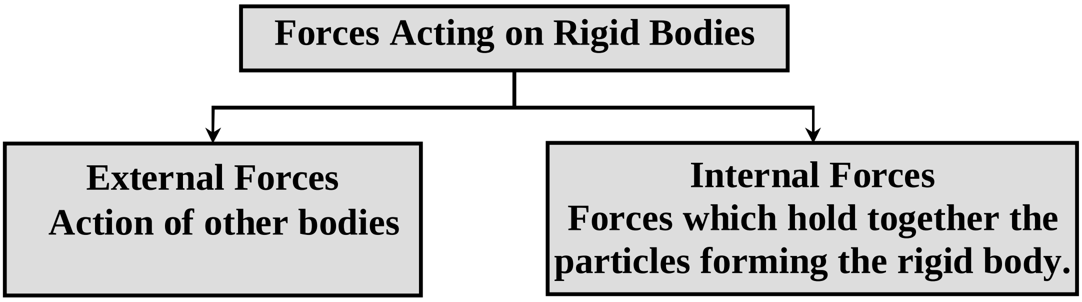
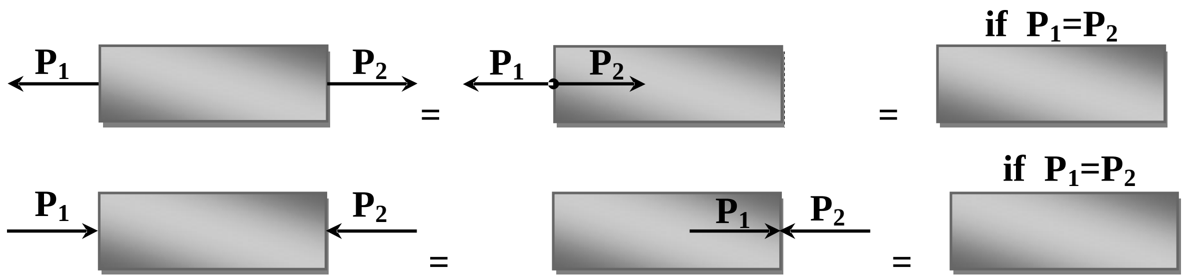
Statics of Rigid Bodies in Two Dimensions:

Rigid Body: defined as one which does not deform (bodies are not absolutely rigid).



Principle of Transmissibility:

(The **external** effect of a force is independent of where it is applied along its line of action).

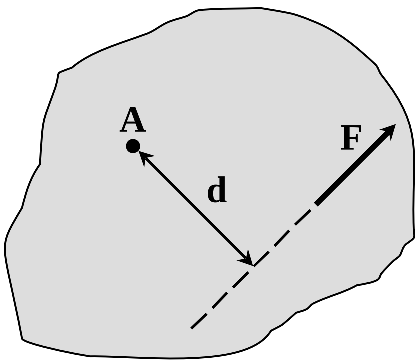


The forces are equivalent if they have the same magnitude , direction & line of action.

Moment of a Force about an Axis:

The tendency of a force to make a rigid body **rotate about an axis** is measured by the moment of the force about that axis.

d: perpendicular distance from "***A***" to the line of action of "***F***"



$M = F * d$ (***N.m***)

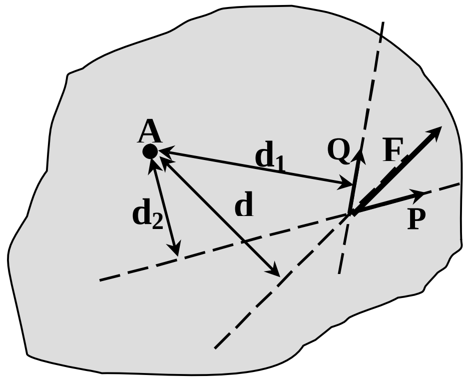
Or (***lb.ft***)

Or (***lb.ft***)

The moment has magnitude and sense (Clockwise - ,and Counterclockwise +)

