Engineering mechanics
"Static"

lecture 1

Force System

Before dealing with a group or system of forces, it is necessary to examine the properties of a single force in some detail, A force has been define as an action of one body on another. In dynamics we will see that a force is defined as an action which tends to cause acceleration of a body. A force is a vector quantity, because its effect depends on the direction as well as on the magnitude of the action. Thus, the forces may be combined according to the parallelogram taw of vector addition.

The action of the cable tension on the bracket in Fig.1a is represented in the side view,.Fig.2b, by the force vector P of magnitude P. The effect of this action on the bracket depends on P, the angle θ, and the location of the point of application A. changing any one of these three specifications will alter the effect on the bracket, such as the forces in one of the bolts which secure the bracket to the base, or the internal the complete specification of the action of a force must include its magnitude, direction, and point application, and therefore we must treat it as a fixed vector.

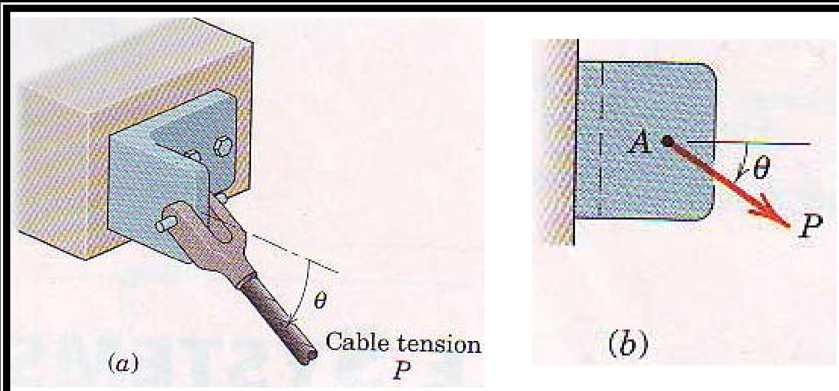


Figure 1

External and internal Effects

We can separate the action of a force on a body into two effects, External and internal , for the bracket of Fig.2 the effects of P external to the bracket are the reactive forces(not shown) exerted on the bracket by the foundation and bolts because of the action of P. forces external to a body can be either applied. forces or reactive forces. The effects of P internal to the bracket are the resulting internal forces and deformations distributed throughout the material of the bracket. The rotation between internal forces and internal deformations depends on the material properties of the body and is studied in strength of materials, elasticity, and plasticity.

Principle of transmissibility

When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force P action on the rigid plate in Fig.2 may be applied at A or at B or at any other point on its line of action, and the net external effects of P on the bracket will not change. The external effect are the force exerted on the plate by the bearing support at 0 and the force exerted on the plate by the roller support at C.

This conclusion is summarized by the principle of transmissibility, which states that a force may be applied at any point on its given line of action without altering the resultant effects of the force external to the rigid body on which it acts. Thus, whenever we are interested in only the resultant external effects of force, the force may be treated as a sliding vector, and we need specify only the magnitude, direction, and line of action of the force, and not its point of application.

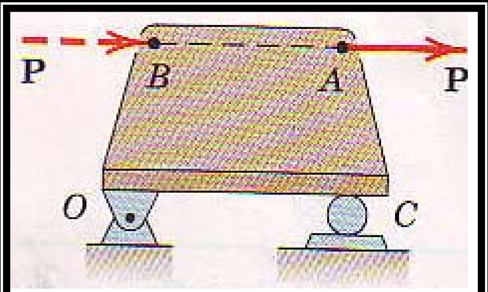


Figure 2

Force Classification

Forces are classified as either contact or body forces. A contact force Is produced by direct physical contact; an example is the force exerted on a body a supporting surface. On the other hand, a body force is generated by virtue of the position of a body within a force field such as A gravitational, electric, or magnetic field. An example of a body force is your weight.

Forces may be further classified as either concentrated or distributed. Every contact force is actually applied over a finite area and is therefore really a distributed force However, when the dimensions of the area are very small compared with the other dimensions of the body, we may consider the force to be concentrated at a point with negligible loss of accuracy. Force can be distributed over an area as in the case of mechanical contact, over a volume when a body force such as weight is acting or over a line, as in the case of the weight of a suspended cable.

The weight of a body is the force of gravitational attraction distributed over its volume and may be taken as a concentrated force acting through the center of gravity. The position of the center of gravity is frequently obvious if the body is symmetric.

We can measure a force either by comparison with other known forces, using a mechanical balance, or by the calibrated movement of an elastic element. AIl such comparisons or calibrations have as their basis a primary standard. The standard unit of force in SI units is the Newton (N) and in the U.S. customary system is the pound (lb).

action and Reaction

According to Newton's third law, the action of a force is always accompanied by an equal and apposite reaction. It is essential to distinguish between the action and the reaction in a pair of forces. To do so, we first isolate the body in question and then identify the force exerted on that body (not the force exerted by the body). It is very easy to mistakenly use the wrong force of the pair unless we distinguish carefully between action and reaction.

Concurrent Forces

Two or more forces are said to be concurrent at a point if their lines of action intersect at that point. The forces F1 and F2 shown in Fig.3a have a common point of application and are concurrent at the point A. Thus, they can he added using the parallelogram law in their common plane to obtain their sum or resultant R, as shown in Fig. 3a. The resultant lies in the same plane as Fl and F2.

Suppose the two concurrent forces lie in the same plane but are applied at two different points as in Fig. 3b. By the principle of transmissibility, we may move them along their lines of action and complete their vector sum R at the point of concurrent A, as shown in Fig. 3b. We can replace F1 and F2 with the resultant R without altering the external effects on the body upon which they act.

We can also use the triangle law to obtain R, but we need to move the line of action of one of the forces, as shown in Fig.3c. If we add the same two forces, as shown in Fig. 3d, we correctly preserve the magnitude and direction of R, but we lose the correct line of action, because R obtained in this way does not pass through A. Therefore this two of combination should be avoided.

We can express the sum of the two forces mathematically by the vector equation

$$\mathbf{R=F1+F2}$$

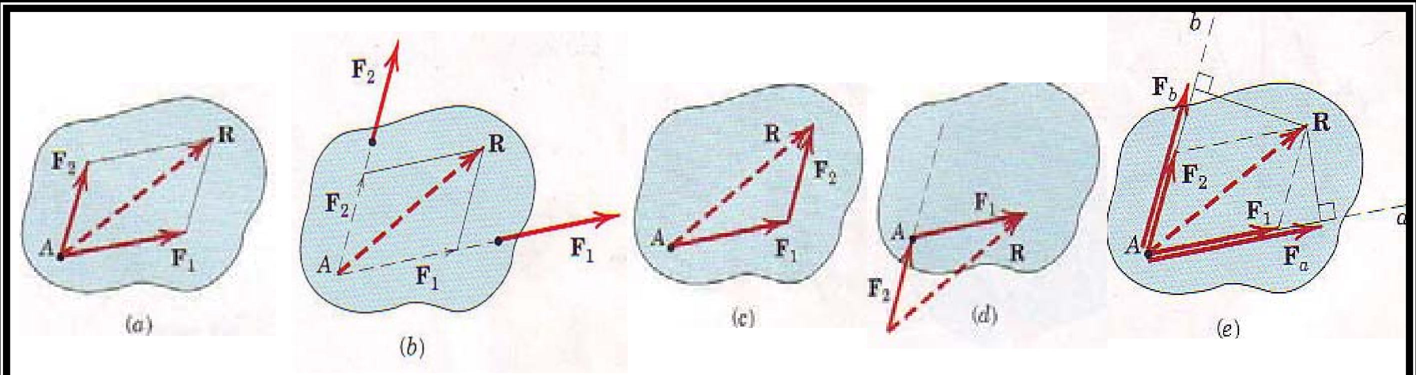


Figure 3

Vector Components

In addition to combining forces to obtain their resultant, we often need to replace a force by its vector components in directions which are convenient for a given application. The vector sum of the components must equal the original vector. Thus, the force R in Fig. 3a may be replaced by, or .resolved into, two vector components F1 and F2 with the specified directions by completing the parallelogram as shown to obtain the magnitudes of F1and F2.

The relationship between a force and its vector components along given axes must not be contused with the relationship between a force and its perpendicular projections onto the same axes. Fig.3e shows the perpendicular projections Fa and Fb of the given force R onto axes a and b, which are parallel to the vector components F1 and F2 of Fig.3a. Figure 3e shows that the components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore, the vector sum of the projections Fa and Fb is not the vector R, because the parallelogram law of vector addition must be used to form the sum. The components and projections of R are equal only when the axes a and b are perpendicular.

A Special Case of Vector Addition

To obtain the resultant when the two forces Fl and F2 are parallel as in Fig. 4, we use a special case of addition. The two vectors are combined by first adding two equal, opposite, and collinear forces F and -F of convenient magnitude, which taken together produce no external effect on the body. adding F1 and F to produce R1 , and combining with the sum R2 of F2 and F yield the resultant R, which is correct in magnitude, direction, and line of action. This procedure is also useful for graphically combining two forces which have a remote and inconvenient point of concurrency because they are almost parallel.

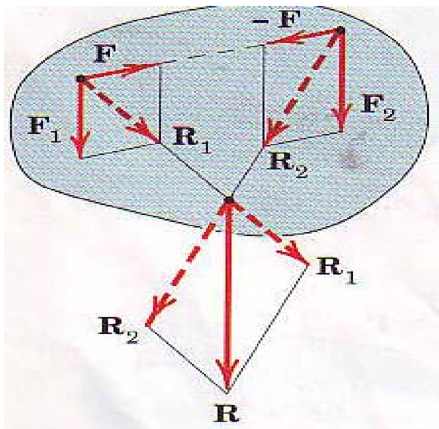


Figure 4

Rectangular Components

The most common two dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector F of Fig. 5 may be written as

F=Fx +Fy

Where Fx and Fy are vector components of F in the x- and y-direction.

For the force vector of Fig. 5, the x and y scalar components arc both positive and are related to the magnitude and direction of F by

$$F_x = F\cos\theta \qquad F = \sqrt{F_x{}^2 + F_y{}^2}$$
$$F_y = F\sin\theta \qquad \theta = \tan^{-1}\frac{F_y}{F_x}$$

…... Eqs.1

Determining the Components of a Force

Dimensions are not always given in horizontal and vertical directions, angles need not be measured counterclockwise from the x-axis, and the origin of coordinate need not be on the line of action of a force. Therefore, it is essential that we be able to determine the correct components of a force no matter how the axes are oriented or how the angles are measured. Figure 6 suggests a few typical examples of vector resolution in two dimensions.

Memorization of Eqs.1 is not a substitute for understanding the parallelogram law and for correctly projecting a vector onto a reference axis. A neatly drawn sketch always he1p6 to clarify the geometry and avoid error.

Rectangular components arc convenient for finding the sum or resultant R of two forces which are concurrent. Consider two forces F1and F2 which are originally concurrent at a point O. Figure 7 shows the line of action of F2 shifted from O to the tip of F1accoding to the triangles rule of Fig. 3 In adding the force vectors F1 and F2, we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\mathbf{j}$$

From which we conclude that

$$R_x = F_{1_x} + F_{2_x} = \Sigma F_x$$
$$R_y = F_{1_y} + F_{2_y} = \Sigma F_y$$

……2

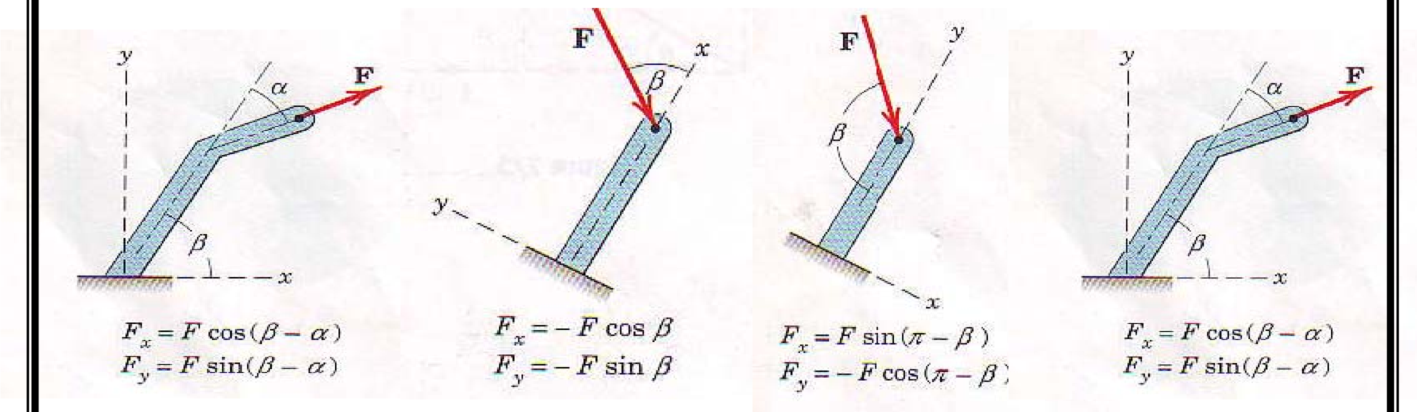


Figure 6

The term ΣFx means "the algebraic sum of the x scalar components". For" the example- shown In Fig. 7, note that the scalar component F_{2y} would be negative.

