**FUNCTIONS OF TWO OR MORE VARIABLES**

For us the distinction for functions of two or more variables is that the domain is a set of n-tuples of numbers. The range remains one dimensional and is referred to an interval of numbers. If n ¼ 2, the domain is pictured as a two-dimensional region. The region is referred to a rectangular Cartesian coordinate system described through number pairs ًx; yق, and the range variable is usually denoted by z. The domain variables are independent while the range variable is dependent.

We use the notation f ًx; yق, Fًx; yق, etc., to denote the value of the function at ًx; yق and write z ¼ f ًx; yق, z ¼ Fًx; yق, etc. We shall also sometimes use the notation z ¼ zًx; yق although it should be understood that in this case z is used in two senses, namely as a function and as a variable.

EXAMPLE. If f ًx; yق ¼ x2 ‏ 2y3, then f ً3;\_1ق ¼ ً3ق2 ‏ 2ً\_1ق3 ¼ 7:

The concept is easily extended. Thus w ¼ Fًx; y; zق denotes the value of a function at [a point in three-dimensional space], etc., the domain for which z is real consists of the set of points ًx; yق such that x2 ‏ y2 @ 1, i.e., the set of points inside and on a circle in the xy plane having center at ً0; 0ق and radius 1.

THREE-DIMENSIONAL RECTANGULAR COORDINATE SYSTEMS

A three-dimensional rectangular coordinate system, as referred to in the previous paragraph, obtained by constructing three mutually perpendicular axes (the x-, y-, and z-axes) intersecting in point O (the origin). It forms a natural extension of the usual xy plane for representing functions of two variables graphically. A point in three dimensions is represented by the triplet ًx; y; zق called coordinates of the point. In this coordinate system z ¼ f ًx; yق [or Fًx; y; zق ¼ 0] represents a surface, in general.

comprises the surface of a hemisphere of radius

1 and center at For functions of more than two variables such geometric interpretation fails, although the terminology is still employed. For example, ًx; y; z; wق is a point in four-dimensional space, and w ¼ f ًx; y; zق [or Fًx; y; z; wق ¼ 0] represents a hypersurface in four dimensions; thus x2 ‏ y2 ‏ z2 ‏ w2 ¼ a2 represents a hypersphere in four dimensions with radius a > 0 and center at describes a function generated from the hypersphere.



**REGIONS**

A point P belonging to a point set S is called an interior point of S if there exists a deleted neighborhood of P all of whose points belong to S. Apoint P not belonging to S is called an exterior point of S if there exists a deleted \_ neighborhood of P all of whose points do not belong to S. Apoint P is called a boundary point of S if every deleted \_ neighborhood of P contains points belonging to S and also points not belonging to S.

If any two points of a set S can be joined by a path consisting of a finite number of broken line segments all of whose points belong to S, then S is called a connected set. Aregion is a connected set which consists of interior points or interior and boundary points. A closed region is a region containing all its boundary points. An open region consists only of interior points. The complement of a set, S, in the x\_y plane is the set of all points in the plane not belonging to S. (See Fig. 6-2.)

Examples of some regions are shown graphically in Figs 6-3(a), (b), and (c) below. The rectangular region of Fig. 6-1(a), including the boundary, represents the sets of points a @ x @ b, c @ y @ d which is a natural extension of the closed interval a @ x @ b for one dimension. The set a < x < b, c < y < d corresponds to the boundary being excluded.

In the regions of Figs 6-3(a) and 6-3(b), any simple closed curve (one which does not intersect itself anywhere) lying inside the region can be shrunk to a point which also lies in the region. Such regions are called simply-connected regions. InFig. 6-3(c) however, a simple closed curve ABCD surrounding one of the ‘‘holes’’ in the region cannot be shrunk to a point without leaving the region. Such regions are called multiply-connected regions.