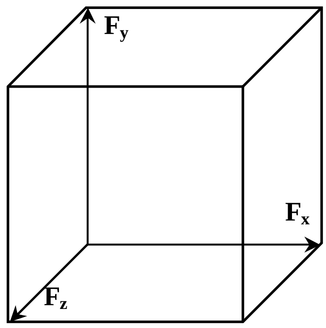
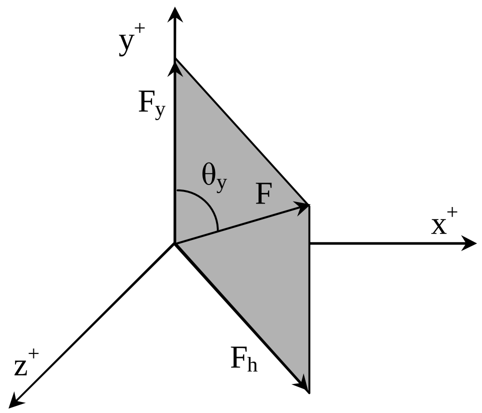
Force in Space:

*Rectangular Components of a Force in Space:

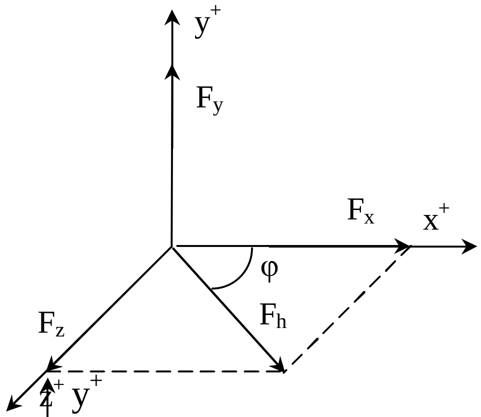


$$F_y = F\cos\theta_y$$
$$F_h = F\sin\theta_y \quad \text{(in the } \boldsymbol{xz} \text{ plane)}$$
$$F^2 = F_y^2 + F_h^2$$



in the ***xz*** plane:

$$F_x = F_h\cos\phi$$
$$F_z = F_h\sin\phi$$
$$F_h^2 = F_x^2 + F_z^2$$

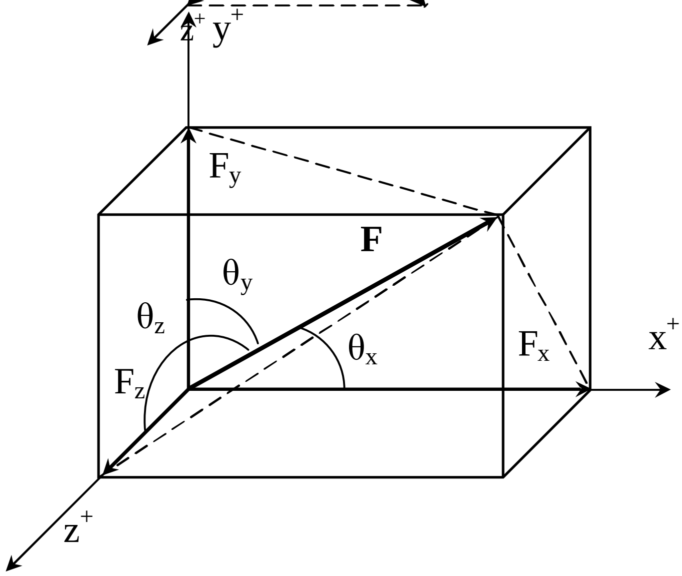


But $F^2 = F_y^2 + F_h^2$

$\therefore\ F^2 = F_x^2 + F_y^2 + F_z^2$

Or $F = \sqrt{F_x^2 + F_y^2 + F_z^2}$ ....**(1)**

$$\left.\begin{aligned} F_x &= F\cos\theta_x \\ F_y &= F\cos\theta_y \\ F_z &= F\cos\theta_z \end{aligned}\right\} \text{........(2)}$$



Sub (2) in (1) get that

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

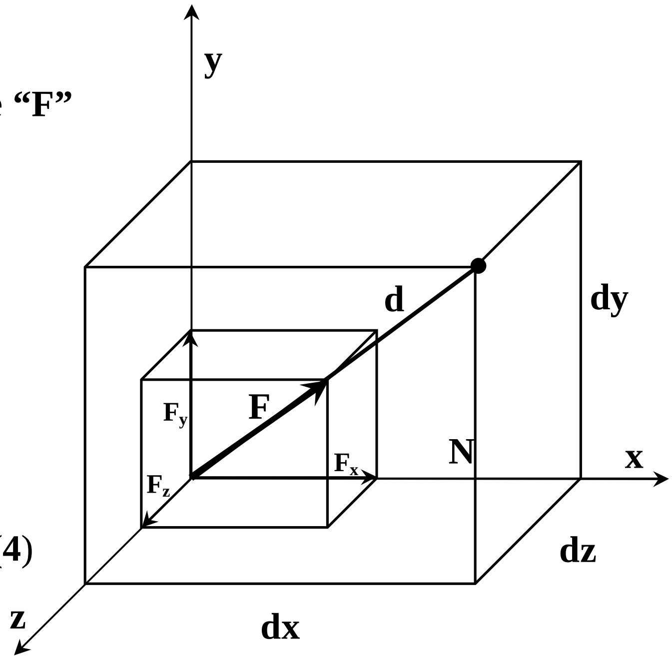
From (2) :

$$\frac{\cos\theta_x}{F_x} = \frac{\cos\theta_y}{F_y} = \frac{\cos\theta_z}{F_Z} = \frac{1}{F}$$

....