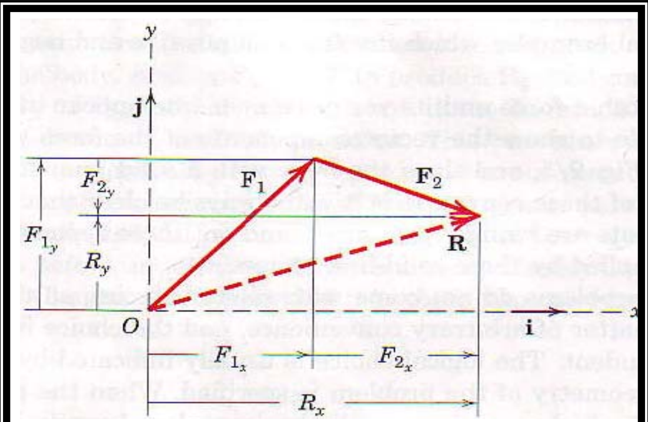
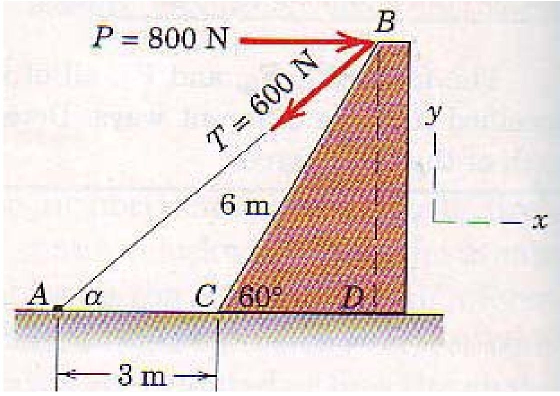


Figure 7

Examples

Example 1

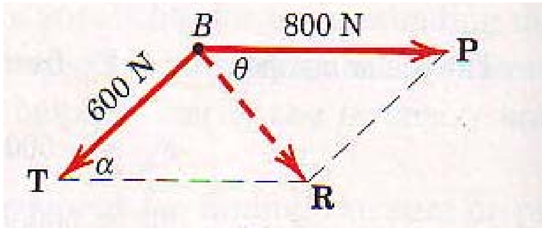
Combine the two forces p and T, which act on the fixed structure at B, into a single equivalent force R.



Graphical solution

The parallogram for the vector addition of forces T and P is constructed as shown in Fig.a . the approxmate scale used here is 1cm=400n; a scale of 1cm = 100 N would be more suitable for regular- size paper and would give greater accuracy. Note that the angle α must be determined prior to construction of the parallogram. From the given figure

$$\tan\alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6\sin 60^\circ}{3 + 6\cos 60^\circ} = 0.866 \qquad \alpha = 40.9^\circ$$



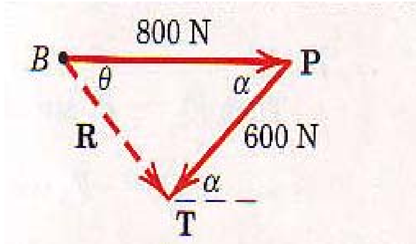
(a)

Measurment of the length R and direction θ of the resultant force R yield the approximate results

$$R = 525\text{ N} \qquad \theta = 49^\circ$$

Geometric solution

The triangle for the vector addition of T and P is shown in Fig, b. the angle α is calculated as above. The law of cosines gives



(b)

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)\cos 40.9^\circ = 274{,}300$$

$$R = 524\text{ N}$$

frome the law sines, we may determine the angle θ which orients R. thuse,

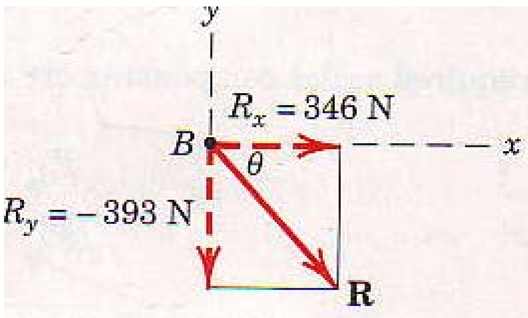
$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9^\circ} \qquad \sin\theta = 0.750 \qquad \theta = 48.6^\circ$$

Algebric solution

By using the x-y coordinate system on the given figure, we may write

$$R_x = \Sigma F_x = 800 - 600\cos 40.9^\circ = 346\text{ N}$$

$$R_y = \Sigma F_y = -600\sin 40.9^\circ = -393\text{ N}$$



(c)

The magintude and direction of the resultant force R as shown in Fig, c are then

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(346)^2 + (-393)^2} = 524\text{ N}$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6^\circ$$

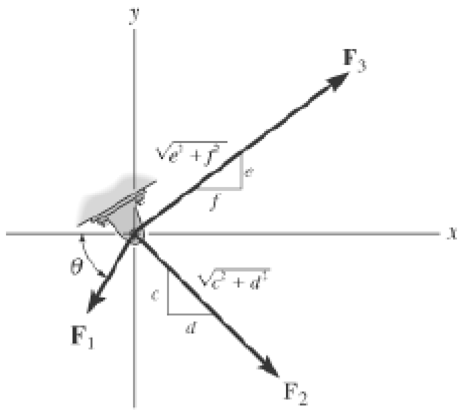
Examples 2:

Determine the magnitude of the resultant force and its direction measured clockwise from the positive *x* axis.

Units Used:

$kN = 10^3$ N

Given:
$F1$ = 20 kN
$F2$ = 40 kN
$F3$ = 50 kN
θ = 60 deg
c = 1
d = 1
e = 3
f = 4



Solution:

$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $$F_{Rx} = F_3\left(\frac{f}{\sqrt{e^2+f^2}}\right) + F_2\left(\frac{d}{\sqrt{c^2+d^2}}\right) - F_1\cos(\theta)$$

$$F_{Rx} = 58.28\,\text{kN}$$

$+\uparrow F_{Ry} = \Sigma F_y;$ $$F_{Ry} = F_3\left(\frac{e}{\sqrt{e^2+f^2}}\right) - F_2\left(\frac{c}{\sqrt{c^2+d^2}}\right) - F_1\sin(\theta)$$

$$F_{Ry} = -15.6\,\text{kN}$$

$$F = \sqrt{F_{Rx}{}^2 + F_{Ry}{}^2}$$

$$F = 60.3\,\text{kN}$$

$$\theta = \operatorname{atan}\left(\frac{|F_{Ry}|}{F_{Rx}}\right)$$

$$\theta = 15\,\text{deg}$$

Example 3

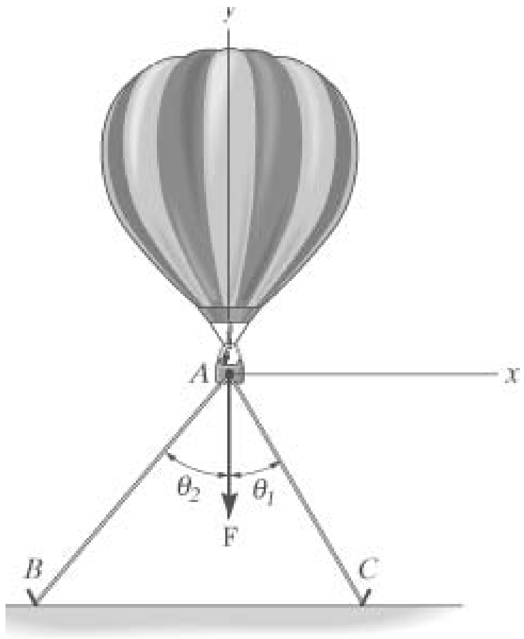
A resultant force **F** is necessary to hold the ballon in place. Resolve this force into components along the tether lines *AB* and *AC*, and compute the magnitude of each component.

Given:

$F = 350 \text{ lb}$

$\theta_1 = 30 \text{ deg}$

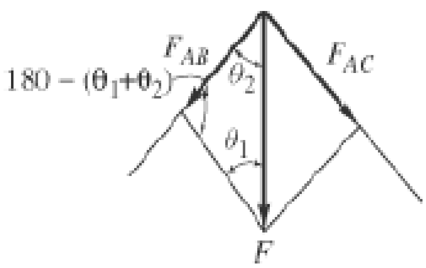
$\theta_2 = 40 \text{ deg}$

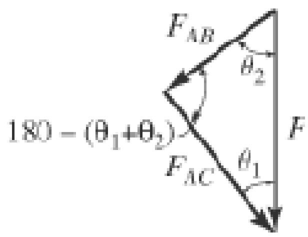


Solution:

$$\frac{F_{AB}}{\sin(\theta_1)} = \frac{F}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]}$$

$$F_{AB} = F\left[\frac{\sin(\theta_1)}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]}\right]$$





$$F_{AB} = 186 \text{ lb}$$

$$\frac{F_{AC}}{\sin(\theta_2)} = \frac{F}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]}$$

$$F_{AC} = F\left[\frac{\sin(\theta_2)}{\sin[180 \text{ deg} - (\theta_1 + \theta_2)]}\right]$$

$$F_{AC} = 239 \text{ lb}$$

Problems

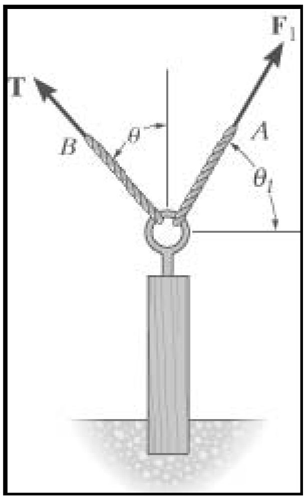
The post is to be pulled out of the ground using two ropes *A* and *B*. Rope *A* is subjected to force $F1$ and is directed at angle θ1 from the horizontal. If the resultant force acting on the post is to be FR, vertically upward, determine the force **T** in rope *B* and the corresponding angle θ.

Given:

FR = 1200 lb

$F1$ = 600 lb

θ1 = 60 deg



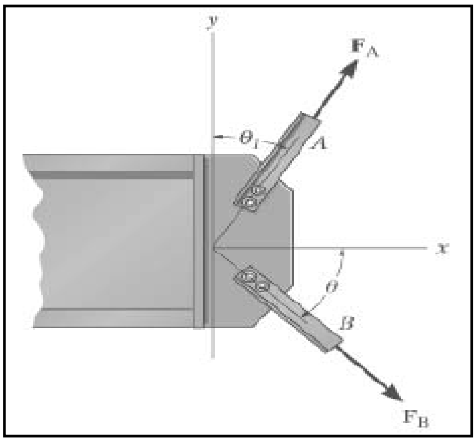
The plate is subjected to the forces acting on members *A* and *B* as shown. Determine the magnitude of the resultant of these forces and its direction measured clockwise from the positive *x* axis. Given:

FA = 400 lb

FB = 500 lb

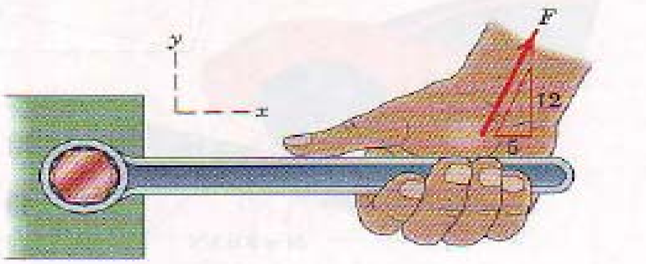
θ1 = 30 deg

θ = 60 deg

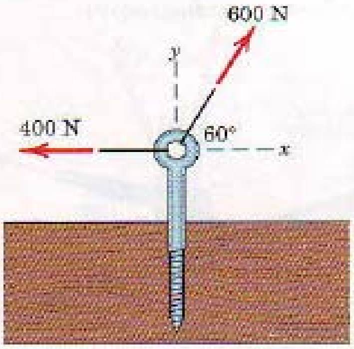


The y-component of the force $\mathbf{F}$ which a person exerts on the handle of the box wrench is known to be 320 N. Determine the x-component and the magnitude of $\mathbf{F}$.

Ans. $F_x = 133.3$ N, $F = 347$ N

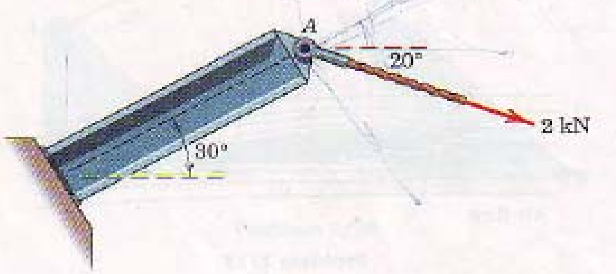


Determine the resultant $\mathbf{R}$ of the two forces shown by (*a*) applying the parallelogram rule for vector addition and (*b*) summing scalar components.

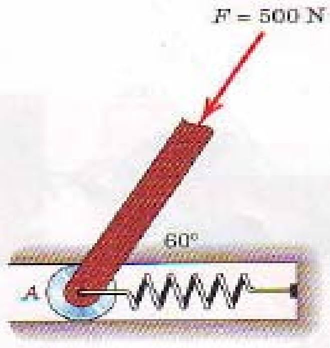


To satisfy design limitations it is necessary to determine the effect of the 2-kN tension in the cable on the shear, tension, and bending of the fixed I-beam. For this purpose replace this force by its equivalent of two forces at A, F_t parallel and F_n perpendicular to the beam. Determine F_t and F_n.

Ans. $F_t = 1.286$ kN, $F_n = 1.532$ kN

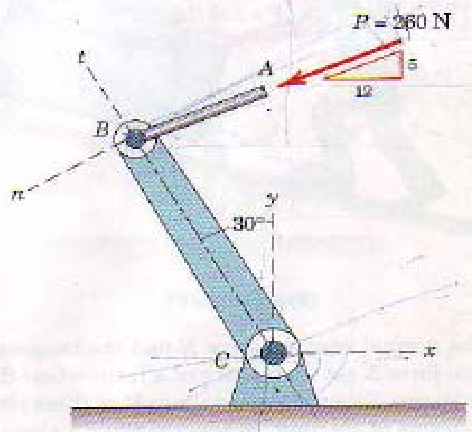


Determine the magnitude F_s of the tensile spring force in order that the resultant of $\mathbf{F}_s$ and $\mathbf{F}$ is a vertical force. Determine the magnitude R of this vertical resultant force.



