1-6 EULER TURBINE EQUATION:

The fluid velocity at the turbine entry and exit can have the fluid velocity at the turbine entry and exit can have three components in the tangential, axial and radial directions of the rotor. This means that the fluid momentum can have three components at the entry and exit. This also means that the force exerted on the runner can have three components. **Out of these the tangential force only can cause the rotation of the runner and produce work.** The axial component produces a thrust in the axial direction, which is taken by suitable thrust bearings. The radial component produces a bending of the shaft which is taken by the journal bearings. Thus it is necessary to consider the tangential component for the determination of work done and power produced. The work done or power produced by the tangential force equals the product of the mass flow, tangential force and the tangential velocity. As the tangential velocity varies with the radius, the work had done also will be vary with the radius. It is not easy to sum up this work. The help of moment of momentum theorem is used for this purpose. It states that **the torque on the rotor equals the rate of change of moment of momentum of the fluid as it passes through the runner.**

Let u_1 be the tangential velocity at entry and u_2 be the tangential velocity at exit.

Let V_{u1} be the tangential component of the absolute velocity of the fluid at inlet and let V_{u2} be the tangential component of the absolute velocity of the fluid at exit. Let r_1 and r_2 be the radii at inlet and exit.

The tangential momentum of the fluid at inlet = $m\ V_{u1}$

The tangential momentum of the fluid at exit = mV_{u2}

The moment of momentum at inlet = $mV_{u1}\, r_1$

The moment of momentum at exit = $mV_{u2}\, r_2$

Torque,

$$\tau = \dot{m}\,(V_{u1}\, r_1 - V_{u2}\, r_2)$$

Depending on the direction of V_{u2} with reference to V_{u1}, the – sign will become + ve sign.

$$\text{Power} = \omega\tau \text{ and } \omega = \frac{2\pi N}{60}$$

where N is rpm.

$$\therefore \qquad \text{Power} = \dot{m}\,\frac{2\pi N}{60}\,(V_{u1}\, r_1 - V_{u2}\, r_2)$$

$$\text{But} \quad \frac{2\pi N}{60}\, r_1 = u_1 \quad \text{and} \quad \frac{2\pi N}{60}\, r_2 = u_2$$

Thus, the Euler Turbine equation becomes:

$$\text{Power} = \dot{m}\,(V_{u1}\, u_1 - V_{u2}\, u_2)$$

1-6-1 Components of Power Produced:

The power produced can be expressed as due to three effects. These are the **dynamic, centrifugal and acceleration effects.** Consider the general velocity triangles at inlet and exit of turbine runner, shown in figure (3).

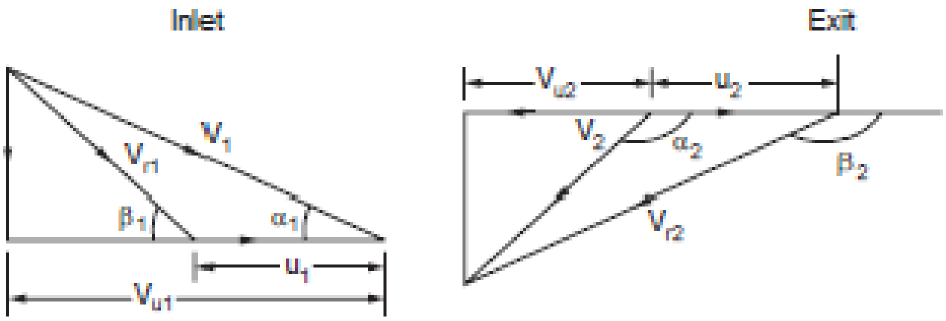

Figure (3) velocity triangle

V_1, V_2 Absolute velocities at inlet and outlet.

V_{r1}, V_{r2} Relative velocities at inlet and outlet.

u_1, u_2 Tangential velocities at inlet and outlet.

V_{u1}, V_{u2} Tangential component of absolute velocities at inlet and outlet.

From inlet velocity triangle, ($V_{u1} = V_1 \cos \alpha_1$))

$$V_{r1}^2 = V_1^2 + u_1^2 - 2u_1 V_1 \cos \alpha_1$$

Or

$$u_1 V_1 \cos \alpha_1 = V_{u1} u_1 = \frac{V_1^2 + u_1^2 - v_{r1}^2}{2}$$

From outlet velocity triangle, ($V_{u2} = V_2 \cos \alpha_2$))

$$V_{r2}^2 = V_2^2 + u_2^2 - 2\, u_2 V_2 \cos \alpha_2$$

Or

$$u_2 V_2 \cos \alpha_2 = u_2 V_{u2} = (V_2^2 - u_2^2 + V_{r2}^2)/2$$

Substituting in Euler equation,

Power per unit flow rate (here the V_{u2} is in the opposite to V_{u1})

$$\dot{m}(u_1 V_{u1} + u_2 V_{u2}) = \dot{m}\frac{1}{2}\left[(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)\right]$$

$\dfrac{V_1^2 - V_2^2}{2}$ is the dynamic component of work done

$\dfrac{u_1^2 - u_2^2}{2}$ is the centrifugal component of work and this will be present only in the radial flow machines

$\dfrac{u_{r2}^2 - V_{r1}^2}{2}$ is the accelerating component and this will be present only in the reaction turbines.

The first term only will be present in Pelton or impulse turbine of tangential flow type.

In pure reaction turbines, the last two terms only will be present.

In impulse reaction turbines of radial flow type, all the terms will be present. (A Francis turbine is of this type).

In impulse reaction turbines, the degree of reaction is defined by the ratio of energy converted in the rotor and total energy converted.

$$R = \frac{(u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}{(V_1^2 - V_2^2) + (u_1^2 - u_2^2) + (V_{r2}^2 - V_{r1}^2)}$$

The degree of reaction is considered in detail in the case of steam turbines where speed reduction is necessary. Hydraulic turbines are generally operated of lower speeds and hence degree of reaction is not generally considered in the discussion of hydraulic turbines.