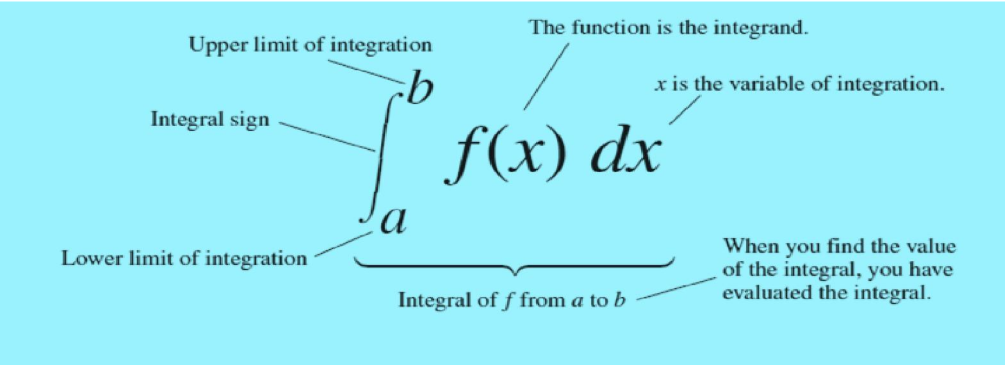
DEFINITION :- The Definite Integral

The symbol for the number I in the definition of the definite integral is

$$\int_a^b f(x)\,dx$$

which is read as "the integral from a to b of f of x dee x" or sometimes as "the ntegral from a to b of f of x with respect to x." The component parts in the integral symbol also have names:



Rules satisfied by definite integrals :-

1. *Order of Integration:* $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$ — A Definition

2. *Zero Width Interval:* $\int_a^a f(x)\,dx = 0$ — Also a Definition

3. *Constant Multiple:* $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ — Any Number k

   $\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$ — $k = -1$

4. *Sum and Difference:* $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. *Additivity:* $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. *Max-Min Inequality:* If f has maximum value max f and minimum value min f on $[a, b]$, then

$$\min f \cdot (b-a) \le \int_a^b f(x)\,dx \le \max f \cdot (b-a).$$

7. *Domination:* $f(x) \ge g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \ge \int_a^b g(x)\,dx$

   $f(x) \ge 0$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \ge 0$ (Special Case)

EXAMPLE 1:- Using the Rules for Definite Integrals

Suppose that

$$\int_{-1}^{1} f(x)\,dx = 5, \qquad \int_{1}^{4} f(x)\,dx = -2, \qquad \int_{-1}^{1} h(x)\,dx = 7.$$

Then

1. $\int_{4}^{1} f(x)\,dx = -\int_{1}^{4} f(x)\,dx = -(-2) = 2$ — Rule 1

2. $\int_{-1}^{1} [2f(x) + 3h(x)]\,dx = 2\int_{-1}^{1} f(x)\,dx + 3\int_{-1}^{1} h(x)\,dx$ — Rules 3 and 4

   $= 2(5) + 3(7) = 31$

3. $\int_{-1}^{4} f(x)\,dx = \int_{-1}^{1} f(x)\,dx + \int_{1}^{4} f(x)\,dx = 5 + (-2) = 3$ — Rule 5

DEFINITION:- Area Under a Curve as a Definite Integral

If $y = f(x)$ is nonnegative and integrable over a closed interval $[a, b]$, then the **area under the curve $y = f(x)$ over $[a, b]$** is the integral of f from a to b,

$$A = \int_{a}^{b} f(x)\,dx.$$

Examples :- Using Properties and Known Values to Find Other Integrals

Suppose that f and g are integrable and that

$$\int_{1}^{2} f(x)\,dx = -4, \int_{1}^{5} f(x)\,dx = 6, \int_{1}^{5} g(x)\,dx = 8.$$

a. $\int_{2}^{2} g(x)\,dx$ **b.** $\int_{5}^{1} g(x)\,dx$

c. $\int_{1}^{2} 3f(x)\,dx$ **d.** $\int_{2}^{5} f(x)\,dx$

e. $\int_{1}^{5} [f(x) - g(x)]\,dx$ **f.** $\int_{1}^{5} [4f(x) - g(x)]\,dx$

Solution:-

(a) $\int_2^2 g(x)\,dx = 0$

(b) $\int_5^1 g(x)\,dx = -\int_1^5 g(x)\,dx = -8$

(c) $\int_1^2 3f(x)\,dx = 3\int_1^2 f(x)\,dx = 3(-4) = -12$

(d) $\int_2^5 f(x)\,dx = \int_1^5 f(x)\,dx - \int_1^2 f(x)\,dx = 6 - (-4) = 10$

(e) $\int_1^5 [f(x) - g(x)]\,dx = \int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = 6 - 8 = -2$

(f) $\int_1^5 [4f(x) - g(x)]\,dx = 4\int_1^5 f(x)\,dx - \int_1^5 g(x)\,dx = 4(6) - 8 = 16$

Example : find the area bounded the curve $y = x^2 + 2$ and the line x=4 and x-axis

Solution :-

$$A = \int_0^4 y\ dx = \int_0^4 (x^2 + 2)dx = \left[\frac{x^3}{3} + 2x\right]_0^4 = 264$$