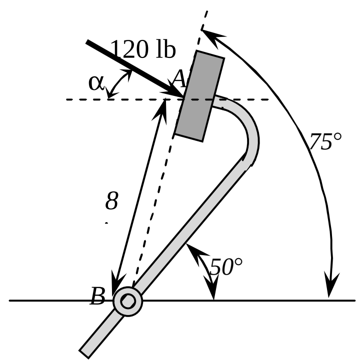
Varignon's Theorem:

The moment of a force about any axis is equal to the sum of the moment of its components about that axis.



$$F * d = Q * d_1 + P * d_2$$

3.1 A 120-lb force is applied to the brake pedal at *A*. Knowing that the distance *AB* is 8 in. , determine the moment of the force about *B* when α is 30°.

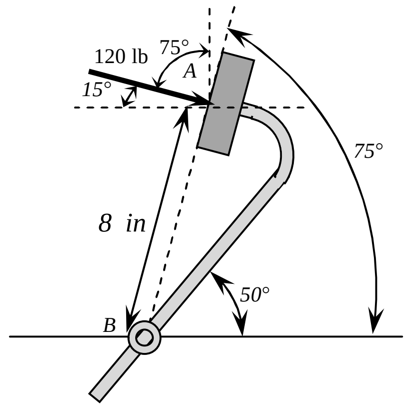


$$\alpha = 30°$$

$$M_B = -120\cos 30° * 8\sin 75° - 120\sin 30° * 8\cos 75°$$

$$M_B = -927.3 lb.in$$

3.2 Knowing that the distance *AB* is 8 in., determine the maximum moment about *B* which can be caused by the 120-lb force. In what direction should the force act.



$$F \perp AB$$

$$\Rightarrow \alpha = 15°$$

$$M_B^{120})_{max.} = -120 * 8 = -960\ lb.in$$

3.5 Compute the moment of the 100-lb force about *A* (*a*) by using the definition of the moment of a force, (*b*) by resolving the force into components along *BD* and *CD*, (*c*) by resolving the force into components along *AD* and in the direction perpendicular to *AD*.

a)

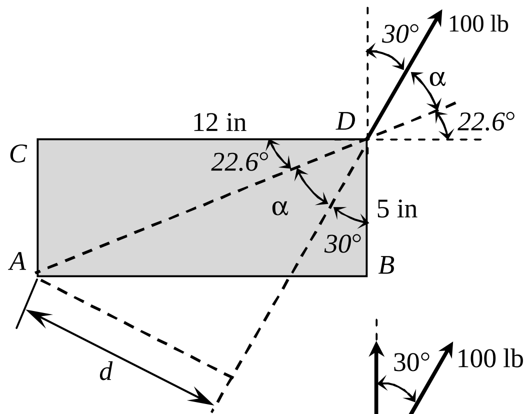
$$\alpha = 90° - 30° - 22.6° = 37.4°$$

$$AD = \sqrt{AB^2 + BD^2} = \sqrt{12^2 + 5^2} = 13 in$$

$$d = AD \sin\alpha = 13\sin 37.4° = 7.9 in$$

$$M_A^{100} = -100 * 7.9$$

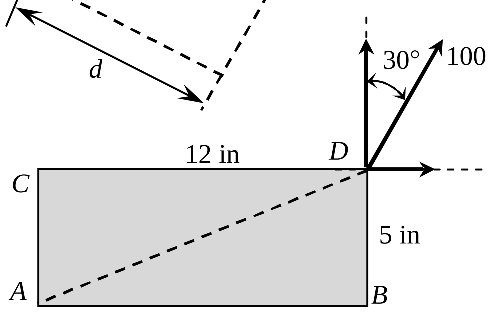
$$M_A^{100} = -790\ lb.in = 790\ lb.in$$



b)

$$M_A^{100} = 100\cos 30° * 12 - 100\sin 30° * 5$$

$$M_A^{100} = -790\ lb.in = 790\ lb.in$$



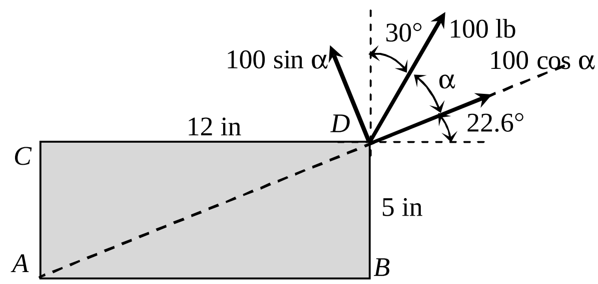
c)

