Normal Distribution

- The most important continuous probability distribution in the entire field of statistics is *normal distribution and it is frequently called Gaussian distribution*.
- The normal distribution depends on the values of,

$$\left(m \;\&\; s^{2}\right), \text{ and denoted by } \mathrm{N}\left(m, s^{2}\right)$$

- The normal density is given by;

$$f(x) = \frac{1}{\sqrt{2ps^{2}}}\, e^{-\frac{1}{2}\left(\frac{X-m}{s}\right)^{2}} \qquad -\infty < X < +\infty$$

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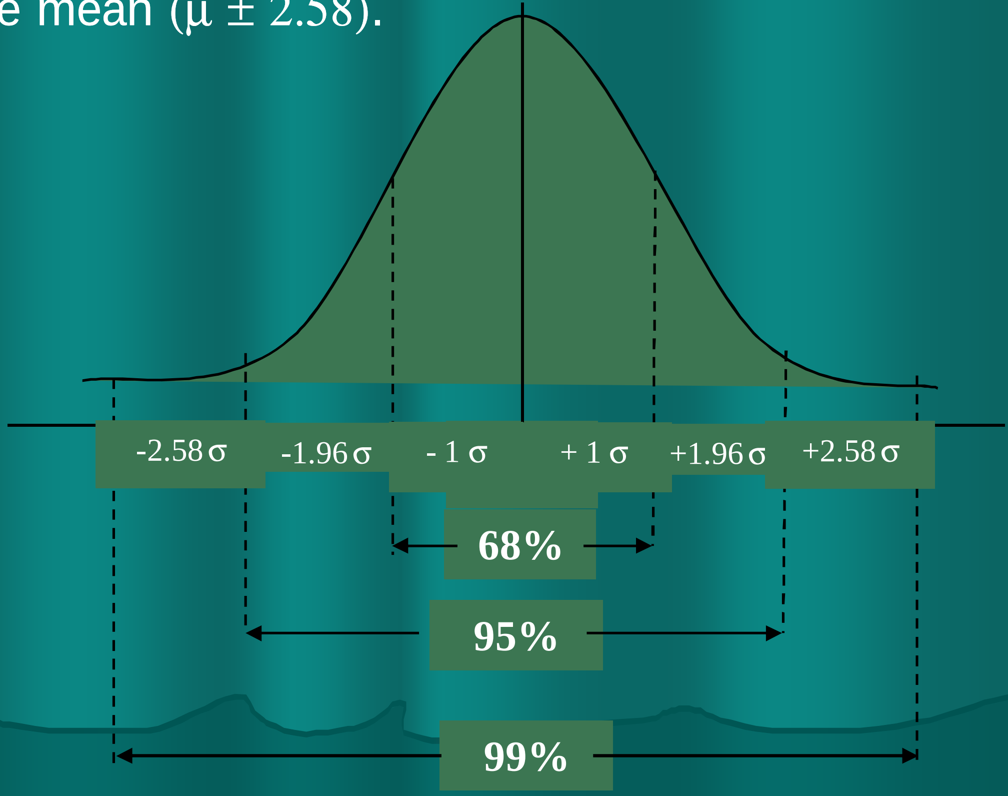
- The properties of Normal Distribution are:
- A normal distribution is “bell shaped “and symmetrical about its mean . 50% of the observations lie above the mean and 50% below it.
- The total area under the curve above the horizontal axis is 1.
- Different values of determine the degree of flatness or peaked ness of the graphs of the distribution.
- Approximately 68% of the observations lie within ±1 standard deviation of the mean ($\mu \pm 1\sigma$), 95% of the observations lie within ±1.96 standard deviation of the mean ($\mu \pm 1.96$), and 99% of the observations lie within ±2.58 standard deviation of the mean ($\mu \pm 2.58$).



