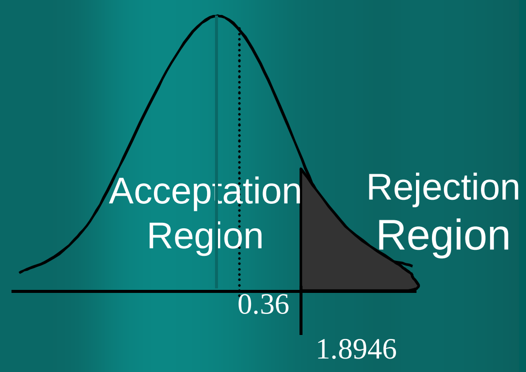
- Sample from Normally Distributed Population, σ^2 Unknown, $n \leq 30$:
- Example: Dissolving (in sec.) of a drug in agitated gastric juice (42.7, 43.4, 44.6, 45.1, 45.6, 45.9, 46.8, 47.6). Do these data provide sufficient evidence to indicate that the populations mean for dissolving a drug greater than 45 sec.? Let $\alpha = 0.05$.
  - $H_0: \mu = 45$, $H_1: \mu > 45$

$$t = \frac{\bar{x} - m_0}{s / \sqrt{n}}$$

$$t = \frac{45.21 - 45}{1.64 / \sqrt{8}} = 0.36$$



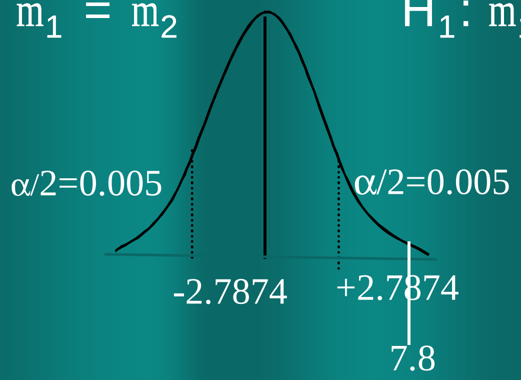
  - From Percentiles of t Distribution Table
  - $t_{(1-\alpha), v} = t_{(1-0.05), 7} = 1.8946$
- The critical value of test statistics is 1.8946
- *Statistical decision*: Accept H0, since t < 1.8946.
- *Clinical decision*: Conclude that, on the bases of these data, there is an indication that the mean for dissolving a drug less than 45 sec. (P>0.05).

Sampling from Normally Distributed Populations: Population Variances, (s_1 and s_2) are Equal but Unknown:

- Example: In order to evaluate the difference in serum Na levels between normotensive and newly diagnosed hypertensive patients not yet on a Na controlled diet, the following data were obtained:

	N	Mean (mEq/liter)	Standard Deviation (mEq/liter)
normotensive	15	144	6.2
Hypertensive	12	160	4.9

- Using α = 0.01 level of significance, can it concluded that there is a difference in Na level for the normotensive and hypertensive groups?
- $H_0: m_1 = m_2$ $H_1: m_1 \neq m_2$



$$Sp = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$Sp = \sqrt{\frac{11\,(4.9)^2 + 14\,(6.2)^2}{12 + 15 - 2}} = 5.665$$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{Sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{160 - 144}{5.665\sqrt{\frac{1}{12} + \frac{1}{15}}} = 7.293$$

- From Percentiles of t Distribution Table
- $t_{(1 - \alpha/2),\, v} = t_{(1 - 0.01/2),\; 12+15-2} = t_{(0.995),\, 25} = 2.7874.$
- *Statistical decision*: Reject H0, since t > 2.7874.
- *Clinical decision*: Conclude that, on the bases of these data, serum Na level is different for the two groups. (P<0.01).