*Force Defined by Its Magnitude and Two Points on Its Line of Action:

N: lies on the line of action of the force **"F"**



$$d_x = d\cos\theta_x$$
$$d_y = d\cos\theta_y$$
$$d_z = d\cos\theta_z$$

$$d = \sqrt{d_x^2 + d_y^2 + d_z^2} \quad \text{..........(3)}$$

$$\frac{\cos\theta_x}{d_x} = \frac{\cos\theta_y}{d_y} = \frac{\cos\theta_z}{d_Z} = \frac{1}{d} \quad \text{..(4)}$$

$$\frac{F_x}{d_x} = \frac{F_y}{d_y} = \frac{F_z}{d_Z} = \frac{F}{d} \quad \text{.....(5)}$$

*Addition of Concurrent Forces in Space:

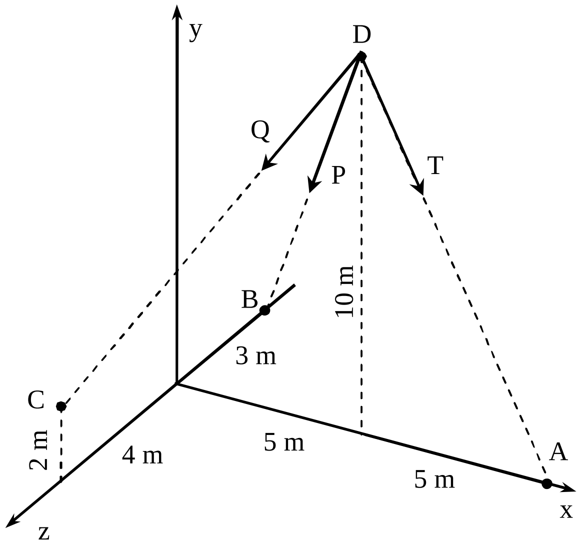
$$R_x = \Sigma F_x$$
$$R_y = \Sigma F_y$$
$$R_z = \Sigma F_z$$
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$
$$\cos\theta_x = \frac{R_x}{R}$$
$$\cos\theta_y = \frac{R_y}{R}$$
$$\cos\theta_z = \frac{R_z}{R}$$

*Equilibrium of a Particle in Space:

$$R_x = \Sigma F_x = 0$$
$$R_y = \Sigma F_y = 0$$
$$R_z = \Sigma F_z = 0$$
$$R = 0$$

Ex:

Find the resultant of the concurrent force system where T = 300 N ; P = 200 N ; Q=500 N , directed from *D* toward *A,B&C* respectively.



Force (N)	Distance Components			Distance $d=\sqrt{d_x^2+d_y^2+d_z^2}$	Force Components		
	d_x	d_y	d_z		$F_x=\frac{d_x}{d}F$	$F_y=\frac{d_y}{d}F$	$F_z=\frac{d_z}{d}F$
T=300	$=x_A-x_D$ $=10-5$ $=5$	$=y_A-y_D$ $=0-10$ $=-10$	$=z_A-z_D$ $=0-0$ $=0$	$d=\sqrt{5^2+10^2+0^2}$ $d=11.18$	$=\frac{d_x}{d}T$ $=\frac{5}{11.18}*300$ $=134.118$	$=\frac{d_y}{d}T$ $=\frac{-10}{11.18}*300$ $=-268$	$=\frac{d_z}{d}T$ $=\frac{0}{11.18}*300$ $=0$
P=200	$=x_B-x_D$ $=0-5$ $=-5$	$=y_B-y_D$ $=0-10$ $=-10$	$=z_B-z_D$ $=-3-0$ $=-3$	$d=\sqrt{5^2+10^2+3^2}$ $d=11.57$	$=\frac{d_x}{d}P$ $=\frac{-5}{11.57}*200$ $=-86.4$	$=\frac{d_y}{d}P$ $=\frac{-10}{11.57}*200$ $=-173$	$=\frac{d_z}{d}P$ $=\frac{-3}{11.57}*200$ $=-51.9$
Q=500	$=x_C-x_D$ $=0-5$ $=-5$	$=y_C-y_D$ $=2-10$ $=-8$	$=z_C-z_D$ $=4-0$ $=4$	$d=\sqrt{5^2+8^2+4^2}$ $d=10.25$	$=\frac{d_x}{d}Q$ $=\frac{-5}{10.24}*500$ $=-244$	$=\frac{d_y}{d}Q$ $=\frac{-8}{10.25}*500$ $=-390$	$=\frac{d_z}{d}Q$ $=\frac{4}{10.25}*500$ $=195$
					$R_x=\Sigma F_x$ $=-196.4$	$R_y=\Sigma F_y$ $=-831$	$R_z=\Sigma F_z$ $=143.1$

$$R=\sqrt{R_x^2+R_y^2+R_z^2}=\sqrt{(-196.4)^2+(-831)^2+(143.1)^2}=865\,N$$

$$\theta_x=\cos^{-1}\frac{R_x}{R}=\cos^{-1}\frac{-196.4}{865}=180-76.8=103.2^\circ$$

$$\theta_y=\cos^{-1}\frac{R_y}{R}=\cos^{-1}\frac{-831}{865}=180-16.1=163.9^\circ$$

$$\theta_z=\cos^{-1}\frac{R_z}{R}=\cos^{-1}\frac{143.1}{865}=80.48^\circ$$