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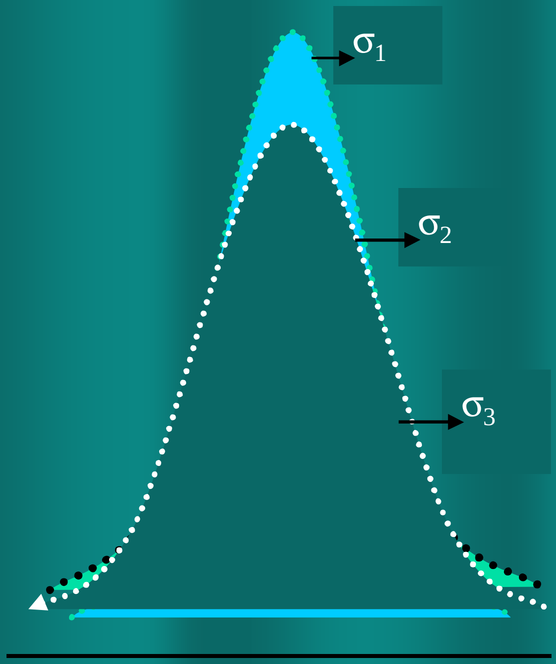
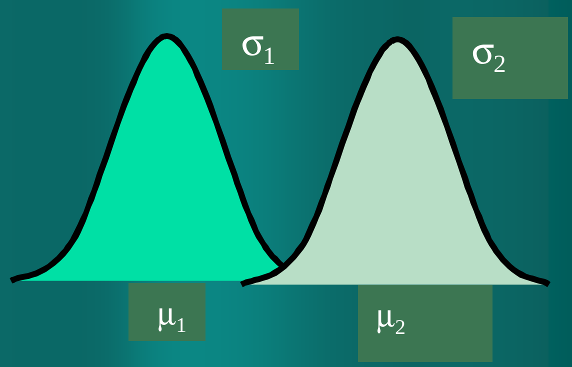


$m_1 = m_2 = m_3 \quad , \quad s_1 < s_2 < s_3$



$m_1 < m_2 \quad , \quad s_1 = s\,2$

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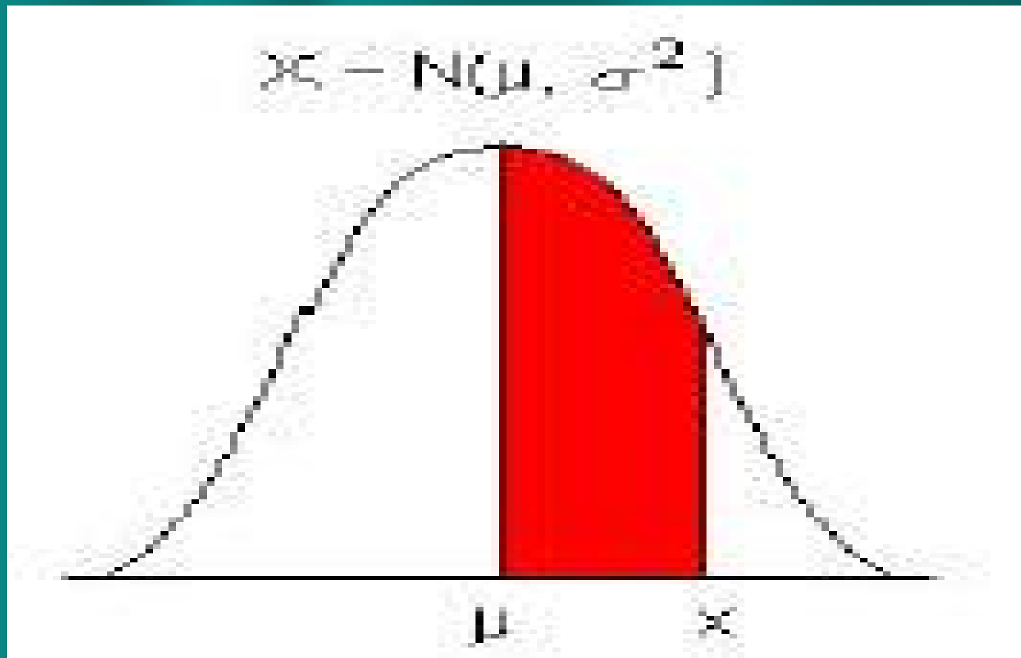
Normal Distribution

• If a random variable X has the Normal distribution with mean μ and variance, s^2 this is denoted by:

- X ~N (μ, s^2).

• In order to obtain probabilities for the Normal distribution (i.e. areas under the curve), it is necessary to express any value of X in terms of the number of standard deviation units it is away from μ.