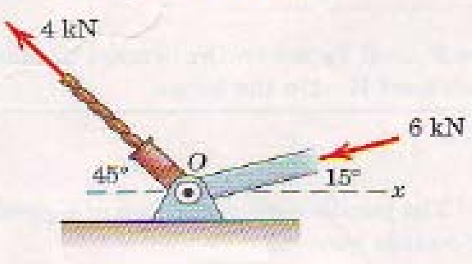
In the design of a control mechanism, it is determined that rod AB transmits a 260-N force $\mathbf{P}$ to the crank BC. Determine the x and y scalar components of $\mathbf{P}$.

Ans. $P_x = -240$ N
$P_y = -100$ N

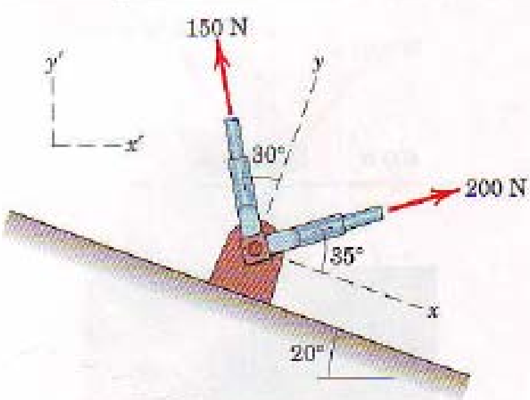


For the mechanism of Prob. 2/11, determine the scalar components P_t and P_n of $\mathbf{P}$ which are tangent and normal, respectively, to crank BC.

The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant $\mathbf{R}$ of the two forces and the angle θ which $\mathbf{R}$ makes with the positive x-axis.

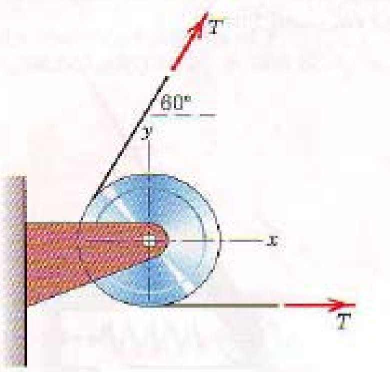


Determine the resultant $\mathbf{R}$ of the two forces applied to the bracket. Write $\mathbf{R}$ in terms of unit vectors along the x- and y-axes shown.

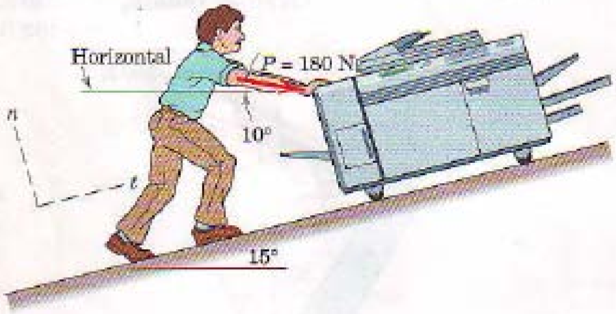


If the equal tensions T in the pulley cable are 400 N, express in vector notation the force **R** exerted on the pulley by the two tensions. Determine the magnitude of **R**.

Ans. $\mathbf{R} = 600\mathbf{i} + 346\mathbf{j}$ N, $R = 693$ N

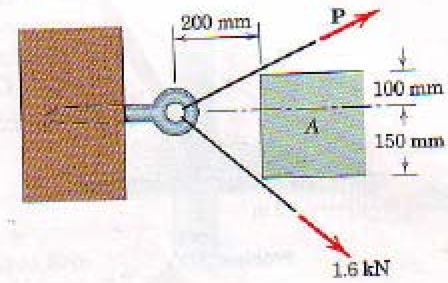


While steadily pushing the machine up an incline, a person exerts a 180-N force **P** as shown. Determine the components of **P** which are parallel and perpendicular to the incline.

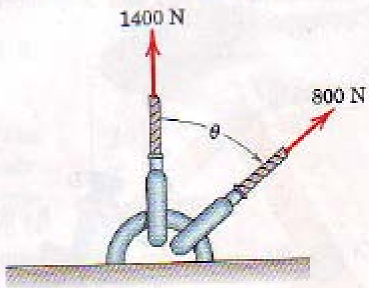


It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction A prevents direct access, so that two forces, one 1.6 kN and the other **P**, are applied by cables as shown. Compute the magnitude of **P** necessary to ensure a resultant **T** directed along the spike. Also find T.

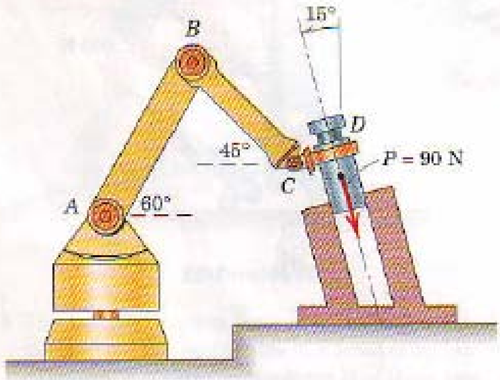
Ans. $P = 2.15$ kN
$T = 3.20$ kN



At what angle θ must the 800-N force be applied in order that the resultant **R** of the two forces has a magnitude of 2000 N? For this condition, determine the angle β between **R** and the vertical.



In the design of the robot to insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90-N force P on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts *on* the robot along axes (*a*) parallel and perpendicular to the arm AB, and (*b*) parallel and perpendicular to the arm BC.



Lecture 2

Moment

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the tine of action of the force. This rotational tendency is known as the moment M of the force. Moment is also refereed to as torque.

As a familiar example of the concept of moment, consider. the pipe wench of Fig. a. One effect of the force applied perpendicular to the handle of the wench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude F of the force and the effective length d of the wrench handle. Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.

Moment about a Point

Figure b shows a two-dimensional body acted on by a force F in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis O-O perpendicular to the plane of the body is proportional both to the magnitude of the force and to the moment arm d, which is the perpendicular distance from the axis to the line of l action of the force. Therefore ,the magnitude of the moment is defined as

$$\boxed{M = Fd}$$

The moment is a vector M perpendicular to the plane of the body. The sense of M depends on the direction in which F tends to rotate the body The right-hand rule, Fig.1c, is used to identify this sense. We represent the moment of F about O-O as a vector pointing in the direction of the thumb, with the finger curled in the direction of the relational tendency.

The moment M obeys all the rules of vector combination and may be considered a sliding vector with a line of action coinciding with the moment axis. The basic units of moment in SI units are Newton-meters (N.m), and in the U.S. customary system are pound-feet (ob-ft).

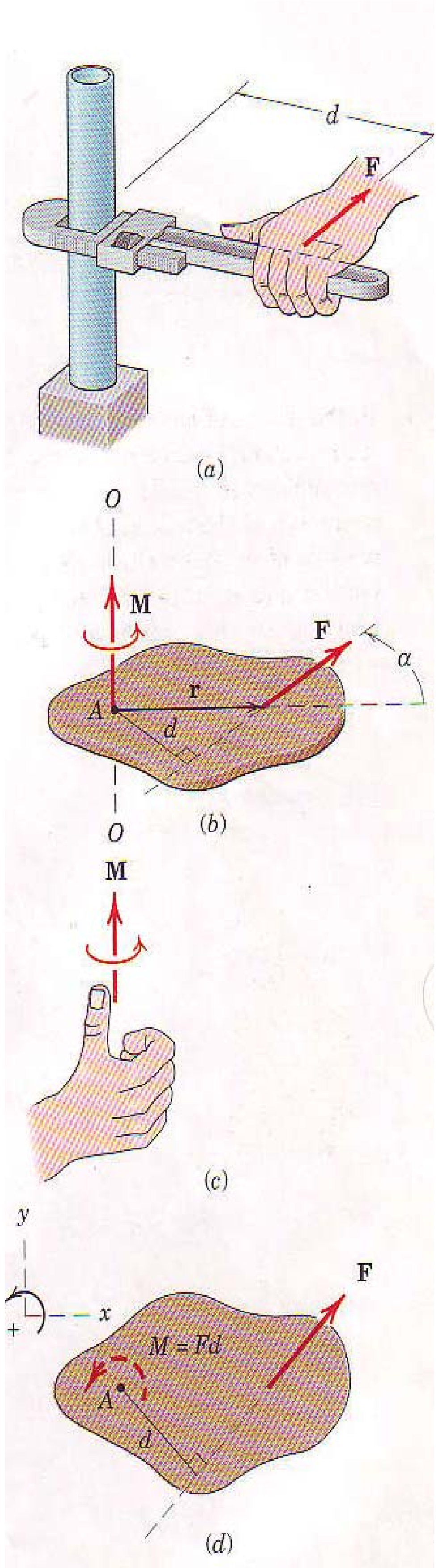


Figure 1

When dealings with forces which all act in a given plane, we customarily speak of the moment about a point. By this we mean the moment with respect to an axis normal to the plane and passing through the point. Thus, the moment of force F about point A in Fig.d has the magnitude M =Fd and is counterclockwise.

Moment directions may be accounted for by using a stated sign convention. such as a plus sign (+) for counterclockwise moment and a minus sign! (+) for clockwise moments, or vice versa. Sign consistency within a given problem is essential. For the sign convention of Fig.d, the moment of F about point A (or about the z-axis passing through point A) is positive. The curved arrow of the figure is a convenient way to represent moments in two-dimensional analysis.

Varignon's theorem

One of the useful principles of mechanics is Varignon's theorem, which states that the moment of a force about any point is equal to the sum of the moment of the components of the force about the same point.

To prove this theorem, consider the force R acting in the plane of the body shown in Fig. 2a. The forces P and Q represent any two nonrectangular components of R. The moment of R about point O is

$$\mathbf{M_o = r \times R}$$

Because R=P + Q, we may write

$$\mathbf{r \times R = r \times (P+Q)}$$

Using the distributive law for cross products, we have

$$\mathbf{M_o = r \times R = r \times P + r \times Q} \qquad \dots\dots\dots\dots$$

which says that the moment of R about O equals the sum of the moments about O of its components P and Q. This proves the theorem.

Varignon's theorem need not be restricted to the case of two component, but it applies equally well to three or more. Thus we could have used any number of concurrent components of R in the foregoing proof

figure 2b illustrates the usefulness of Varignon's theorem. The moment of R about point O is Rd. However, if d is more difficult to determine than p and q, we can resolve R into the components P and Q, and compute the moment as

$$\mathbf{M_o = Rd = -pP + qQ}$$

where we take the clockwise moment sense to be positive. Sample Problem 1 shows how Varignon's theorem can help us to calculate moments.

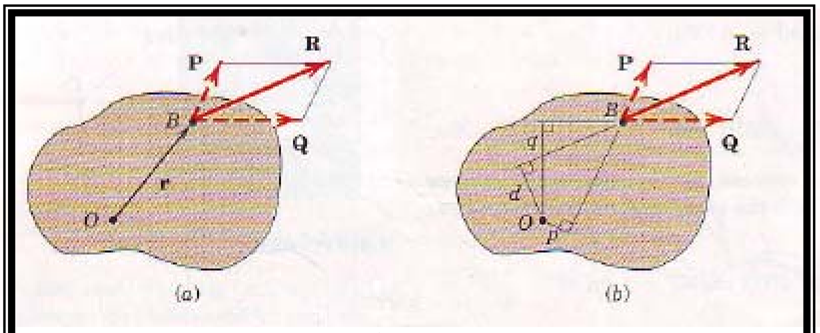
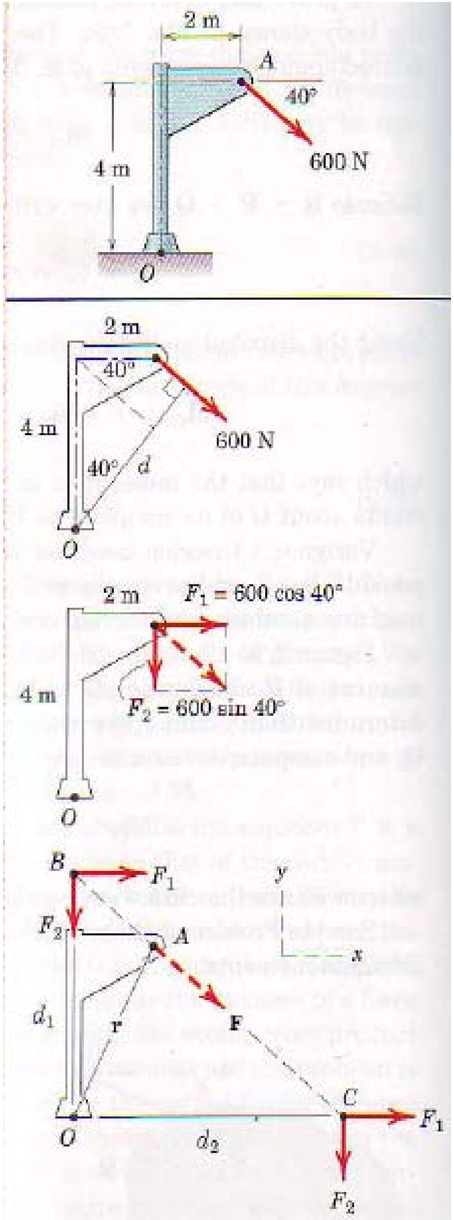