$$\alpha = 90° - 30° - 22.6 = 37.4°$$

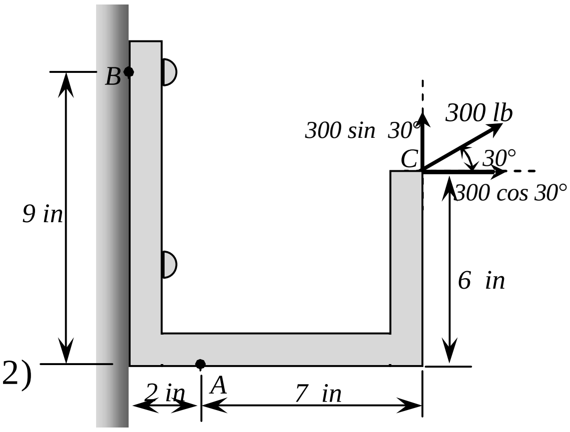
$$AD = \sqrt{12^2 + 5^2} = 13\ in$$

$$M_A^{100} = -100\sin 37.4° * 13$$

$$M_A^{100} = -790\ lb.in = 790\ lb.in$$



3.6 Determine the moment of the 300-lb force about *A*.



$$M_A^{300} = -300\cos 30^\circ * 6 + 300\sin 30^\circ * 7$$
$$M_A^{100} = -509\ lb.in$$
$$M_A^{300} = 509\ lb.in$$

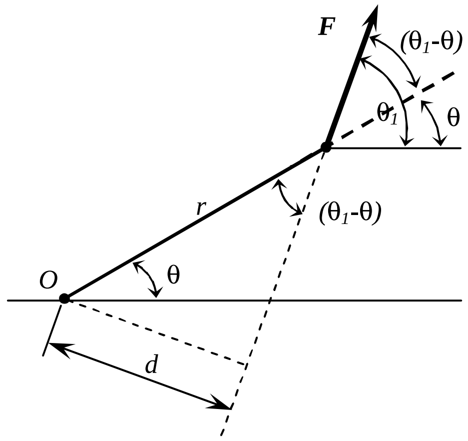
....

3.7 Determine the moment of the 300-lb force about *B*.

$$M_B^{300} = -300\cos 30^\circ * (9-6) + 300\sin 30^\circ * (7+2)$$
$$M_B^{300} = +2129.4 lb.in$$
$$M_B^{300} = 2129.4 lb.in$$

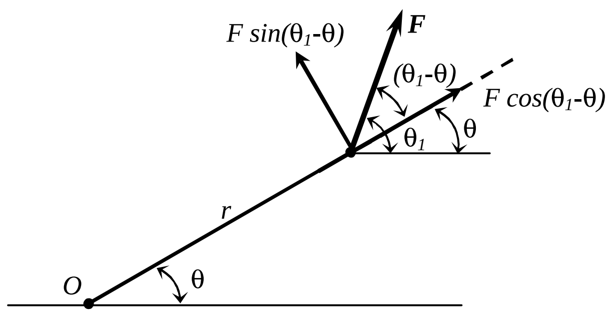
....

3.10 A force ***F*** acts at a point of coordinates *r* and θ as shown. The force forms an angle θ_1 with a line parallel to the horizontal reference axis. Show that the moment of the force about the origin of coordinates *O* is $F\, r\sin(\theta_1-\theta)$.