- i.e; X=μ+z s
- or Z=(X- μ)/ s



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Standard Normal distribution

- The Standard Normal distribution has a mean of 0 and a SD of 1,

$$Z \sim N(0,1)$$

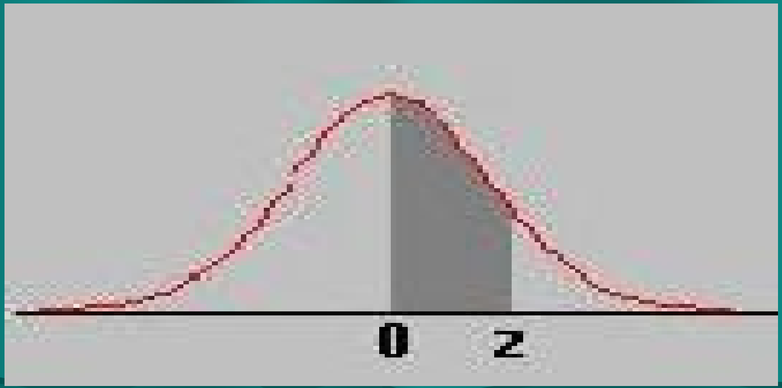
- Probabilities for Normal distributions other than the standard Normal distribution N (0, 1) are obtained by using the formula,

$$Z = X - \mu / \sigma$$

to convert from normal distribution $X \sim N(\mu, \sigma)$ to standard normal distribution $Z \sim N(0,1)$ and then using the table of probabilities for N(0,1).

Total area under the curve=1

The values inside the given table represent the areas under the standard normal curve for values between 0 and the relative z-score.



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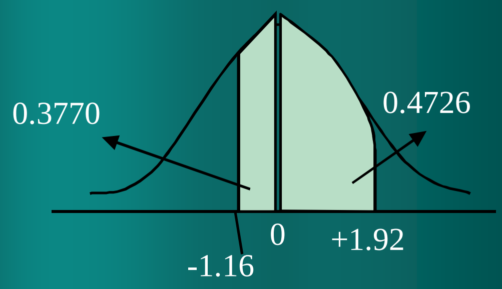
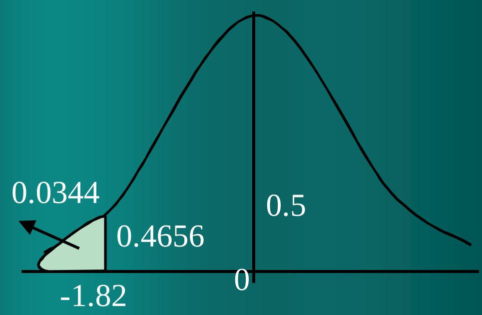
- Example: Assume the mean and standard deviation of the distribution of serum Cholesterol level in normal men are 242.2 and 45.4 respectively. Find the probability of a normal man will have a cholesterol value:
- 1- between 189.5 and 329.5

$$= P\left(\frac{189.5-242.2}{45.4} \le Z \le \frac{329.5-242.2}{45.4}\right)$$
$$= P(-1.16 \le Z \le 1.92)$$
$$= 0.3770 + 0.4726 = 0.8496$$



- 2- P(X< 159.5)

$$= P\left(Z \le \frac{159.5-242.2}{45.4}\right)$$
$$= P(Z \le -1.82)$$
$$= 0.5 - 0.4656 = 0.0344$$



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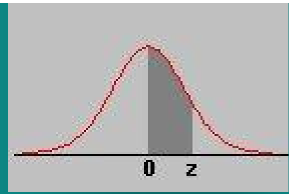
Area between 0 and z



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936

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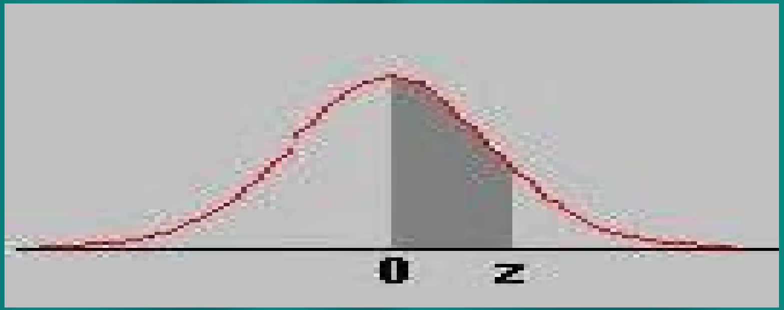