Figure 2

Examples

Example 1

Calculate the magnitude of the moment about the base point O of the 600N force in five different way



Solution

(I) The moment arm to the 600-N force is

$$\mathbf{d = 4\cos 40^\circ + 2\sin 40^\circ = 4.35\ m}$$

(1) By M = rd the moment is clockwise and his the magnitude

$$\mathbf{M_o = 600(4.\ 35) = 2610\ N.m}$$

(II) Replace the force by its rectangular components at A

$$\mathbf{F1 = 600\cos 40^\circ = 460\ N, \quad F2 = 600\sin 40^\circ = 386N}$$

(2)By Varignon's theorem, the moment becomes

$$\mathbf{M_o = 460(4) + 386(2) = 2610N}$$

(III) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F2. The moment arm of F1 becomes

$$\mathbf{d1 = 4 + 2\tan 40^\circ = 5.68\ m}$$

and the moment is

$$\mathbf{M_o = 460(5.68) = 2610\ N.m}$$

(3) (IV) Moving the force to point C eliminates the moment of the component F1. The moment am of F2 becomes

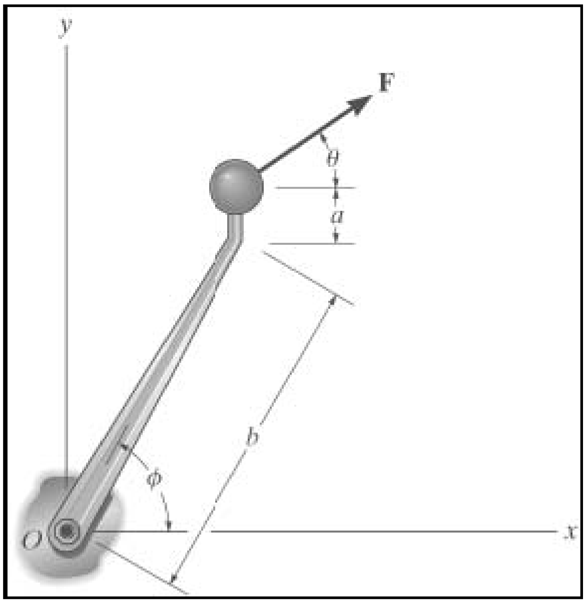
$$\mathbf{d2 = 2 + 4\cos 40^\circ = 6.77\ m}$$

and the moment is

$$\mathbf{M_o = 386(6.77) + 2610\ N.m}$$

Example 2

Determine the angle θ (0 <= θ <= 90 deg) so that the force **F** develops a clockwise moment *M* about point *O*.



Given:

$F = 100$ N $\varphi = 60$ deg

M 20 = N.m $a = 50$ mm

$\theta = 30$ deg $b = 300$ mm

Solution:

Initial Guess $\theta = 30$ deg

Given

$M=F\cos(\theta)(a+b\sin(\varphi))-F\sin(\theta)(b\cos(\varphi))$

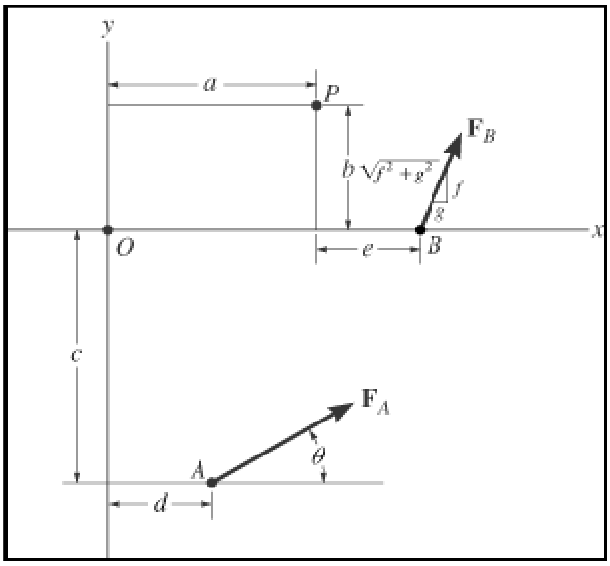
$\theta = \text{Find}(\theta)$ $\theta = 28.6$ deg

Example 3

Determine the magnitude and directional sense of the moment of the forces

(1) about point *O*.

(2) about point *P*.



Given:

$F_B = 260$ N $e = 2$ m

$a = 4$ m $f = 12$

$b = 3$ m $g = 5$

$c = 5$ m $\theta = 30$ deg

$d = 2$ m $F_A = 400$ N

Solution:

(1)

$$M_O = F_A\sin(\theta)d + F_A\cos(\theta)c + F_B\frac{f}{\sqrt{f^2+g^2}}(a+e)$$

$M_O = 3.57\,\text{kN}\cdot\text{m}$ (positive means counterclockwise)

$$M_P = F_B\frac{g}{\sqrt{f^2+g^2}}b + F_B\frac{f}{\sqrt{f^2+g^2}}e - F_A\sin(\theta)(a-d) + F_A\cos(\theta)(b+c)$$

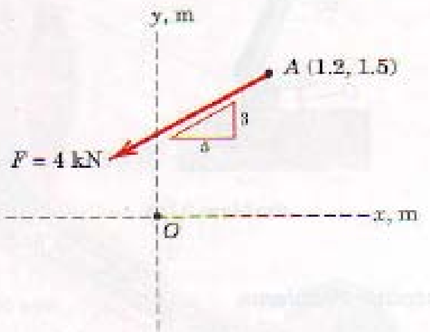
(2)

$M_P = 3.15\,\text{kN}\cdot\text{m}$ (positive means counterclockwise)

Problems

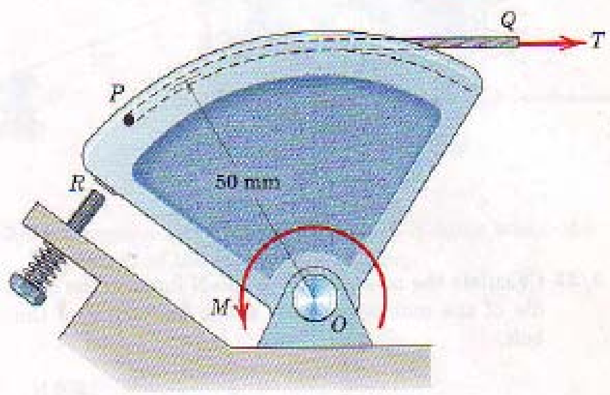
The 4-kN force **F** is applied at point A. Compute the moment of **F** about point O, expressing it both as a scalar and as a vector quantity. Determine the coordinates of the points on the x- and y-axes about which the moment of **F** is zero.

Ans. $M_O = 2.68$ kN·m CCW, $\mathbf{M}_O = 2.68\mathbf{k}$ kN·m
$(x, y) = (-1.3, 0)$ and $(0, 0.78)$ m

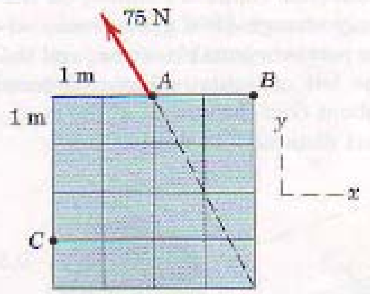


The throttle-control sector pivots freely at O. If an internal torsional spring exerts a return moment $M = 2$ N·m on the sector when in the position shown, for design purposes determine the necessary throttle–cable tension T so that the net moment about O is zero. Note that when T is zero, the sector rests against the idle-control adjustment screw at R.

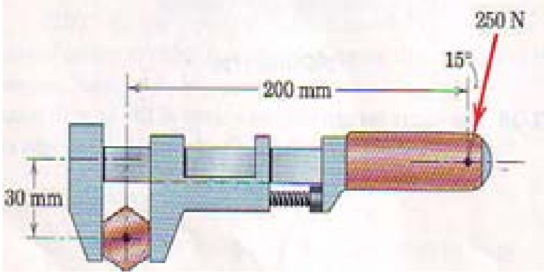
Ans. $T = 40$ N



The rectangular plate is made up of 1-m squares as shown. A 75-N force is applied at point A in the direction shown. Determine the moment of this force about point B and about point C.

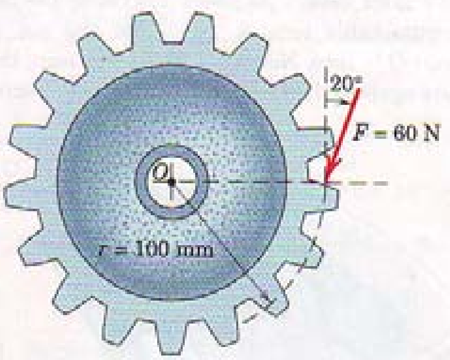


Calculate the moment of the 250-N force on the handle of the monkey wrench about the center of the bolt.



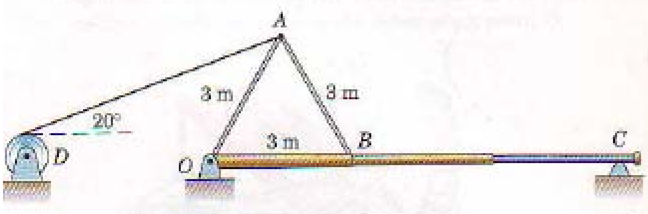
A force **F** of magnitude 60 N is applied to the gear. Determine the moment of **F** about point O.

Ans. $M_O = 5.64$ N·m CW



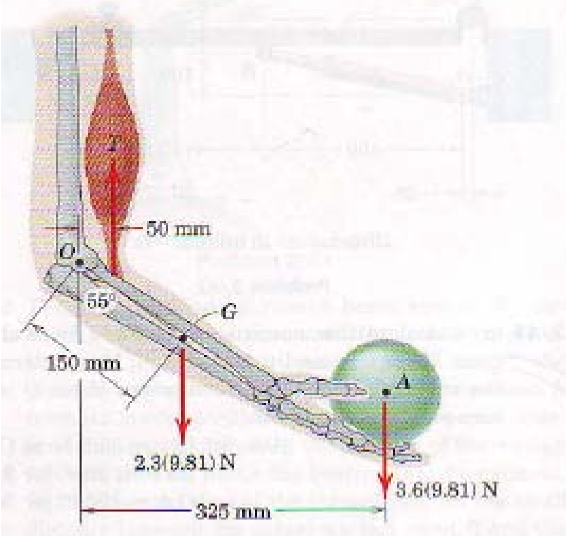
In order to raise the flagpole OC, a light frame OAB is attached to the pole and a tension of 3.2 kN is developed in the hoisting cable by the power winch D. Calculate the moment M_O of this tension about the hinge point O.

Ans. $M_O = 6.17$ kN·m CCW

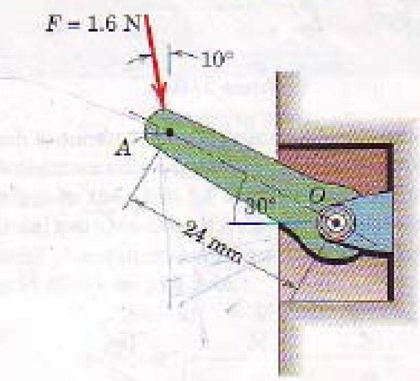


Elements of the lower arm are shown in the figure. The mass of the forearm is 2.3 kg with mass center at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the 3.6-kg homogeneous sphere. What must the biceps tension force T be so that the overall moment about O is zero?

Ans. $M_O = 14.25\ \text{N}\cdot\text{m CW}$, $T = 285\ \text{N}$

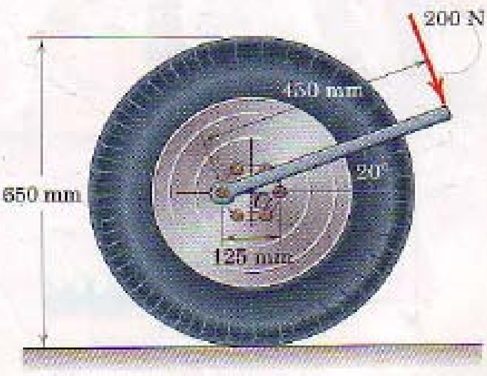


Compute the moment of the 1.6-N force about the pivot O of the wall-switch toggle.

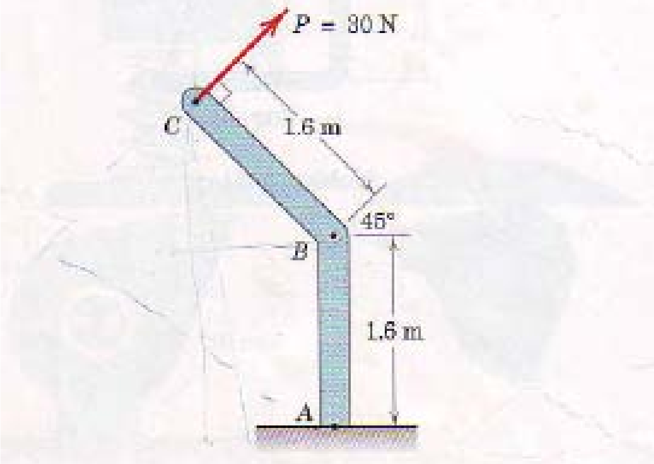


A force of 200 N is applied to the end of the wrench to tighten a flange bolt which holds the wheel to the axle. Determine the moment M produced by this force about the center O of the wheel for the position of the wrench shown.