Paired t- test:

In previous discussion involving the difference between two population means, it was assumed the samples were independent. A method employed for assessing the effectiveness of treatment or experimental procedure is one that makes use of related observations resulting from nonindependent samples. A hypothesis test based on this type of data is known as *paired comparisons test*.

$$t = \frac{\overline{D} - m_d}{s_d / \sqrt{n}}$$

Where $\overline{D}$ samples mean differences,

m_d population means difference,

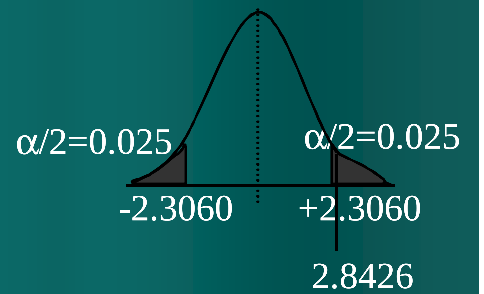
s_d Standard deviation of the difference between samples.

- Example: Serum digoxin levels were determined for nine healthy males aged 20-24 years following rapid intravenous injection of the drug. The measurements were made 4-hours after the injection and again at the end of an 8-hours period.
- Serum Digoxin (ng/ml) 4-H_o.: 1.0, 1.3, 0.9, 1.0, 1.0, 0.9, 1.3, 1.1, 1.0
- Serum Digoxin (ng/ml) 8-H_o.: 1.0, 1.3, 0.7, 1.0, 0.9, 0.8, 1.2, 1.0, 1.0
- Is there a statistically significant difference in the serum digoxin concentration at the end of 4 hours and concentration at the end of 8 hours? Let (α = 0.05).
- Differences (D): 0.0, 0.0, 0.2, 0.0, 0.1, 0.1, 0.1, 0.1, 0.0
  - H_o: $\mu_d = 0$, H1: $\mu_d \neq 0$

$$\bar{D} = \frac{D}{n} = \frac{0.6}{9} = 0.067$$

$$s_d = \sqrt{\frac{1}{9-1}\left[0.08 - \frac{(0.6)^2}{9}\right]} = 0.071$$

$$t = \frac{\bar{D} - m_d}{s_d / \sqrt{n}} = \frac{0.067 - 0}{0.071 / \sqrt{9}} = 2.8426$$



From Percentiles of t Distribution Table,

$$(t_{(1-\alpha/2),\, v} = t_{(1-0.05/2),\, 8} = 2.3060)$$

Statistical decision: Reject H_0, since t > 2.3060.

Clinical decision: Conclude that, on the bases of these data, serum digoxin concentration is different for the two times.

- <u>Hypothesis Testing for a Single Population Proportion:</u>
- <u>Example</u>: A survey was conducted to study the dental health practices of a certain urban adult population. Of 300 adults interviewed, 123 said that regularly had a dental checkup twice a year. Test the null hypothesis that P=0.5. Let α =0.01.

$$H_0 : P = 0.5 \qquad , \qquad H_1 : P \neq 0.5$$

$$\hat{p} = \frac{123}{300} = 0.41$$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.41 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{300}}} = -3.11$$

- *Statistical decision*: Reject H_0, since Z<-2.58.
- *Clinical decision*: Conclude that, on the bases of these data, not 50% of the population regularly have a dental checkup twice a year. (P<0.01).

Hypothesis Testing for the Difference between Two Populations Proportions:

- Example: In a study designed to assess the side effects of two drugs, 50 animals were given drug A and 100 animals were given drug B. Of the 50 receiving drug A, 11 showed undesirable side effects, while 9 of those receiving drug B reacted similarly. Test whether the theoretical probabilities of side effects are the same for the two drugs. Let α =0.05.

$$H_0: P_1 = P_2 \quad , \quad H_1: P_1 \neq P_2$$

$$\hat{p}_1 = \frac{11}{50} = 0.22 \quad , \quad \hat{p}_2 = \frac{9}{100} = 0.09$$

$$\bar{p} = \frac{11+9}{50+100} = 0.133 \; , \; \bar{q} = 1 - 0.133 = 0.867$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}};$$

$$Z = \frac{0.22 - 0.09}{\sqrt{\frac{(0.133)(0.867)}{50} + \frac{(0.133)(0.867)}{100}}} = 2.210$$

- *Statistical decision*: Reject H0, since Z>1.96
- *Clinical decision*: Conclude that, on the bases of these data, the probabilities of side effects of the two drugs are not the same. (P<0.05).