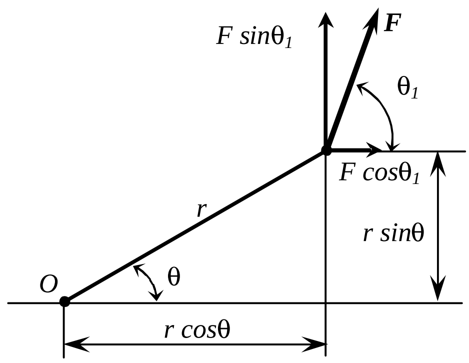
$$d = r\sin(\theta_1 - \theta)$$
$$M_O^F = F * d$$
$$M_O^F = Fr\sin(\theta_1 - \theta)$$

or



$$M_O^F = F\sin(\theta_1 - \theta) * r + F\cos(\theta_1 - \theta) * 0$$
$$M_O^F = Fr\sin(\theta_1 - \theta)$$

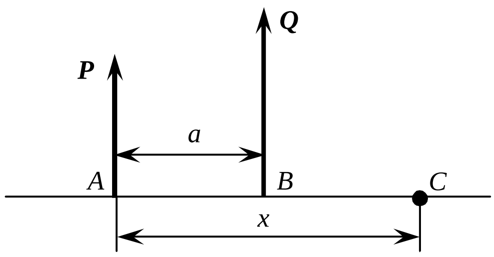
or



$$M_O^F = F\sin\theta_1 * r\cos\theta - F\cos\theta_1 * r\sin\theta$$
$$M_O^F = Fr(\sin\theta_1\cos\theta - \cos\theta_1\sin\theta)$$
$$M_O^F = Fr\sin(\theta_1 - \theta)$$

....

3.12 Two forces **P** and **Q** have parallel lines of action and act at *A* and *B*, respectively. The distance between *A* and *B* is a. Fined the distance *x* from *A* to point *C* about which both forces have the same moment. Check the formula obtained by assuming a = 10 in. and (a) P = 20 lb up, Q = 10 lb up; (b) P = 10 lb up, Q = 20 lb up; (c) P = 20 lb up, Q = 10 lb down; (d) P = 10 lb up, Q = 20 lb down.



$$M = F * d$$

$$M_C^P = M_C^Q$$

$$P * x = Q * (x - a)$$

$$P * x = Q * x - Q * a \Rightarrow Q * a = x(Q - P)$$

$$\therefore x = \frac{Q * a}{Q - P}$$

$$a) a = 10\,in;\ P = 20\,lb \uparrow;\ Q = 10\,lb \uparrow$$

$$x = \frac{10 * 10}{10 - 20} = -10\,in$$

$$M_C^P = P * x = 20 * (-10) = -200\,lb.in = 200\,lb.in \curvearrowright$$

$$M_C^Q = Q * (x - a) = 10 * (-10 - 10) = -200\,lb.in = 200\,lb.in \curvearrowright$$

$$b) a = 10\,in;\ P = 10\,lb \uparrow;\ Q = 20\,lb \uparrow$$

$$x = \frac{20 * 10}{20 - 10} = 20\,in$$

$$M_C^P = P * x = 10 * (20) = -200\,lb.in = 200\,lb.in \curvearrowright$$

$$M_C^Q = Q * (x - a) = 20 * (20 - 10) = -200\,lb.in = 200\,lb.in \curvearrowright$$

$$c) a = 10\,in;\ P = 20\,lb \uparrow;\ Q = 10\,lb \downarrow = -10\,lb$$

$$x = \frac{-10 * 10}{-10 - 20} = 3.333\,in$$

$$M_C^P = P * x = 20 * (3.333) = -66.666\,lb.in = 66.666\,lb.in \curvearrowright$$

$$M_C^Q = Q * (x - a) = 10 * (3.333 - 10) = -66.666\,lb.in = 66.666\,lb.in \curvearrowright$$

$$d) a = 10\,in;\ P = 10\,lb \uparrow;\ Q = 20\,lb \downarrow = -20\,lb$$

$$x = \frac{-20 * 10}{-20 - 10} = 6.667\,in$$

$$M_C^P = P * x = 10 * (6.667) = -66.666\,lb.in = 66.666\,lb.in \curvearrowright$$

$$M_C^Q = Q * (x - a) = -20 * (6.667 - 10) = -66.666\,lb.in = 66.666\,lb.in \curvearrowright$$