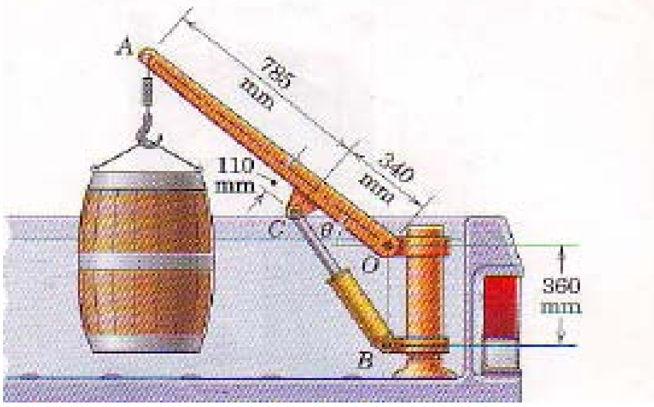
Ans. $M = 78.3\ \text{N}\cdot\text{m CW}$



The 30-N force **P** is applied perpendicular to the portion BC of the bent bar. Determine the moment of **P** about point B and about point A.

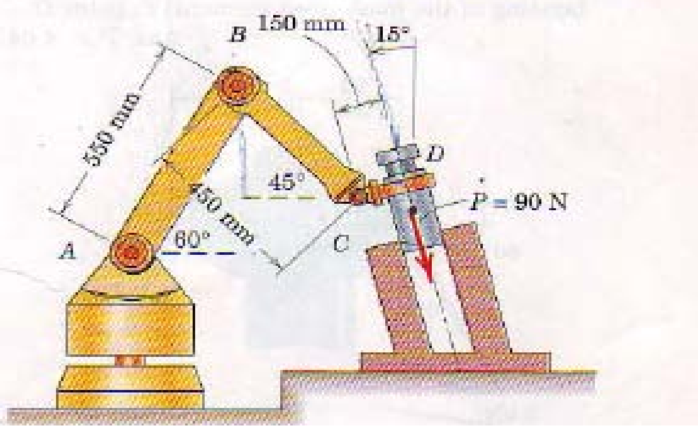


The small crane is mounted along the side of a pickup bed and facilitates the handling of heavy loads. When the boom elevation angle is $\theta = 40°$, the force in the hydraulic cylinder BC is 4.5 kN, and this force applied at point C is in the direction from B to C (the cylinder is in compression). Determine the moment of this 4.5-kN force about the boom pivot point O.



Design criteria require that the robot exert the 90-N force on the part as shown while inserting a cylindrical part into the circular hole. Determine the moment about points A, B, and C of the force which the part exerts on the robot.

Ans. $M_A = 68.8\ \text{N}\cdot\text{m}$, $M_B = 33.8\ \text{N}\cdot\text{m}$
$M_C = 13.50\ \text{N}\cdot\text{m}$ (all CCW)



Lecture 3

Couples

The moment produced by two equal, opposite, and noncollinear forces is called a couple. couples have certain unique properties and have important applications in mechanics.

Consider the action of two equal and opposite forces F and -F a distance d apart, as shown in lfig.1a . These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple M. This couple has a magnitude

$$\mathbf{M= F(a+b)-Fa}$$

Or $$\mathbf{M= Fd}$$

Its direction is counterclockwise when viewed from above for the case Illustrate. Note especially that the magnitude of the couple is dependent of the distance a which locates the forces with respect to the moment center 0. lt follows that the moment of a couple has the same value for all moment centers.

Vector Algebra Method

We may also express the moment of a couple by using vector algebra. With the cross product Eq. the combined moment about point 0 of the couple of Fig. 1b is

$$\mathbf{M = r_A \times F + r_B \times (-F) = (r_A - r_B) \times F}$$

where r_A and r_B are position vector which run from point O to arbitrary points A and B on the tines of action of F and -F, respectively. Because $r_A - r_B = r$, we can express M as

$$\mathbf{M = r \times F}$$

Here again, the moment expression contains no reference to the moment center 0 and, therefore, is the same for all moment centers. Thus, we may represent M by a free vector, as shown in Fig. 1c, where the direction of M is normal to the plane of the couple and sense of M is established by the right-hand rule.

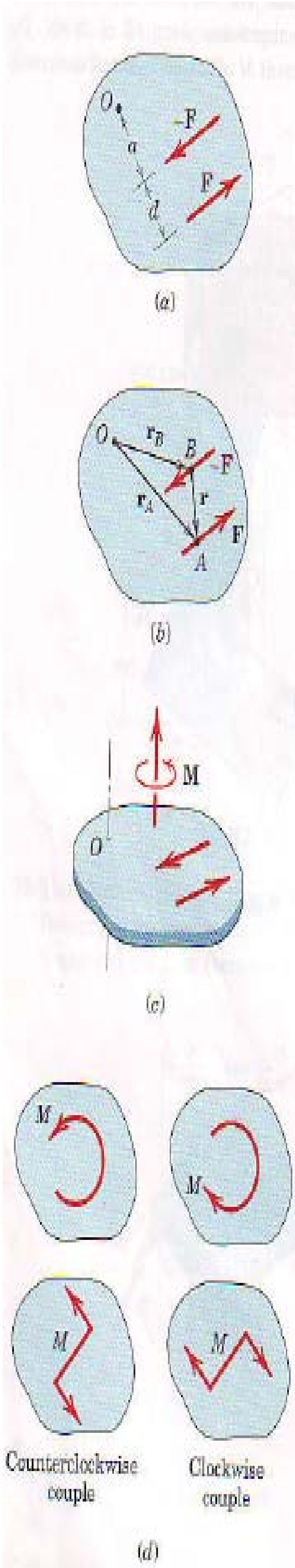


Figure 1

Because the couple vector M is always perpendicular to the plane of the forces which constitute the couple, in two dimensional analysis we can represent the sense of couple vector as clockwise or counterclockwise by one of the convention shown in fig.1d. later, when we deal with couple vectors in three-dimensional problems, we will make full use of vector notation to represent them, and the mathematics will automatically account for their sense.

Equivalent Couples

Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. Figure 2 shows four different configurations of the same couple M. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

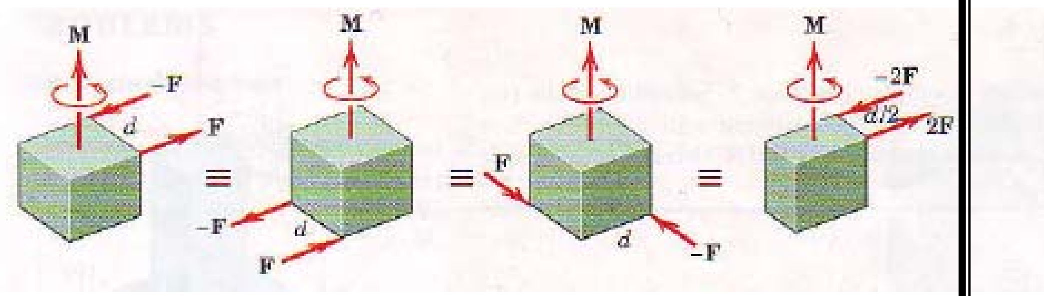


Figure 2

Force-Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in Fig. 3, where the given force F acting at point A is replaced by an equal force F at some point B and the counterclockwise couple M = Fd. The transfer is seen in the middle figure, where the equal and opposite forces F and -F are added at point B without introducing any net external effects on the body. We now see that the original force at A and the equal and opposite one at B constitute the couple M = Fd, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at A by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig.3 is referred to as a force-couple system.

By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force. Replacement of a force by an equivalent force-couple system, and the reverse procedure, have many applications in mechanics and should be mastered.

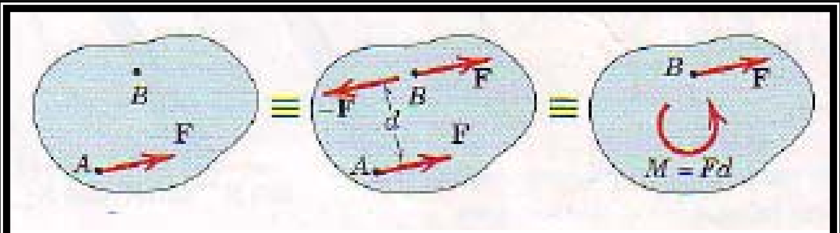
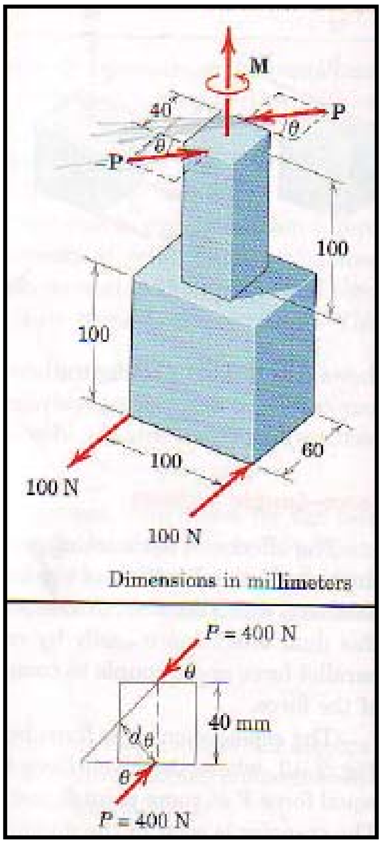


Figure 3

Examples

Example 1

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces P and –P, each of which has a magnitude of 400 N. Determine the proper angle θ.



Solution

The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$$[M = F\ d]$$

$$M = 100(0.1) = 10\ \text{N.m}$$

The forces P and –P produce a counterclockwise couple

$$M = 400(0.040)\cos\theta$$

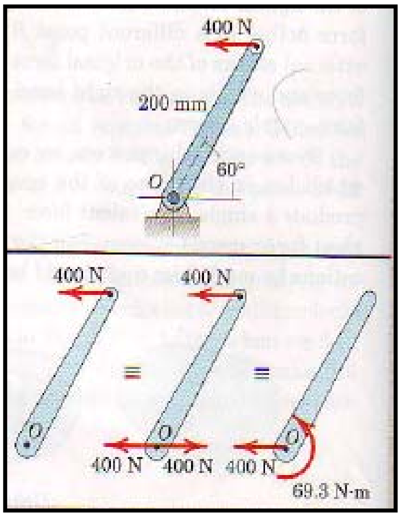
Equating the two expression gives

$$10 = 400(0.040)\ \cos\theta$$

$$\theta = \cos\ (10/16) = 51.3^{\circ}$$

Example 2

Replace the horizontal 400-n force acting on the lever by an equivalent system consisting of a force at O and a couple.



Solution

We apply two equal and opposite 400-N forces at o and identify counterclockwise couple

$$[M = F\ d]$$

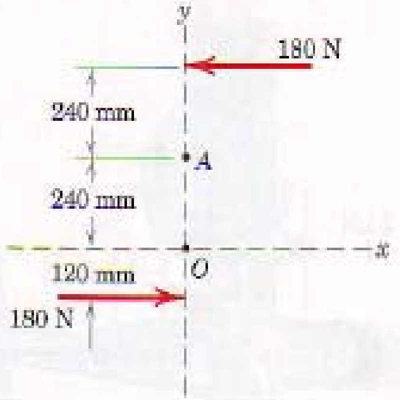
$$M = 400(0.200\sin 60^{\circ}) = 69.3\ \text{N.m}$$

Thus, the original force is equivalent to the 400-n forces at 0 and the 69.3 N.m couple as shown in third of the three equivalent figures

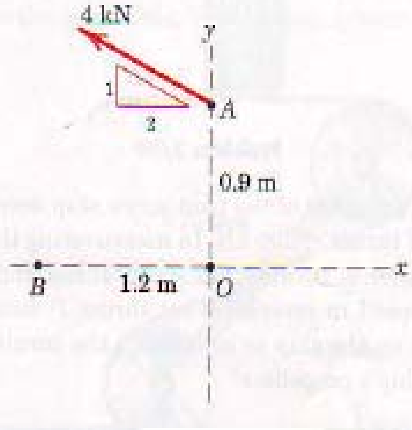
PROBLEMS

Compute the combined moment of the two 180-N forces about (*a*) point *O* and (*b*) point *A*.

Ans. (*a*) $M_O = 108$ N·m CCW
(*b*) $M_A = 108$ N·m CCW

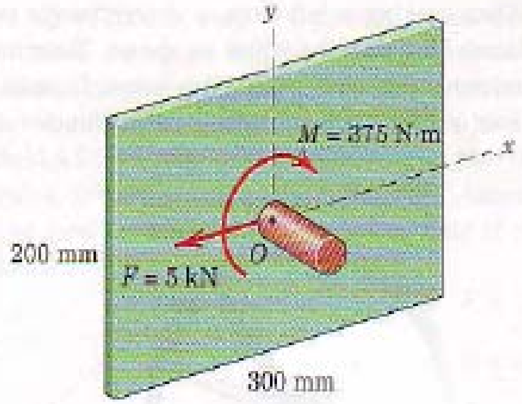


Replace the 4-kN force acting at point *A* by a force-couple system at (*a*) point *O* and (*b*) point *B*.

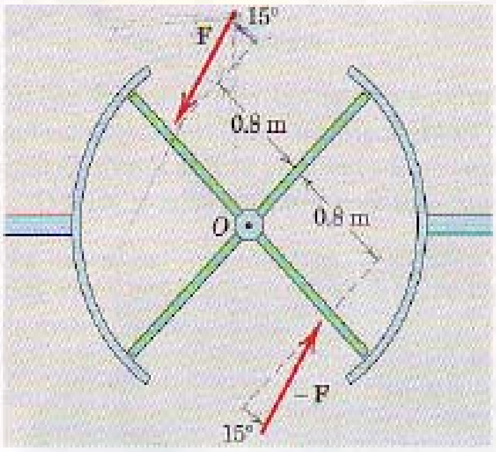


The indicated force-couple system is applied to a small shaft at the center of the rectangular plate. Replace this system by a single force and specify the coordinate of the point on the *y*-axis through which the line of action of this resultant force passes.

