EX :- Find the average value of the following function over it's interval

1- $f(x) = x^2 - 1 \quad \text{on} \quad [0, \sqrt{3}]$

2- $f(x) = -\frac{x^2}{2} \quad \text{on} \quad [0, 3] \qquad$ **57.** $f(x) = -3x^2 - 1 \quad \text{on} \quad [0, 1]$

3- $f(x) = 3x^2 - 3 \quad \text{on} \quad [0, 1]$

4- $f(t) = (t - 1)^2 \quad \text{on} \quad [0, 3]$

5- $f(t) = t^2 - t \quad \text{on} \quad [-2, 1]$

6- $g(x) = |x| - 1 \quad \text{on} \quad$ **a.** $[-1, 1]$, **b.** $[1, 3]$, and **c.** $[-1, 3]$

7- $h(x) = -|x| \quad \text{on} \quad$ **a.** $[-1, 0]$, **b.** $[0, 1]$, and **c.** $[-1, 1]$

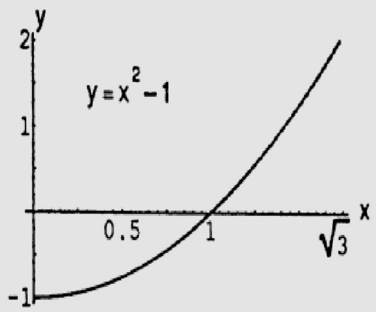
SOL :

1-

$$\text{av}(f) = \left(\frac{1}{\sqrt{3}-0}\right) \int_0^{\sqrt{3}} (x^2 - 1)\,dx$$

$$= \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} x^2\,dx - \frac{1}{\sqrt{3}} \int_0^{\sqrt{3}} 1\,dx$$

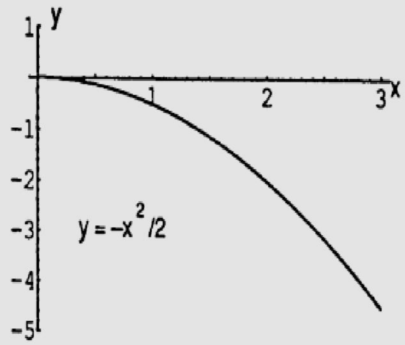
$$= \frac{1}{\sqrt{3}} \left(\frac{(\sqrt{3})^3}{3}\right) - \frac{1}{\sqrt{3}} \left(\sqrt{3} - 0\right) = 1 - 1 = 0.$$



2-

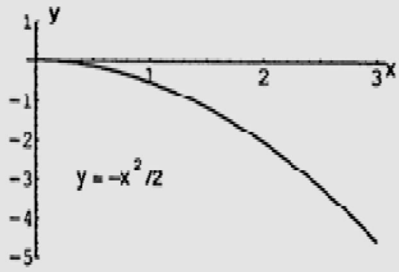
$$\text{av}(f) = \left(\frac{1}{3-0}\right) \int_0^3 \left(-\frac{x^2}{2}\right) dx = \frac{1}{3}\left(-\frac{1}{2}\right) \int_0^3 x^2\,dx$$

$$= -\frac{1}{6}\left(\frac{3^3}{3}\right) = -\frac{3}{2};\ -\frac{x^2}{2} = -\frac{3}{2}.$$



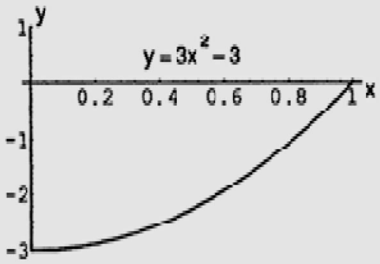
3-

$$\text{av}(f) = \left(\tfrac{1}{1-0}\right)\int_0^1 (-3x^2 - 1)\,dx =$$
$$= -3\int_0^1 x^2\,dx - \int_0^1 1\,dx = -3\left(\tfrac{1^3}{3}\right) - (1-0)$$
$$= -2.$$



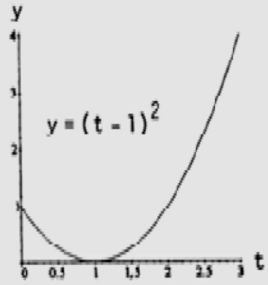
4-

$$\text{av}(f) = \left(\tfrac{1}{1-0}\right)\int_0^1 (3x^2 - 3)\,dx =$$
$$= 3\int_0^1 x^2\,dx - \int_0^1 3\,dx = 3\left(\tfrac{1^3}{3}\right) - 3(1-0)$$
$$= -2.$$



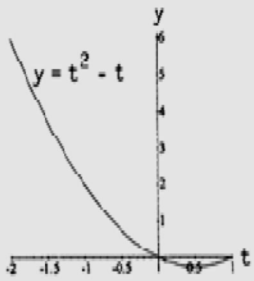
5-

$$\text{av}(f) = \left(\tfrac{1}{3-0}\right)\int_0^3 (t-1)^2\,dt$$
$$= \tfrac{1}{3}\int_0^3 t^2\,dt - \tfrac{2}{3}\int_0^3 t\,dt + \tfrac{1}{3}\int_0^3 1\,dt$$
$$= \tfrac{1}{3}\left(\tfrac{3^3}{3}\right) - \tfrac{2}{3}\left(\tfrac{3^2}{2} - \tfrac{0^2}{2}\right) + \tfrac{1}{3}(3-0) = 1.$$



6-

$$\text{av}(f) = \left(\tfrac{1}{1-(-2)}\right)\int_{-2}^1 (t^2 - t)\,dt$$
$$= \tfrac{1}{3}\int_{-2}^1 t^2\,dt - \tfrac{1}{3}\int_{-2}^1 t\,dt$$
$$= \tfrac{1}{3}\int_0^1 t^2\,dt - \tfrac{1}{3}\int_0^{-2} t^2\,dt - \tfrac{1}{3}\left(\tfrac{1^2}{2} - \tfrac{(-2)^2}{2}\right)$$
$$= \tfrac{1}{3}\left(\tfrac{1^3}{3}\right) - \tfrac{1}{3}\left(\tfrac{(-2)^3}{3}\right) + \tfrac{1}{2} = \tfrac{3}{2}.$$



7-

(a) $$\text{av}(g) = \left(\tfrac{1}{1-(-1)}\right)\int_{-1}^1 (|x| - 1)\,dx$$
$$= \tfrac{1}{2}\int_{-1}^0 (-x-1)\,dx + \tfrac{1}{2}\int_0^1 (x-1)\,dx$$
$$= -\tfrac{1}{2}\int_{-1}^0 x\,dx - \tfrac{1}{2}\int_{-1}^0 1\,dx + \tfrac{1}{2}\int_0^1 x\,dx - \tfrac{1}{2}\int_0^1 1\,dx$$
$$= -\tfrac{1}{2}\left(\tfrac{0^2}{2} - \tfrac{(-1)^2}{2}\right) - \tfrac{1}{2}(0-(-1)) + \tfrac{1}{2}\left(\tfrac{1^2}{2} - \tfrac{0^2}{2}\right) - \tfrac{1}{2}(1-0)$$
$$= -\tfrac{1}{2}.$$