 = min (a1 - a1, b2) in the cell (1, 2) or x21 = min (a2, b1 - b1) in the cell (2, 1)

### Step 3

Start from the new north-west corner of the transportation table and repeat steps 1 and 2 until all the requirements are satisfied.

### Find the initial basic feasible solution by using North-West Corner Rule

1.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| W→  F  ↓ | W1 | W2 | W3 | W4 | Factory Capacity |
| F1 | 19 | 30 | 50 | 10 | 7 |
| F2 | 70 | 30 | 40 | 60 | 9 |
| F3 | 40 | 8 | 70 | 20 | 18 |
| Warehouse Requirement | 5 | 8 | 7 | 14 | 34 |

1

### Solution

W1 W2 W3 W5 Availability

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 5 |  | 2 |  |  | |  | |
|  | (19) |  | (30) |
|  | | 6 |  | 3 |  |  | |
|  | (30) |  | (40) |
|  | |  | | 4 |  | 14 | |
|  | (70) |  | (20) |

F1 7 2 0



F2 9 3 0

F3 18 14 0

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | 5 | 8 | 7 | 14 |
| Requirement | 0 | 6 | 4 | 0 |
|  |  | 0 | 0 |  |

Initial Basic Feasible Solution

x11 = 5, x12 = 2, x22 = 6, x23 = 3, x33 = 4, x34 = 14

The transportation cost is 5 (19) + 2 (30) + 6 (30) + 3 (40) + 4 (70) + 14 (20) = Rs. 1015

2.

D1 D2 D3 D4 Supply O1 34

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 5 | 3 | 3 |  |
| 3 | 3 | 1 | 2 |
| 0 | 2 | 2 | 3 |
| 2 | 7 | 2 | 4 |

O2 15

O3 12

O4 19

Demand 21 25 17 17 80

### Solution

D1 D2 D3 D4 Supply

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 21 | 13 | |  |  | |
| (1) |  | (5) |
|  | 12 | | 3 |  | |
|  | (3) | (1) |
|  |  | | 12  (2) |  | |
|  |  | | 2 | 17 | |
| (2) |  | (4) |

O1 34 13 0



O2 15 3 0

O3 12 0

O4

Demand 21 25

0 12

0

17 17

14 0

2

0

19 17

2

Initial Basic Feasible Solution

x11 = 21, x12 = 13, x22 = 12, x23 = 3, x33 = 12, x43 = 2, x44 = 17

The transportation cost is 21 (1) + 13 (5) + 12 (3) + 3 (1) + 12 (2) + 2 (2) + 17 (4) = Rs. 221

3.

From To Supply

4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 2 | 11 | 10 | 3 | 7 |
| 1 | 4 | 7 | 2 | 1 |
| 3 | 1 | 4 | 8 | 12 |

8

9

Demand 3 3 4 5 6

### Solution

From To Supply

4 1 0



|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 3 |  | 1 |  |  | |  | |  | |
|  | (2) |  | (11) |
|  | | 2 |  | 4 |  | 2 |  |  | |
|  | (4) |  | (7) |  | (2) |
|  | |  | |  | | 3 |  | 6 |  |
|  | (8) |  | (12) |

8 6 2 0

3 3 4 5 6

Demand 0 2 0 3 0

0 0

9 6 0

Initial Basic Feasible Solution

x11 = 3, x12 = 1, x22 = 2, x23 = 4, x24 = 2, x34 = 3, x35 = 6

The transportation cost is 3 (2) + 1 (11) + 2 (4) + 4 (7) + 2 (2) + 3 (8) + 6 (12) = Rs. 153

3

## 2 - Lowest Cost Entry Method (Matrix Minima Method)

### Step 1

Determine the smallest cost in the cost matrix of the transportation table. Allocate xij = min (ai, bj) in the cell (i, j)

### Step 2

* If xij = ai, cross out the ith row of the table and decrease bj by ai. Go to step 3.
* If xij = bj, cross out the jth column of the table and decrease ai by bj. Go to step 3.
* If xij = ai = bj, cross out the ith row or jth column but not both.

### Step 3

Repeat steps 1 and 2 for the resulting reduced transportation table until all the requirements are satisfied. Whenever the minimum cost is not unique, make an arbitrary choice among the minima.

### Find the initial basic feasible solution using Matrix Minima method

1.

F1 F2 F3

Requirement 5 8 7 14

Availability 7

9

|  |  |  |  |
| --- | --- | --- | --- |
| W1 | W2 | W3 | W4 |
| 19 | 30 | 50 | 10 |
| 70 | 30 | 40 | 60 |
| 40 | 8 | 70 | 20 |

18

### Solution

W1 W2 W3 W4

|  |  |  |  |
| --- | --- | --- | --- |
| (19) | (30) | (50) | (10) |
| (70) | (30) | (40) | (60) |
| (40) | 8 | (70) | (20) |
| (8) |

F1 7

F2 9

F3 10

5 X 7 14

W1 W2 W3 W4

|  |  |  |  |
| --- | --- | --- | --- |
| (19) | (30) | (50) | 7  (10) |
| (70) | (30) | (40) | (60) |
| (40) | 8  (8) | (70) | (20) |

F1 X

F2 9

F3 10

5 X 7 7

W1 W2 W3 W4

|  |  |  |  |
| --- | --- | --- | --- |
| (19) | (30) | (50) | 7  (10) |
| (70) | (30) | (40) | (60) |
| (40) | 8  (8) | (70) | 7  (20) |

F1 X

F2 9

F3 3

5 X 7 X

W1 W2 W3 W4

|  |  |  |  |
| --- | --- | --- | --- |
| (19) | (30) | (50) | 7  (10) |
| (70) | (30) | (40) | (60) |
| 3  (40) | 8  (8) | (70) | 7  (20) |

F1 X

F2 9

F3 X

2 X 7 X

W1 W2 W3 W4

|  |  |  |  |
| --- | --- | --- | --- |
| (19) | (30) | (50) | 7  (10) |
| 2  (70) | (30) | 7  (40) | (60) |
| 3  (40) | 8  (8) | (70) | 7  (20) |

F1 X

F2 X

F3 X

X X X X

Initial Basic Feasible Solution

x14 = 7, x21 = 2, x23 = 7, x31 = 3, x32 = 8, x34 = 7

The transportation cost is 7 (10) + 2 (70) + 7 (40) + 3 (40) + 8 (8) + 7 (20) = Rs. 814

2.

From

To Availability

4

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 2 | 11 | 10 | 3 | 7 |
| 1 | 4 | 7 | 2 | 1 |
| 3 | 9 | 4 | 8 | 12 |

8

9

Requirement 3 3 4 5 6

### Solution

To

From

3 3 4

0 0 0

4 0

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  | 4 |  |
| (3) |
| 3 |  |  |  | 5 |
| (1) | (1) |
|  | 3 | 4 | 1 | 1 |
| (9) | (4) | (8) | (12) |

8 5 0

9 5 4 1 0

5 6

1 1

0 0

Initial Basic Feasible Solution

x14 = 4, x21 = 3, x25 = 5, x32 = 3, x33 = 4, x34 = 1, x35 = 1

The transportation cost is 4 (3) + 3 (1) + 5(1) + 3 (9) + 4 (4) + 1 (8) + 1 (12) = Rs. 78