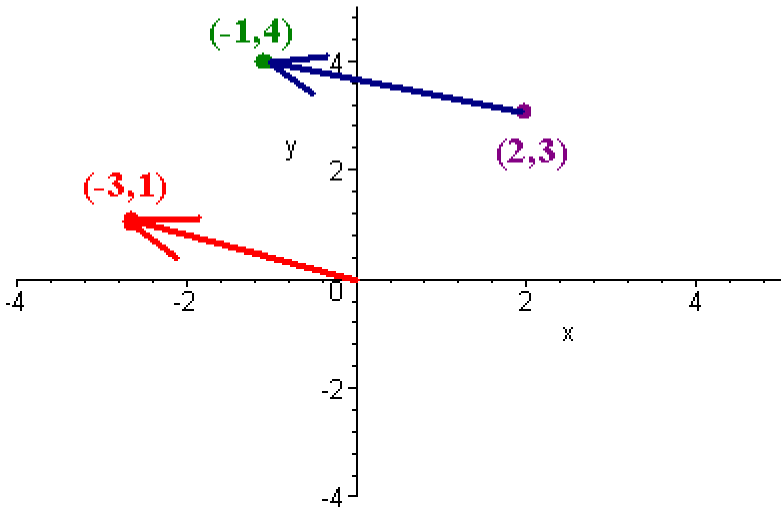
VECTORS



Directed Line Segments and Vectors:

Definition : A directed line segment is defined as an initial point P , and a terminal point Q.

Example : **P = (2,3) and Q = (-1,4)**

Definition of a Vector :

A vector is the equivalence class of all directed segments of the same length and direction.

We can represent a vector by writing the unique directed line segment that has its initial point at the origin.

Example : P = (2,3) and Q = (-1,4) is equivalent to the directed line segment

"Q - P" = <-3, 1>

When we write the < -3,1 > we mean that the vector has initial point at the origin and terminal point at (-3,1). This notation is called the *component* form of the vector.

The *length* of the vector < x , y > is called the *norm or magnitude*. (طول، معيار،مرتبة)

Length of a Vector

$$\|\langle x, y\rangle\| = \sqrt{x^2 + y^2}$$

Example: $\|\langle 3,1\rangle\| = \sqrt{3^2+1^2} = \sqrt{10}$

NOTE : We also can use the notation -3i + j to denote the vector <-3,1>.

Example :

A vector that has length 10 and makes an angle of x/6 with the x-axis. Find its components.

Solution: X = r cos x , Y = r sin x

So that : X = (10)($\sqrt{3}$ / 2) = 5 $\sqrt{3}$, Y = 10 (1/2) = 5

We can write the vector as 5 $\sqrt{3}$ i + 5 j

Unit Vectors in the Direction of v :

Definition : A vector is called a *unit vector* (متجه الوحدة) if it has magnitude = 1 .

If v = < a , b > then the unit vector in the direction of v can be found by :

Unit Vector in the Direction of v

$$u = \frac{1}{\| v \|} v$$

Example: **The unit vector in the direction of < -3 , 1 > is** $\left\langle -\frac{3}{\sqrt{10}}, \frac{1}{\sqrt{10}} \right\rangle$

NOTE : We can use the < > notation and the i j notation interchangeably.

Algebra Of Vectors :

Definition : If v = < a , b > and w = < c , d > and k is a constant, then we can define the sum as : v + w = <a + c,b + d>

and scalar multiplication as : kv = < ka , kb >

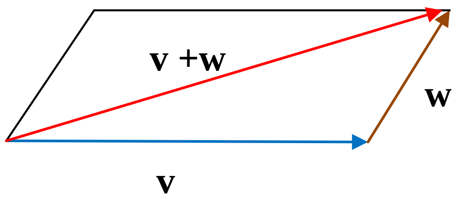
Example : Find the value of : 3 < 2 , 1 > – 2 < – 1 , 3 >

SOL : $3 < 2, 1 > - 2 < -1, 3 > = < 6, 3 > + < 2, -6 >$

$= < 6 + 2, 3 - 6 >$

$= < 8, -3 > = 8i - 3j$

NOTE : Geometrically : v + w is the vector that corresponds to the diagonal of the parallelogram with two sides v and w.



NOTE : The appropriate diagram can also be drawn to show how

v - w = v + (-w).

Properties of Vector Addition and Subtraction :

We have the following four properties of vectors: If u , v , and w are vectors , and a , b are numbers then :

1. **(u + v) + w = u + (v + w)**
2. **a (u + v) = au + av**

3. $\mathbf{a\ (bv) = (ab)\ v}$

4. $\mathbf{u + v = v + u}$