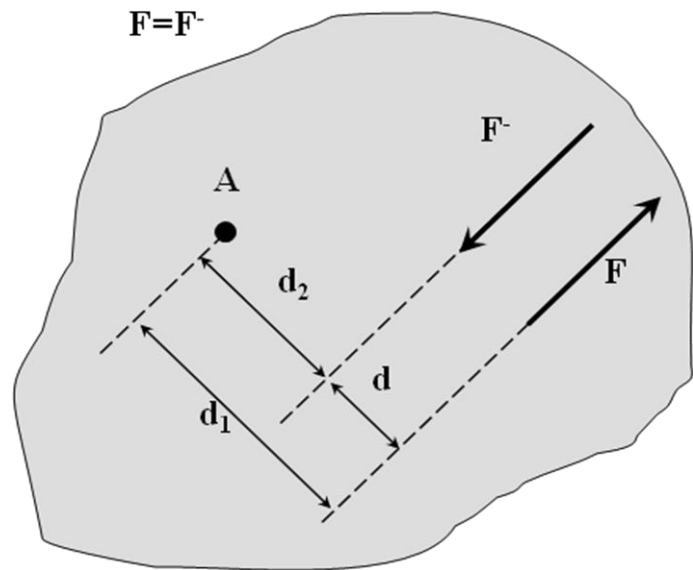


**Moment of a Couple:**

Couple: Two forces having the same magnitude, parallel lines of action, and opposite sense.

F & F^{-} form a couple

$$\begin{aligned} \curvearrowright M_A &= F \cdot d_1 - F \cdot d_2 \\ &= F (d_1 - d_2) \\ &= F \cdot d \end{aligned}$$



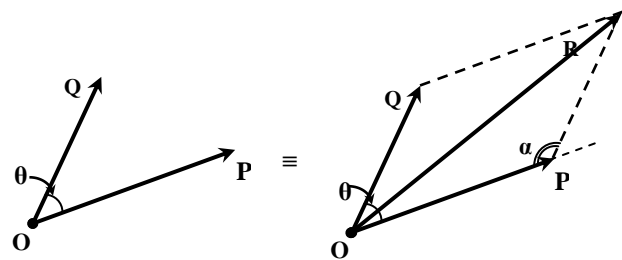
M : is called the moment of couple.

- It have the same magnitude and same sense regardless of the location of “A”.
- M is constant $= F \cdot \text{distance between the lines of action}$.
- The sense of “ M ” is obtained by direct observation.

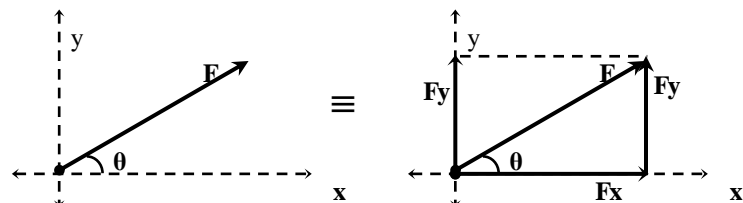
Equivalent Couples:

Two system of forces are equivalent (have the same effect on a rigid body) if we can transform one of them into the other by means of one of the following operations (or several):

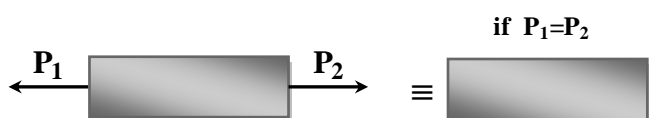
- 1- Replacing the forces by their resultant.



- 2- Resolving into components.

