Sampling Distributions

- From a given population we can take series of different *samples* and calculate a sample statistics (eg., sample mean) for each sample.
- Properties of Sampling Distribution of Means:

  1). The mean of sampling distribution is similar to population mean μ

  2). SD of the distribution of the sample means is equal to

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

  This is called SE of the mean, It is measured the variability of mean of the samples.

  3). Provided n is large enough, $(n \geq 30)$ the shape of the sampling distribution is approximately normal. The best estimate of the population standard error of the mean, $s_{\bar{x}}$, is

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

Descriptive Statistics of SBP from 15 Samples-

Variable	N	N*	Mean	Median	StDev
Population	491	9	121.96	119.00	16.97
samp1	30	0	120.37	116.50	16.05
samp2	30	0	126.57	127.50	22.88
Samp3	30	0	124.73	120.00	16.36
samp4	30	0	122.03	117.00	21.18
samp5	30	0	121.37	119.50	18.00
samp6	30	0	117.53	114.00	14.65
samp7	30	0	122.90	121.00	16.50
samp8	30	0	119.20	116.00	17.39
samp9	30	0	116.47	116.00	15.02
samp10	30	0	125.63	122.50	16.38
samp11	30	0	115.43	114.50	11.11
samp12	30	0	124.55	123.00	20.44
samp13	30	0	122.30	116.50	18.62
samp14	30	0	121.47	116.50	17.00
samp15	30	0	119.63	116.50	14.95

* Missing values

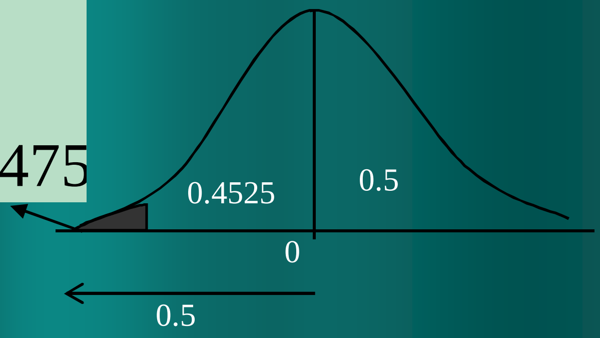
SBP Example

	N	Mean	SD
Population	491	121.96	16.97
Sample (size=30)	15	121.35	SE=16.97/ $\sqrt{15}$ = 4.

- Just as $Z = \left(\frac{X_i - m}{s} \right)$ normal deviate that refers to normal distribution of X_i values,
- $$Z = \left(\frac{\overline{x}_i - m}{s_{\overline{x}}} \right)$$ is a normal deviate referring to normal distribution of means (X values).
- Example: The mean and standard deviation of systolic blood pressure for normal men are 130 and 9 mm Hg respectively, what is the probability of drawing from them a random sample of 9 measurements which has a *mean,*
- 1- Less than 125 mm Hg?

$$P(\overline{x} < 125) = P\left(Z \le \frac{125 - 130}{9/\sqrt{9}} \right) = P(Z < -1.67)$$

$$= 0.5 - 0.4525 = 0.0475$$

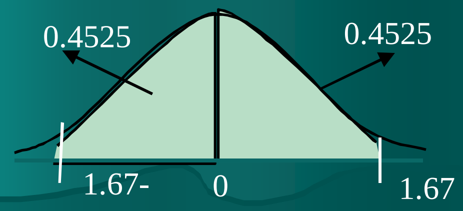


2- between 125 and 135 mm Hg?

$$P(125 \le \overline{X} \le 135) = P\left(\frac{125 - 130}{9/\sqrt{9}} \le Z \le \frac{135 - 130}{9/\sqrt{9}} \right)$$

$$P(-1.67 \le Z \le 1.67)$$

$$0.4525 + 0.4525 = 0.9050$$



- Distribution of the Difference between Two Sample Means:

If independent samples of size n_1 and n_2 are drawn at random from two populations, discrete or continuous, with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively, then the sampling distribution of the differences of means, $\bar{x}_1 - \bar{x}_2$, is approximately normally distributed with mean and variance given by;

$$m_{\bar{x}_1 - \bar{x}_2} = m_1 - m_2$$

$$s^2_{\bar{x}_1 - \bar{x}_2} = \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}$$

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (m_1 - m_2)}{\sqrt{(s_1^2 / n_1) + (s_2^2 / n_2)}}$$

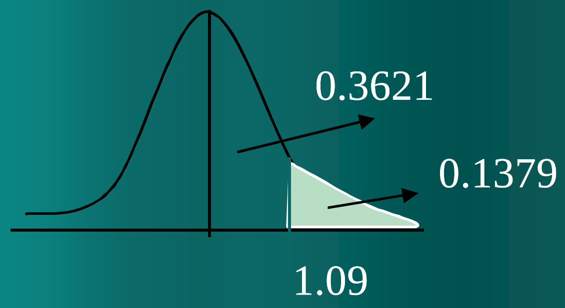
is approximately a standard normal variable.

<u>NOTE</u>: If both n_1 and n_2 are greater than or equal to 30, the normal approximation for the distribution of $\bar{x}_1 - \bar{x}_2$ is very good.