Ans. $y = -75$ mm

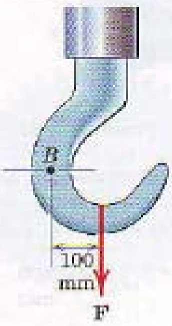


The top view of a revolving entrance door is shown. Two persons simultaneously approach the door and exert forces of equal magnitudes as shown. If the resulting moment about the door pivot axis at *O* is 25 N·m, determine the force magnitude *F*.

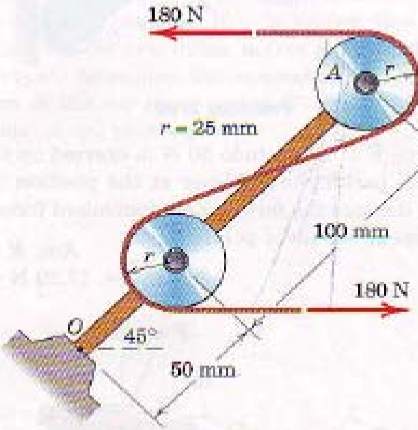


In the design of the lifting hook the action of the applied force **F** at the critical section of the hook is a direct pull at B and a couple. If the magnitude of the couple is 4000 N·m, determine the magnitude of **F**.

Ans. $F = 40$ kN

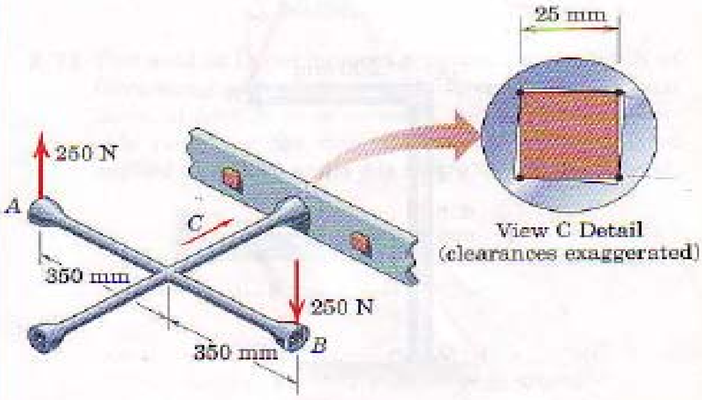


The system consisting of the bar OA, two identical pulleys, and a section of thin tape is subjected to the two 180-N tensile forces shown in the figure. Determine the equivalent force–couple system at point O.

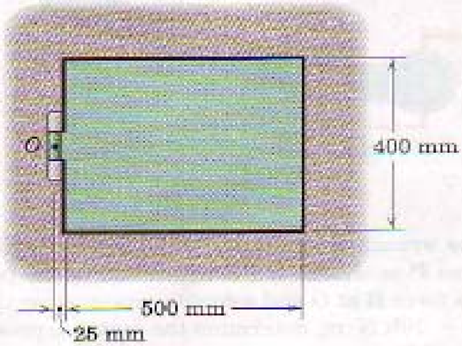


A lug wrench is used to tighten a square-head bolt. If 250-N forces are applied to the wrench as shown, determine the magnitude F of the equal forces exerted on the four contact points on the 25-mm bolt head so that their external effect on the bolt is equivalent to that of the two 250-N forces. Assume that the forces are perpendicular to the flats of the bolt head.

Ans. $F = 3500$ N

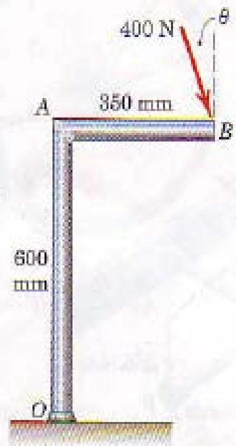


The inspection door shown is constructed of sheet steel which is 3 mm thick. Determine the force–couple system located at the hinge center O which is equivalent to the weight of the door. State any assumptions.

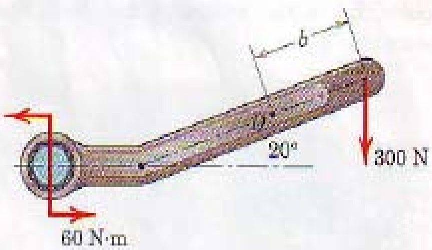


A 400-N force is applied to the welded slender bar at an angle $\theta = 20^\circ$. Determine the equivalent force–couple system acting on the weld at (*a*) point *A* and (*b*) point *O*. For what value of θ would the results of parts (*a*) and (*b*) be identical?

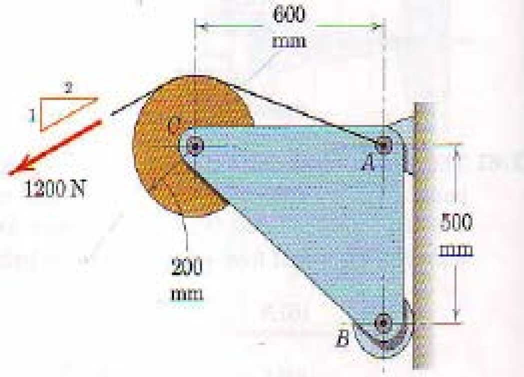
Ans. (*a*) $F = 400$ N, $M_A = 131.6$ N·m CW
(*b*) $F = 400$ N, $M_O = 214$ N·m CW
$\theta = 0$ or 180°



Replace the couple and force shown by a single force **F** applied at a point *D*. Locate *D* by determining the distance *b*.

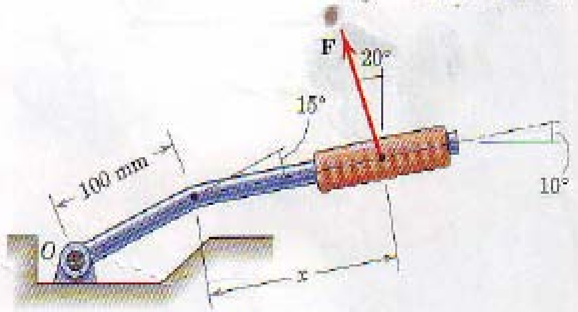


Calculate the moment of the 1200-N force about pin *A* of the bracket. Begin by replacing the 1200-N force by a force–couple system at point *C*.



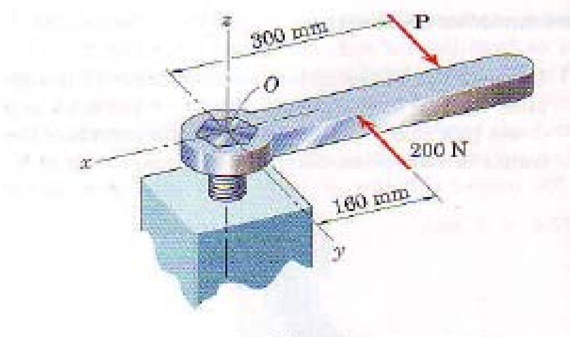
A force **F** of magnitude 50 N is exerted on the automobile parking-brake lever at the position $x = 250$ mm. Replace the force by an equivalent force–couple system at the pivot point *O*.

Ans. $R = 50$ N
$M_O = 17.29$ N·m CCW

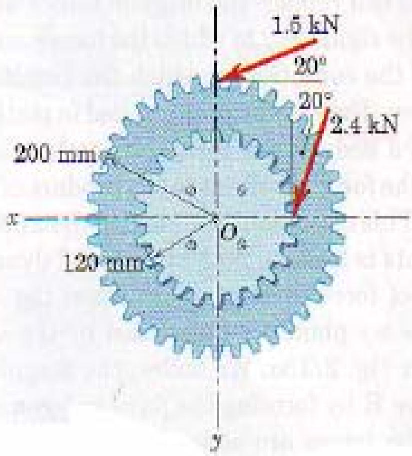


The wrench is subjected to the 200-N force and the force **P** as shown. If the equivalent of the two forces is a force **R** at *O* and a couple expressed as the vector $\mathbf{M} = 20\mathbf{k}$ N·m, determine the vector expressions for **P** and **R**.

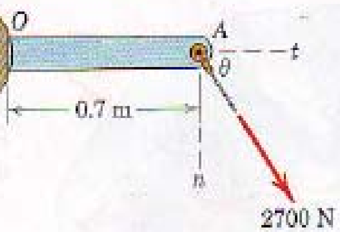
Ans. $\mathbf{P} = 40\mathbf{j}$ N
$\mathbf{R} = -160\mathbf{j}$ N



The figure represents two integral gears subjected to the tooth-contact forces shown. Replace the two forces by an equivalent single force **R** at the rotation axis O and a corresponding couple **M**. Specify the magnitudes of **R** and **M**. If the gears were to start from rest under the action of the tooth loads shown, in what direction would rotation take place?

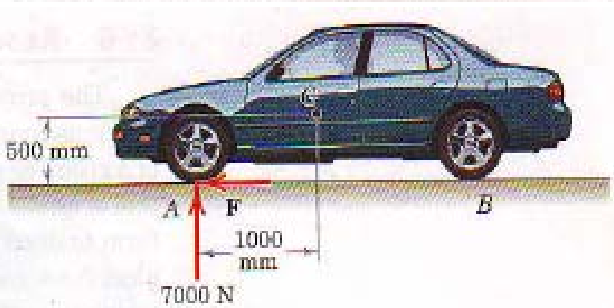


The weld at O can support a maximum of 2500 N of force along each of the n- and t-directions and a maximum of 1400 N·m of moment. Determine the allowable range for the direction θ of the 2700-N force applied at A. The angle θ is restricted to $0 \leq \theta \leq 90°$.



The combined drive wheels of a front-wheel-drive automobile are acted on by a 7000-N normal reaction force and a friction force **F**, both of which are exerted by the road surface. If it is known that the resultant of these two forces makes a 15° angle with the vertical, determine the equivalent force–couple system at the car mass center G. Treat this as a two-dimensional problem.

Ans. $R = 7250$ N
$M_G = 7940$ N·m CW



Lecture 4
Resultant

The properties of force, moment, and couple were developed in the previous four lecture. Now we are ready to describe the resultant action of a group or system of forces. Most problems in mechanics deal with a system of forces, and it is usually necessary to reduced the system to its simplest form to describe its action. The .resultant of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied. Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics

The most common type of force system occurs when the forces all act in a single plane, say, the x-y plane, as illustrated by the system of three forces F1, F2, and F3 in Fig. 1. We obtain the magnitude and direction of the resultant force R by forming the force polygon shown in part b of the figure, where the forces are added head to-tail in any sequence. Thus, for any system of coplanar forces we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F}$$

$$R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}$$

Graphically, the correct line of action of R may be obtained bv preserving the correct lines of action of the forces and adding them by the parallelogram law. We see this in part a of the figure for the case of three forces where the sum R1 of F2 and F3 is added to F1 to obtain R. The principle of transmissibility has been used in this process.

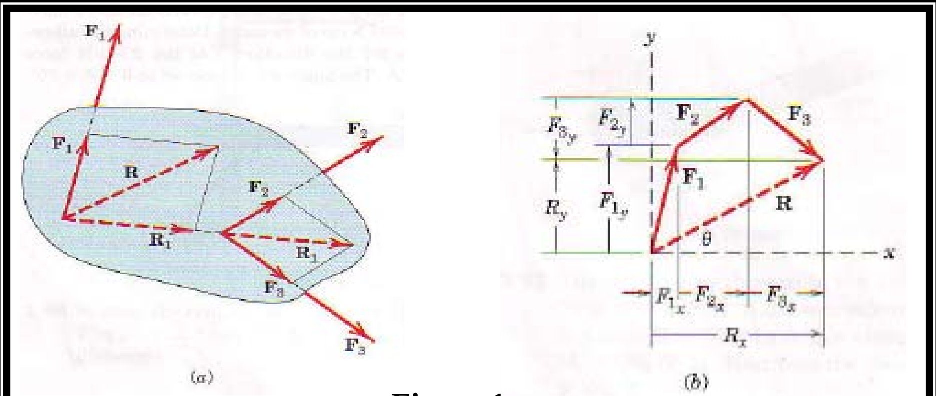


Figure 1

Algebraic. Method

We can use algebra to obtain the resultant force and its line of action

1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Figs.2a and b, where M1, M2, and M3 are the couples resulting from the transfer of forces F1, F2, and F3 from their respective original lines of action to lines of action through point O.

2. Add all forces at O to form the resultant force R, and add all couples to form the resultant couple Mo. We now have the single force-couple system, as shown in Fig. 2c.