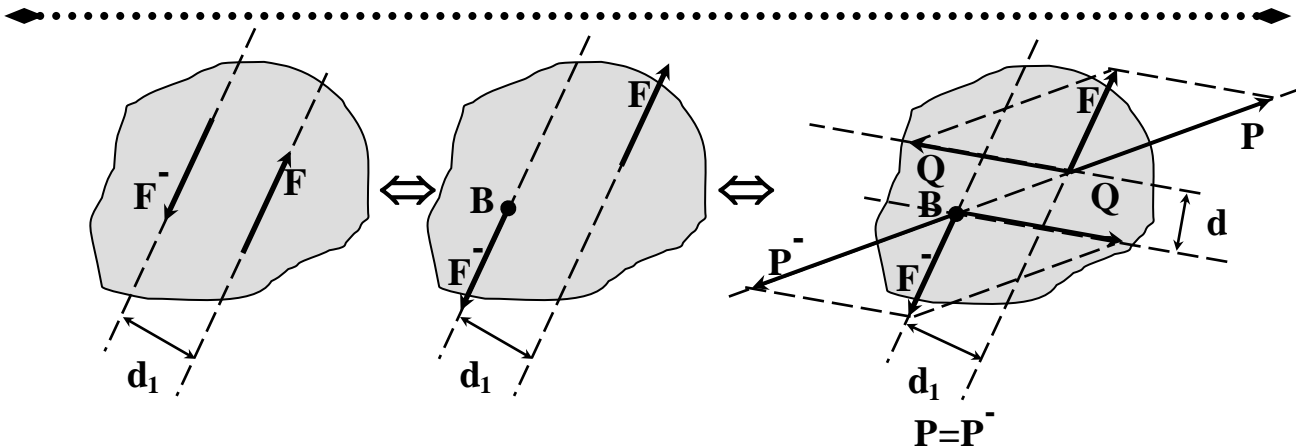


- 3- Canceling two equal & opposite forces acting on the same particle (on the same line).



- 4- Attaching to the same particle two equal & opposite forces.

- 5- Moving a force along its line of action.



Moment about “B”

$$M_B = F d_1$$

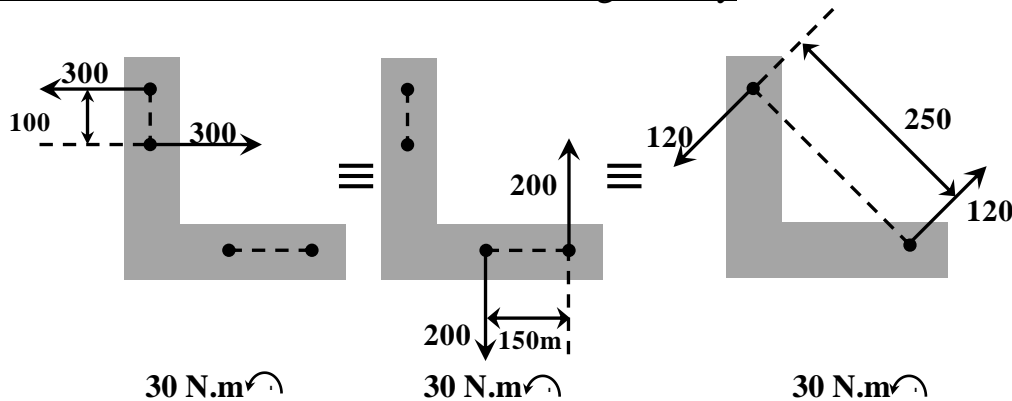
& by Varignon's theorem

$$M_B = Q d_2 - P \cdot 0$$

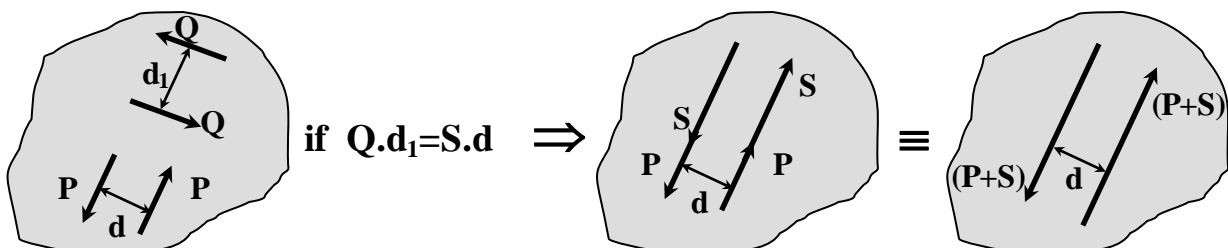
$$M_B = Q d_2$$

$\Rightarrow F d_1 = Q d_2$ in magnitude & sense

When a couple acts on a rigid body, it does not matter where the two forces forming the couple act or what magnitude and direction they have. The only thing which counts in the moment of the couple (magnitude & sense). Couples with the same moment will have the same effect on the rigid body.