- Example: A researcher is willing to assume that levels of vitamin A in the livers of two human populations are each normally distributed. The variances for the two populations are assumed to be 19600 and 8100 respectively. What is the probability that a random sample of size 15 from the first population and 10 from the second population will yield a value of the difference between samples means greater than or equal to 50 if there is no difference in the population means?

$$P(\bar{x}_1 - \bar{x}_2 \geq 50) = P\left(Z \geq \frac{50-0}{\sqrt{\frac{19600}{15} + \frac{8100}{10}}} \right) = P(Z \geq 1.09)$$

$$0.5 - 0.3621 = 0.1379$$



Distribution of the Sample Proportion:

- This distribution is obtained from Binomial distribution with parameter *P*. *is a best and unbiased estimate of P and is approximately normally distributed with a mean and standard error denoted by;*

$$m_{\hat{P}} = P \quad \text{and} \quad s_{\hat{P}} = \sqrt{\frac{pq}{n}}$$

- This distribution would be constructed experimentally in exactly the same manner as was suggested in the case of the mean and the difference between two means. *A widely used criterion is that both np and nq are greater than 5.*

$$Z = \frac{\hat{P} - P}{s_{\hat{P}}}$$

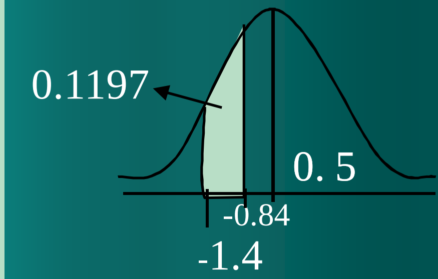
Example: If, in a population of adults, 0.15 are on some sort of diet, what is the probability that a random sample of size 100 will yield a proportion who are on a diet:

1- Between 0.10 and 0.12?

$$P\left(0.10 \le \hat{p} \le 0.12\right)$$

$$= P\left(\frac{0.10 - 0.15}{\sqrt{(0.15)(0.85)/100}} \le Z \le \frac{0.12 - 0.15}{\sqrt{(0.15)(0.85)/100}} \right)$$

$$= P\left(-1.40 \le Z \le -0.84\right) = 0.4192 - 0.2995 = 0.1197$$



2- No greater than 0.12?

$$P\left(\hat{p} \le 0.12\right) = P\left(Z \le \frac{0.12 - 0.15}{\sqrt{(0.15)(0.85)/100}} \right)$$

$$= P\left(Z \le -0.84\right) = 0.5 - 0.2995 = 0.2005$$

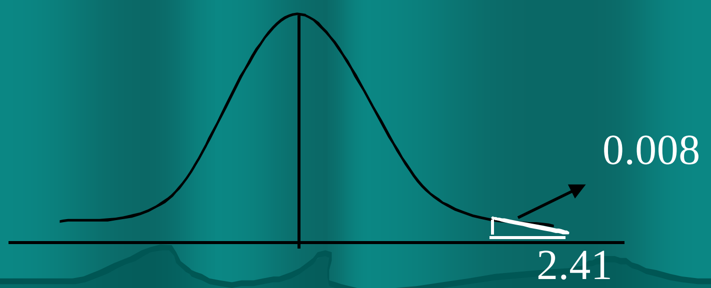
- <u>Distribution of the Difference between Two Sample Proportions</u>:
- This distribution is obtained from two Binomial distributions populations with parameters P_1 and P_2. *are best and unbiased estimate of* P_1 and P_2 , *and are approximately normally distributed with a means and standard errors denoted by*

$$m_{\hat{P}_1-\hat{P}_2} = p_1 - \mathrm{p}_2 \quad \text{and} \quad s_{\hat{P}_1-\hat{P}_2} = \sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}} \quad ,$$

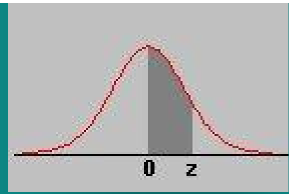
$$Z = \frac{(\hat{p}_1-\hat{p}_2)-(p_1-p_2)}{\sqrt{\frac{p_1q_1}{n_1}+\frac{p_2q_2}{n_2}}}$$

Example: In a certain population of retarded children, it is known that the proportion who is hyperactive is 0.40. A random sample of size 120 was drawn from this population, and a random sample of size 100 was drawn from another population of retarded children. If the proportion of hyperactive children is the same in both populations, what is the probability that the sample would yield a difference, $\hat{P}_1 - \hat{P}_2$, of 0.16 or more?

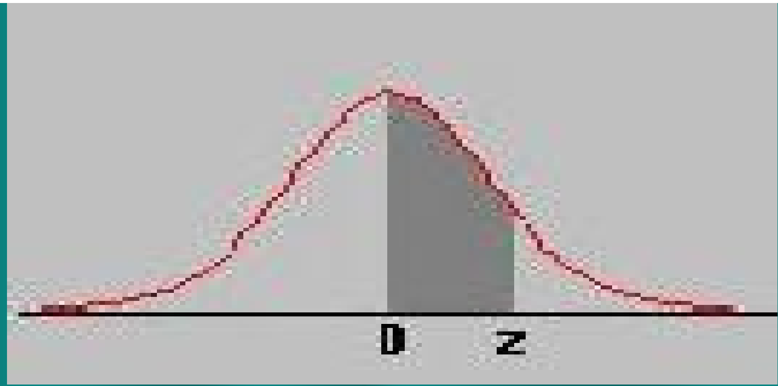
$$= P\left(Z \geq \frac{0.16 - 0}{\sqrt{\frac{(0.4)(0.6)}{120} + \frac{(0.4)(0.6)}{100}}} \right) = P(Z \geq 2.41) = 0.5 - 0.4920 = 0.008$$



Area between 0 and z



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936



	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990