3. In Fig. 2d, find the line of action of R by requiring R to have a moment of Mo about point O. Note that the force systems of Figs.2a and ,2d. are equivalent, and that Σ(Fd) in Fig. 2a is equal to Rd in Fig. 2d

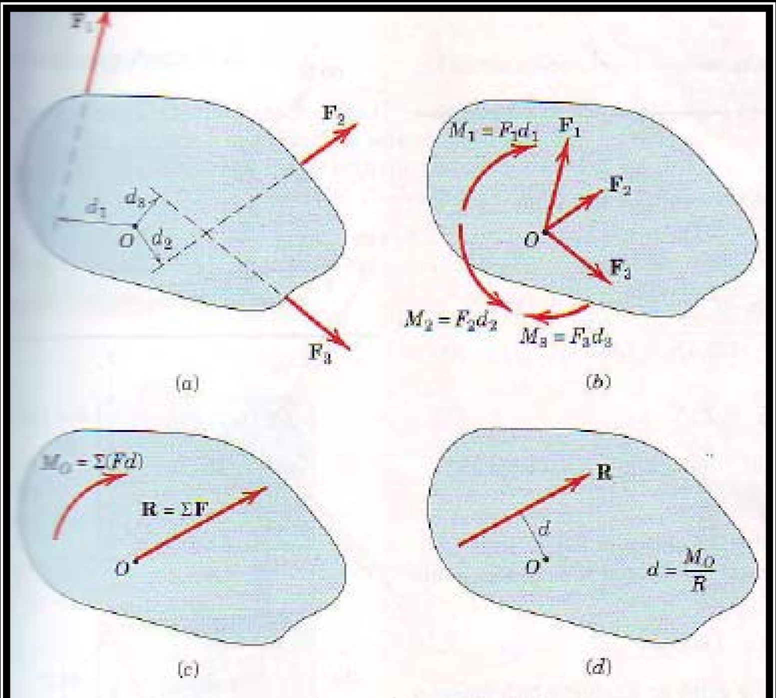


Figure 2

principle of Moments

This process is summarized in equation form by

$$\mathbf{R} = \Sigma\mathbf{F}$$
$$M_O = \Sigma M = \Sigma(Fd) \qquad \ldots\ldots\ldots 2$$
$$Rd = M_O$$

The first two of Eqs.2 reduce a given system of forces to a force- couple system at an arbitrarily chosen but convenient point O. The last equation specifies the distance d from point O to the line of action of R, and states that the moment of the resultant force about any point O equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of nonconcurrent force system; we call this extension the principle of moments . for a concurrent system of forces where the lines of action of all forces pass through a common point O, the moment sum ΣMo about that point is zero. Thus, the line of action of the resultant R = ΣF, determined by the first of Eqs. 2, passes though point O. For a parallel force system, select a coordinate axis in the direction of the forces. If the resultant force R for a given force system is zero, the resultant of the system need not be zero because the resultant may be a couple. The three forces in Fig. 3, for instance, have a zero resultant force but have a resultant clockwise couple M=F3d

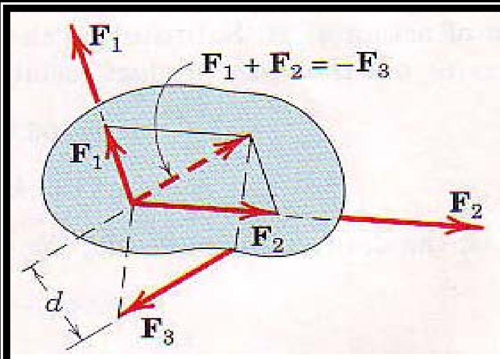
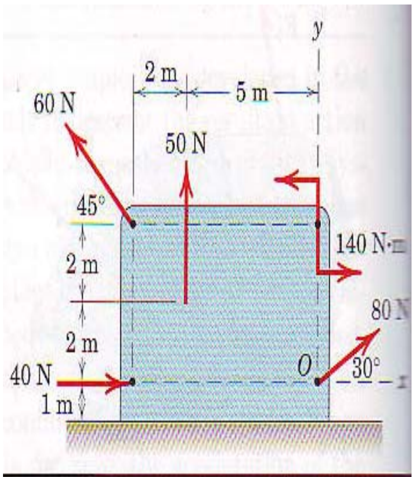


Figure 3

Examples

<u>Example1</u>

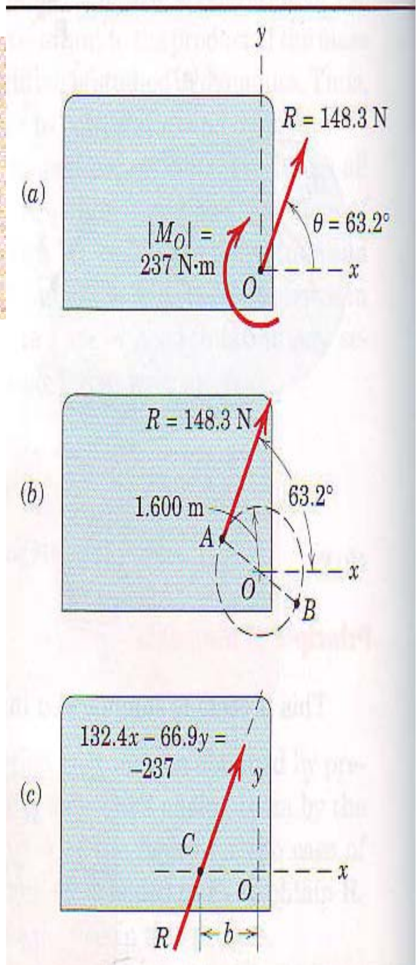
Determine the resultant of the four forces and one couple which act on the plate shown.



Solution

Point 0 is selected as a convenient reference point for the force-couple system that is to represent the given system

$[R_x = \Sigma F_x]$	$R_x = 40 + 80\cos 30° - 60\cos 45° = 66.9\ \text{N}$
$[R_y = \Sigma F_y]$	$R_y = 50 + 80\sin 30° + 60\cos 45° = 132.4\ \text{N}$
$[R = \sqrt{R_x^2 + R_y^2}]$	$R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3\ \text{N}$ *Ans.*
$\left[\theta = \tan^{-1}\dfrac{R_y}{R_x}\right]$	$\theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$ *Ans.*
$[M_O = \Sigma(Fd)]$	$M_O = 140 - 50(5) + 60\cos 45°(4) - 60\sin 45°(7) = -237\ \text{N·m}$



The force-couple system consisting of R and Mo is shown in Fig.a

We now determine the final line of action of R such that R alone represents the original system

$[Rd = |M_O|]$ $\quad 148.3d = 237 \quad d = 1.600\ \text{m}$ *Ans.*

Hence, the resultant R may be applied at point on the line which makes a 63.2˚ angle with the x-axis and is tangent at point A to a circle of 1.6m radius with center 0, as shown in part b of the figure. We apply the equation

Rd=Mo in an absolute-value sense (ignoring any sign of Mo) and let the physics of the situation, as depicted in Fig.a, dictate the final placement of R. had Mo been counterclockwise, the correct line of action of R would have been the tangent at point B.

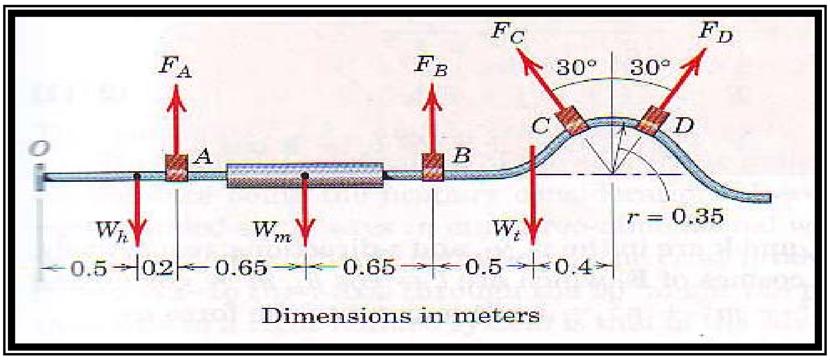
The resultant R may also be located by determining its intercept distance b to point C on the x-axis, Fig.c. with R_x and R_y acting through point C, only R_y exerts a moment about 0 so that

$$R_y b = |M_O| \quad \text{and} \quad b = \frac{237}{132.4} = 1.792 \text{ m}$$

Alternatively, the y-intercept could have been obtained by noting that the moment about 0 would be due to R_x only.

Example 2

An exhaust system for pickup truck is shown in the Figure. The weights W_h, W_m, and W_t of the headpipe, muffler, and tajlpipe are 10, 100. and 50 N, respectively, and act at the indicated points. If the exhaust pipe hanger at point A is adjusted so that its tension F_A is 50 N, determine the required forces in the hangers at points B, C, and D so that the force couple system at point O is zero. Why is a zero force couple system at O desirable?



Solution

For a zero force-couple system at point O:

$$R_x = \Sigma F_x = -F_C \sin 30^\circ + F_D \sin 30^\circ = 0$$

$$F_D \sin 30^\circ = F_C \sin 30^\circ$$

$$F_D = F_C = F$$

$$R_y = \Sigma F_y = 50 - 10 - 100 - 50 + F_B + F_C \cos 30^\circ + F_D \cos 30^\circ = 0$$

$$0 = -110 + F_B + 2F \cos 30$$

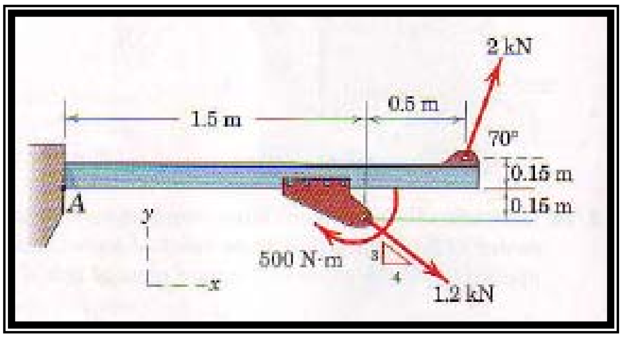
$$F_B = 110 - 2F \cos 30^\circ$$

$$\circlearrowleft M_O = -10(0.5) + 50(0.7) - 100(1.35) + F_B(2) - 50(2.5) + 2F\cos 30^\circ(2.9) = 0$$

$$F = F_C = F_B = \underline{6.42 \text{ N}}, \quad F_B = \underline{98.9 \text{ N}}$$

Example 3

The flanged steel cantilever beam with riveted bracket is subjected to the couple and two forces shown, and their effect on the design of the attachment at A must be determined. Replace the two forces and couple by an equivalent couple M and resultant force R at A



Solution

$$R_x = \Sigma F_x$$
$$= 2\cos 70^\circ + 1.2\left(\frac{4}{5}\right) = \underline{1.664 \text{ KN}}$$

$$R_y = \Sigma F_y = 2\sin 70^\circ - 1.2\left(\frac{3}{5}\right) = \underline{1.159 \text{ KN}}$$

$$\overset{+}{\circlearrowleft} M_A = -2\cos 70^\circ(0.15) + 2\sin 70^\circ(1.5+0.5) + 1.2\left(\frac{4}{5}\right)(0.15)$$
$$- 1.2\left(\frac{3}{5}\right)(1.5) - 0.5 \Rightarrow \underline{2.22 \text{ kN.m CCW}}$$

The force –couple system is

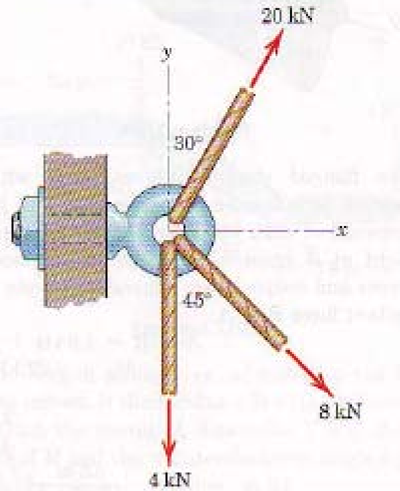
$$R = \sqrt{(1.644)^2 + (1.159)^2} = \text{ KN}$$

$$M_A = 2.22 \text{ KN.m CCW}$$

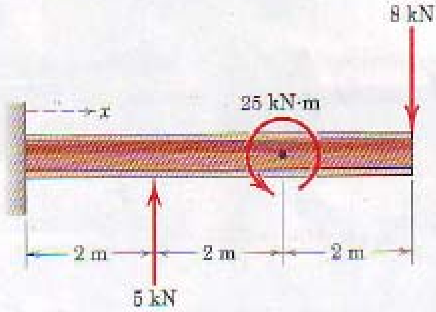
Problems

Determine the resultant **R** of the three tension forces acting on the eye bolt. Find the magnitude of **R** and the angle θ_x which **R** makes with the positive x-axis.

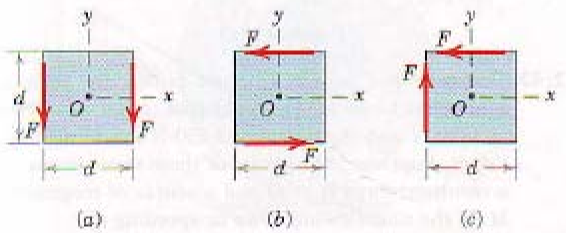
Ans. $R = 17.43$ kN, $\theta_x = 26.1°$



Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.

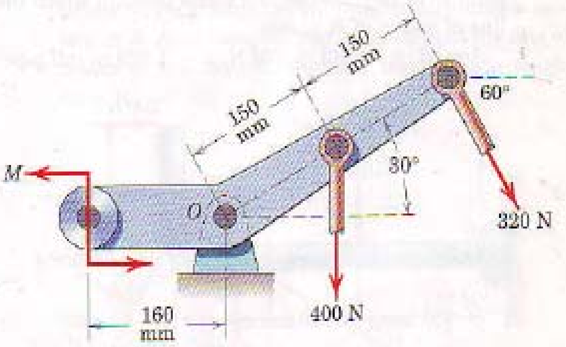


Determine the equivalent force–couple system at the center O for each of the three cases of forces being applied along the edges of a square plate of side d.



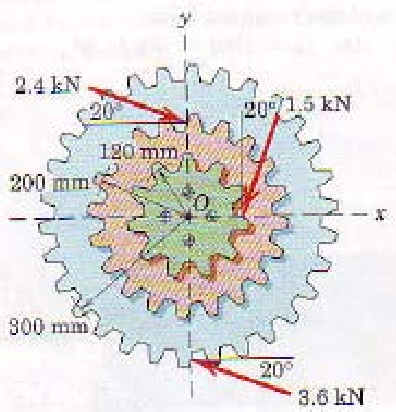
If the resultant of the two forces and couple M passes through point O, determine M.