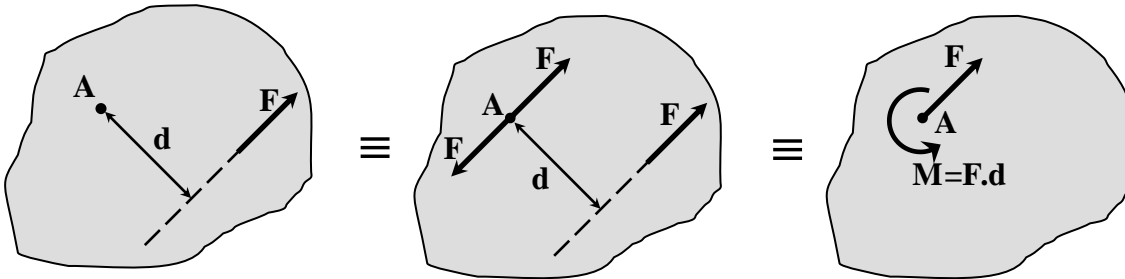
Addition of Couples:



Two couples may be replaced by a single couple of moment equal to the algebraic sum of the moments of the given couples.



A given force can be resolved in to a force acting at a given point and a Couple:



3.13 The two couples shown are applied to a 6- by 8- in plate. Knowing that $P_1 = P_2 = 30$ lb and $Q_1 = Q_2 = 40$ lb, prove that their sum is zero (a) by adding their moments, (b) by combining P_1 and Q_1 into their resultant R_1 , combining P_2 and Q_2 into their resultant R_2 , and then showing that R_1 and R_2 are equal and opposite and have the same line of action.

a)

$$M_1 = P_1 * 8 = 30 * 8 = 240 \text{ lb.in} \curvearrowright$$

$$M_2 = Q_1 * 6 = 40 * 6 = 240 \text{ lb.in} \curvearrowleft$$

$$M = M_1 + M_2 = +240 - 240$$

$$M = 0$$

b)

$$R_1 = \sqrt{P_1^2 + Q_1^2} = \sqrt{(-30)^2 + (40)^2}$$

$$R_1 = 50 \text{ lb}$$

$$\theta_1 = \tan^{-1} \frac{P_1}{Q_1} = \tan^{-1} \frac{30}{40} = 36.87^\circ$$

$$R_2 = \sqrt{P_2^2 + Q_2^2} = \sqrt{(30)^2 + (-40)^2}$$

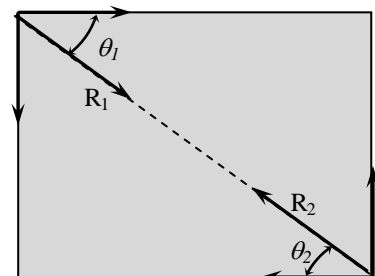
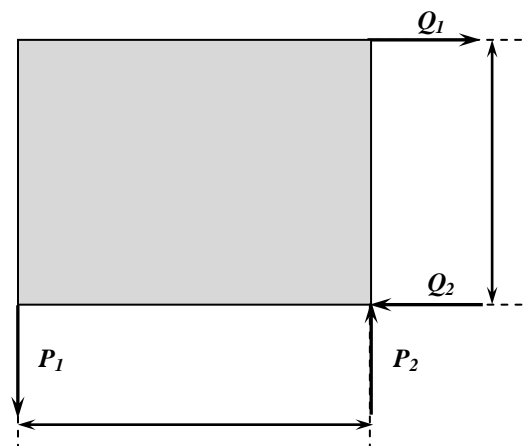
$$R_2 = 50 \text{ lb}$$

$$\theta_2 = \tan^{-1} \frac{P_2}{Q_2} = \tan^{-1} \frac{30}{40} = 36.87^\circ$$

$$\alpha = \tan^{-1} \frac{6}{8} = 36.87^\circ$$

$$\therefore R_1 = R_2 \text{ \& } \theta_1 = \theta_2 = \alpha$$

$\therefore R_1$ and R_2 are equal and opposite and have the same line of action





3.16 Four pegs are attached to a board as shown. Two strings are passed around the pegs and pulled with forces of magnitude $P = 16 \text{ lb}$ and $Q = 40 \text{ lb}$. Determine the required diameter of the pegs if the resultant couple applied to the board is to be 300 lb.in. clockwise.