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.5	**0.4938**	**0.4940**	**0.4941**	**0.4943**	**0.4945**	**0.4946**	**0.4948**	**0.4949**	**0.4951**	**0.4952**
2.6	**0.4953**	**0.4955**	**0.4956**	**0.4957**	**0.4959**	**0.4960**	**0.4961**	**0.4962**	**0.4963**	**0.4964**
2.7	**0.4965**	**0.4966**	**0.4967**	**0.4968**	**0.4969**	**0.4970**	**0.4971**	**0.4972**	**0.4973**	**0.4974**
2.8	**0.4974**	**0.4975**	**0.4976**	**0.4977**	**0.4977**	**0.4978**	**0.4979**	**0.4979**	**0.4980**	**0.4981**
2.9	**0.4981**	**0.4982**	**0.4982**	**0.4983**	**0.4984**	**0.4984**	**0.4985**	**0.4985**	**0.4986**	**0.4986**
3.0	**0.4987**	**0.4987**	**0.4987**	**0.4988**	**0.4988**	**0.4989**	**0.4989**	**0.4989**	**0.4990**	**0.4990**

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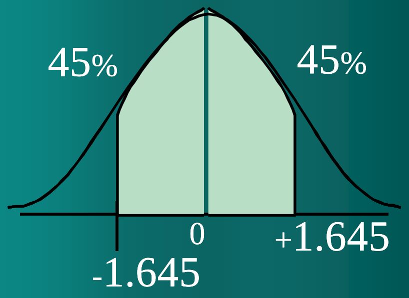
- 3-between what two cholesterol values will 90% of all cholesterol values for a normal man. From standard normal table, find the area
- *between (0 to Z, or 0 to - Z) is 0.45.* Thus, Z = ±1.645.

$$Z = \frac{X_i - m}{s},$$

$$-1.645 = \frac{X_1 - 242.2}{45.4}, \qquad 1.645 = \frac{X_2 - 242.2}{45.4}$$

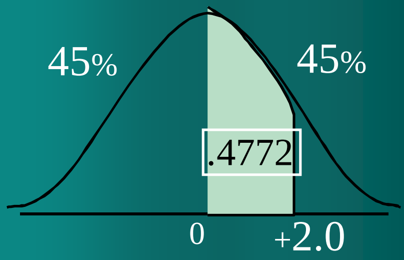
$$X_1 = 167.517, \qquad X_2 = 316.883$$

$$Thus \quad P(167.5 \le X \le 316.883)$$



<u>Example</u>: Find the area under the curve above the Z-axis,
1.between 0 and 2,from standard normal table,

$$P(0 \le Z \le 2) = 0.4772$$



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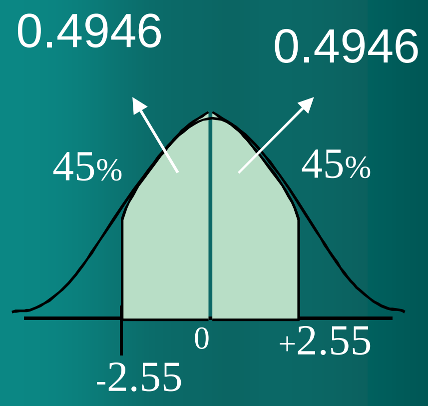
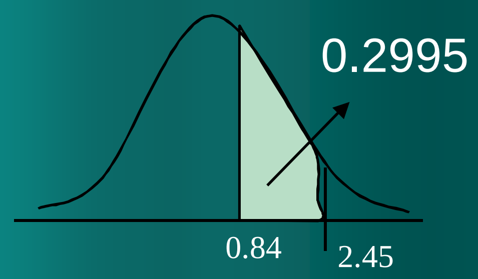
- 2- between –2.55 and 2.55, *from standard normal able*,0.502.550.49460.4946-2.55



- $$P(-2.55 \le Z \le 2.55) = 2P(Z \le 2.55)$$
$$= 2(0.4946) = 0.9892$$

- 3- between 0.84 and 2.45

$$P(0.84 < Z \le 2.45)$$
$$= P(0 \le Z \le 2.45) - P(0 \le Z \le 0.84)$$
$$= 0.4929 - 0.2995 = 0.1934$$



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