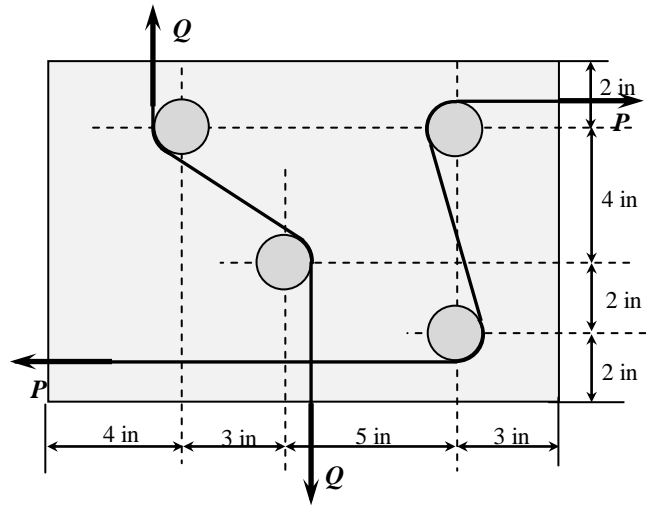
$$P = 16 \text{ lb}; Q = 40 \text{ lb}; M = 300 \text{ lb.in}$$

$$M = -P * (4 + 2 + 2 * \frac{d}{2}) - Q * (3 + 2 * \frac{d}{2})$$

$$300 = -16 * 6 - 16d - 120 - 40d$$

$$300 = 56d$$

$$d = 1.5 \text{ in}$$



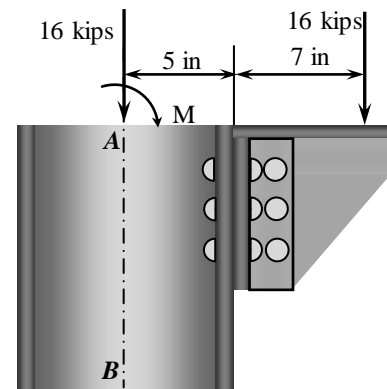
3.18 A crane column supports a 16-kip load as shown. Reduce the load to an axial force along AB and a couple.

$$F = 16 \text{ kips}$$

$$M = F * d = 16 * 12$$

$$M = 192 \text{ kip.in} \curvearrowright$$

$$\& F = 16 \text{ kips}$$



3.20 A 65-lb force is applied to a bent plate as shown. Determine an equivalent force-couple system (a) at A (b) at B.

(a) at A

$$M_A = F * d$$

$$M_A = 65 \sin 30^\circ * 5 - 65 \cos 30^\circ * (8 + 3)$$

$$M_A = -456.7 \text{ lb.in} = 456.7 \text{ lb.in} \curvearrowright$$

$$\& F = 65 \text{ lb} \quad \swarrow 60^\circ$$

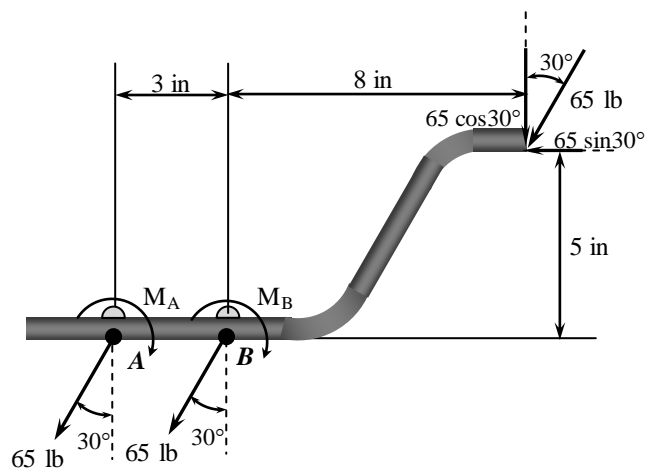
(b) at B

$$M_B = F * d$$

$$M_B = 65 \sin 30^\circ * 5 - 65 \cos 30^\circ * 8$$

$$M_B = -288 \text{ lb.in} = 288 \text{ lb.in} \curvearrowright$$

$$\& F = 65 \text{ lb} \quad \swarrow 60^\circ$$





3.22 The force and couple shown are to be replaced by an equivalent single force. Determine the required value of α so that the line of action of the single equivalent force will pass through point B .

$$M_O^{30} = F * d = 30 * 2r = 240 \text{ lb.in} \curvearrowleft$$

$$d_1 = r \sin \alpha$$

$$M_C^{160} = 160 * r \sin \alpha = 160 * 4 \sin \alpha$$

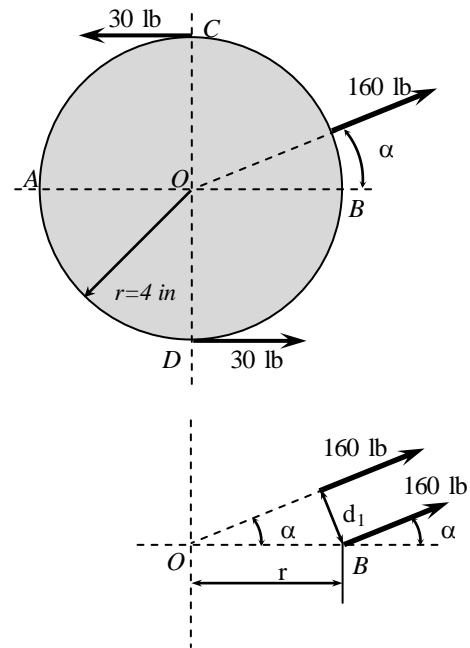
$$M_C^{160} = 640 \sin \alpha$$

$$M_C^{160} = M_O^{30}$$

$$640 \sin \alpha = 240$$

$$\sin \alpha = \frac{240}{640}$$

$$\alpha = 22.0^\circ$$



3.23 A hook is held by screws at A and B . (a) Replace the 50-lb load shown by an equivalent force-couple system at B . (b) Find two horizontal forces at A and B which form a couple equivalent to the couple found in part a .

(a) at B

$$M_B^{50} = F * d = 50 * 2 \curvearrowright$$

$$M_B^{50} = 100 \text{ lb.in}$$

$$F = 50 \text{ lb} \downarrow$$

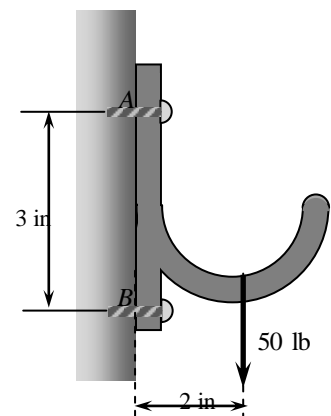
(b)

$$M_B^{50} = F * d = 50 * 2 = 100 \text{ lb.in}$$

$$M = F * d$$

$$100 = F * 3$$

$$F = 33.33 \text{ lb}$$





3.24 A force P is applied to beam AB at point C as shown. Find the vertical forces F_A and F_B applied respectively, at A and B, which form a system equivalent to P .

$$F_A + F_B = P \dots\dots\dots(1)$$

$$M_B^P = M_B^{F_A}$$

$$P * b = F_A (a + b)$$

$$F_A = \frac{b}{(a + b)} * P \dots\dots\dots(2)$$

sub(2)in(1)

$$\frac{b}{(a + b)} * P + F_B = P$$

$$F_B = (1 - \frac{b}{a + b}) * P$$

$$F_B = (\frac{a + b - b}{a + b}) * P$$

$$F_B = (\frac{a}{a + b}) * P$$