Ans. $M = 148.0$ N·m CCW

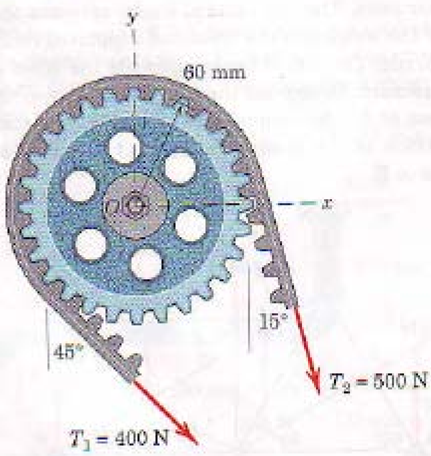


Determine the x- and y-axis intercepts of the line of action of the resultant of the three loads applied to the gearset.

Ans. $x = 1.637$ m, $y = -0.997$ m

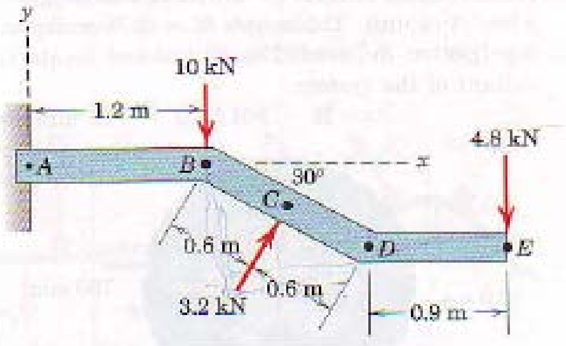


As part of a design test, the camshaft-drive sprocket is fixed and then the two forces shown are applied to a length of belt wrapped around the sprocket. Find the resultant of this system of two forces and determine where its line of action intersects both the x- and y-axes.

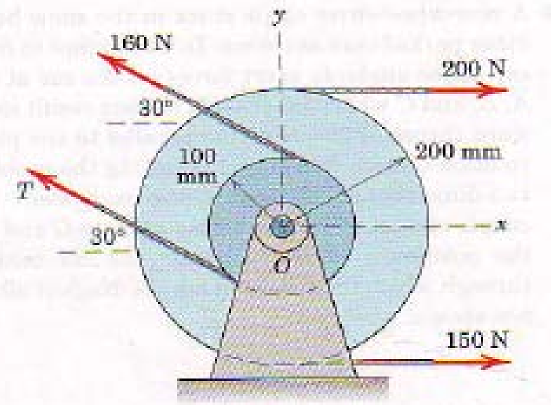


Replace the three forces which act on the bent bar by a force–couple system at the support point A. Then determine the x-intercept of the line of action of the stand-alone resultant force $\mathbf{R}$.

Ans. $\mathbf{R} = 1.6\mathbf{i} - 12.03\mathbf{j}$ kN
$M_A = 21.8$ kN·m CW
$x = 1.814$ m

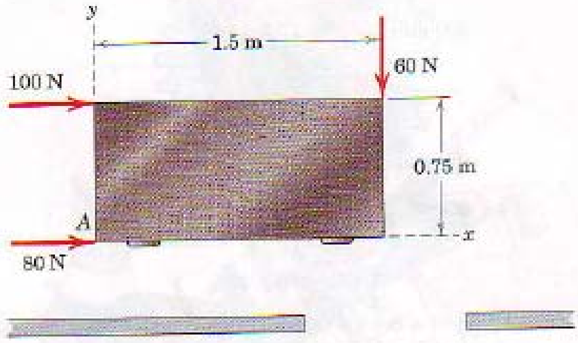


Two integral pulleys are subjected to the belt tensions shown. If the resultant $\mathbf{R}$ of these forces passes through the center O, determine T and the magnitude of $\mathbf{R}$ and the counterclockwise angle θ it makes with the x-axis.

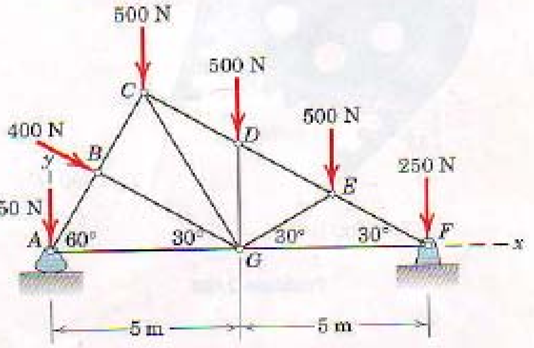


While sliding a desk toward the doorway, three students exert the forces shown in the overhead view. Determine the equivalent force–couple system at point A. Then determine the equation of the line of action of the resultant force.

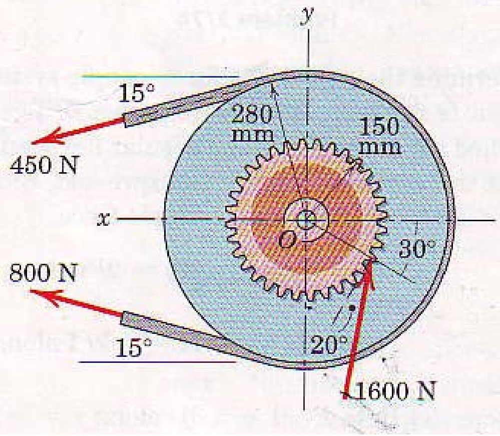
Ans. $\mathbf{R} = 180\mathbf{i} - 60\mathbf{j}$ N, $\mathbf{M}_A = -165\mathbf{k}$ N·m
$y = -\frac{1}{3}x + \frac{11}{12}$ m



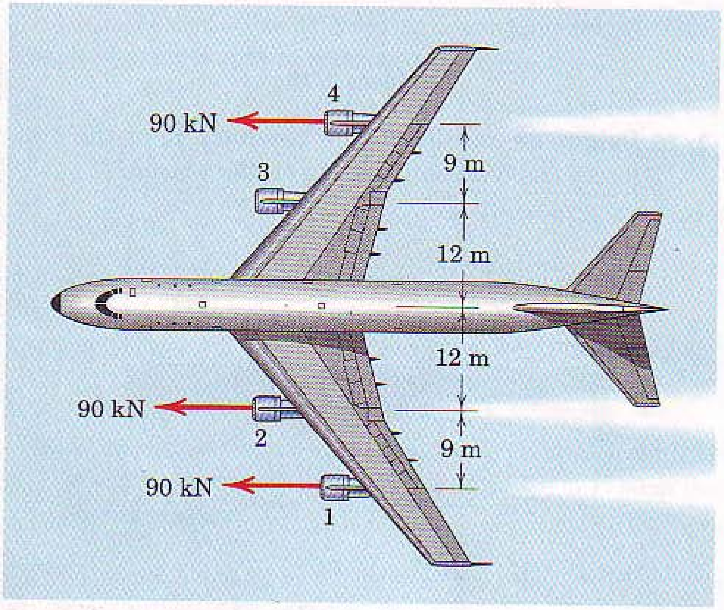
The asymmetric roof truss is of the type used when a near normal angle of incidence of sunlight onto the south-facing surface ABC is desirable for solar energy purposes. The five vertical loads represent the effect of the weights of the truss and supported roofing materials. The 400-N load represents the effect of wind pressure. Determine the equivalent force–couple system at A. Also, compute the x-intercept of the line of action of the system resultant treated as a single force $\mathbf{R}$.



The gear and attached V-belt pulley are turning counterclockwise and are subjected to the tooth load of 1600 N and the 800-N and 450-N tensions in the V-belt. Represent the action of these three forces by a resultant force $\mathbf{R}$ at O and a couple of magnitude M. Is the unit slowing down or speeding up?

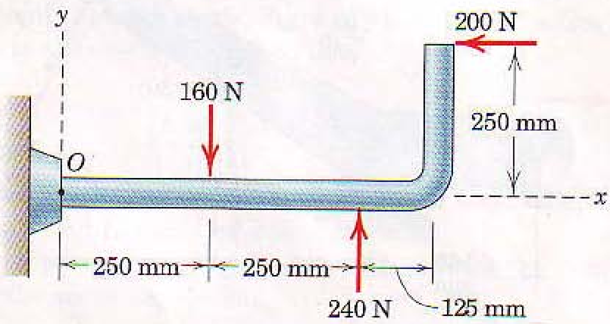


A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two-dimensional problem.

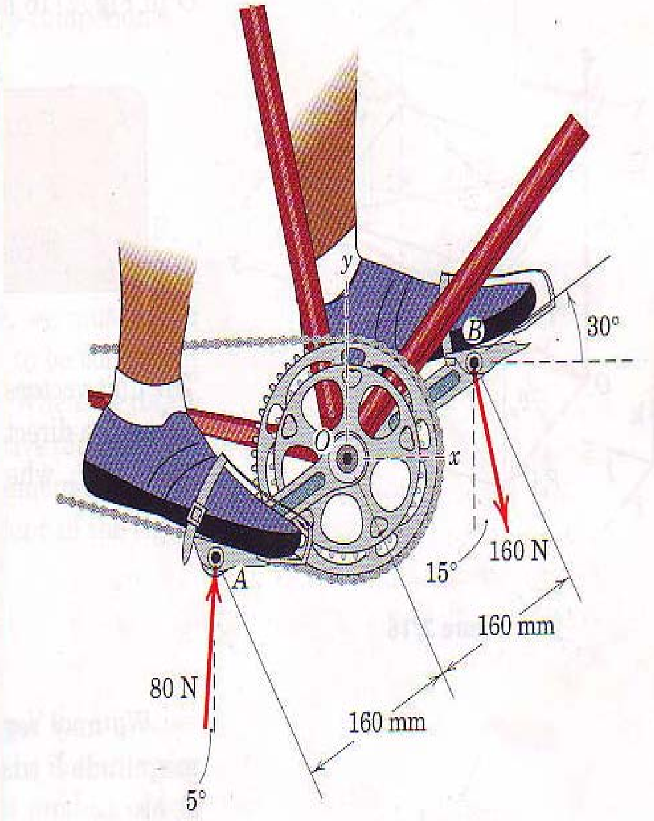


Replace the three forces acting on the bent pipe by a single equivalent force **R**. Specify the distance x from point O to the point on the x-axis through which the line of action of **R** passes.

Ans. $\mathbf{R} = -200\mathbf{i} + 80\mathbf{j}$ N, $x = 1.625$ m (off pipe)



The pedal-chainwheel unit of a bicycle is shown in the figure. The left foot of the rider exerts the 160-N force, while the use of toe clips allows the right foot to exert the nearly upward 80-N force. Determine the equivalent force–couple system at point O. Also, determine the equation of the line of action of the system resultant treated as a single force **R**. Treat the problem as two-dimensional.



Lecture 5

Equilibrium

Static deals primarily with the description of the force conditions necessary and sufficient to maintain the equilibrium of engineering structures.

When body is equilibrium, the resultant of all forces acting on it is zero. Thus, the resultant force R and the resultant couple m are both zero, and we have the equilibrium equations

$$\boxed{\mathbf{R} = \Sigma \mathbf{F} = \mathbf{0} \qquad \mathbf{M} = \Sigma \mathbf{M} = \mathbf{0}}$$

These requirements are both necessary and sufficient conditions for equilibrium.

All physical bodies are three-dimensional, but we can treat many of them as two-dimensional when the forces to which they are subjected act in a single plane or can be projected onto a single plane. When this simplification is not possible, the problem must be treated as three

System Isolation And The Fee- body Diagram

Before we apply Eqs.3/1, we must define unambiguously the particular body or mechanical system to be analyzed and represent clearly and completely all forces actins oz the body. Omission of a force which acts on the body in question, or inclusion of a force which does not act on the body, will give erroneous results. A mechanical system is defined as a body or group of bodies which can be conceptually isolated from all other bodies. A system may be a single body or a combination of connected bodies. The bodies may be rigid or non rigid. The system may also be an identifiable fluid mass, either liquid or gas, or a combination of fluids and solids. In statics we study primarily forces which act on rigid bodies at rest, although we also study forces actins on fluids in equilibrium. Once we decide which body or combination of bodies to analyze, we then treat this body or combination as a single body isolated from all surrounding bodies. This isolation is accomplished by means of the free body diagram, which is a diagrammatic representation of the isolated system treated as a single body. The diagram shows all forces applied to the system by mechanical contact with other bodies, which are imagined to be removed If appreciable body forces are present. Such as gravitational or magnetic attraction, then these forces must also be shown on the free-body diagram of the isolated system.

Only after such a diagram has been carefully drawn should the equilibrium equations be written.

Because of its critical importance, we emphasize here that

the free-body diagram is the most important single step in the solution of problems in mechanics.

Before attempting to &aw a free-body diagram, we must recall the basic characteristics of force. These chrematistics were described in Art. 2/2, with primary attention focused on the vector properties of force. Forces can be applied either by direct physical contact or by remote action. Forces car be either internal or external to the system under consideration. Application of force is accompanied by reactive force, and both applied and reactive forces may be either concentrated or distributed. The principle of transmissibility permits the treatment of force as a sliding vector as far as its external effects on a rigid body are concerned.

We will now use these force characteristics to develop conceptual models of isolated mechanical systems. These models enable us to write the appropriate equations of equilibrium, which can then be analyzed.

Modeling the Action of Forces

Figure 1 shows the common types of force application on mechanical systems for analysis in two dimensions. Each example shows the force exerted on the body to be isolated, by the body to be removed. Newton's third law, which notes the existence of an equal and opposite reaction to every action, must be carefully observed. The force exerted on the body in question by a contacting or supporting member is always in the sense to oppose the movement of the isolated body which would occur if the contacting or supporting body were removed.

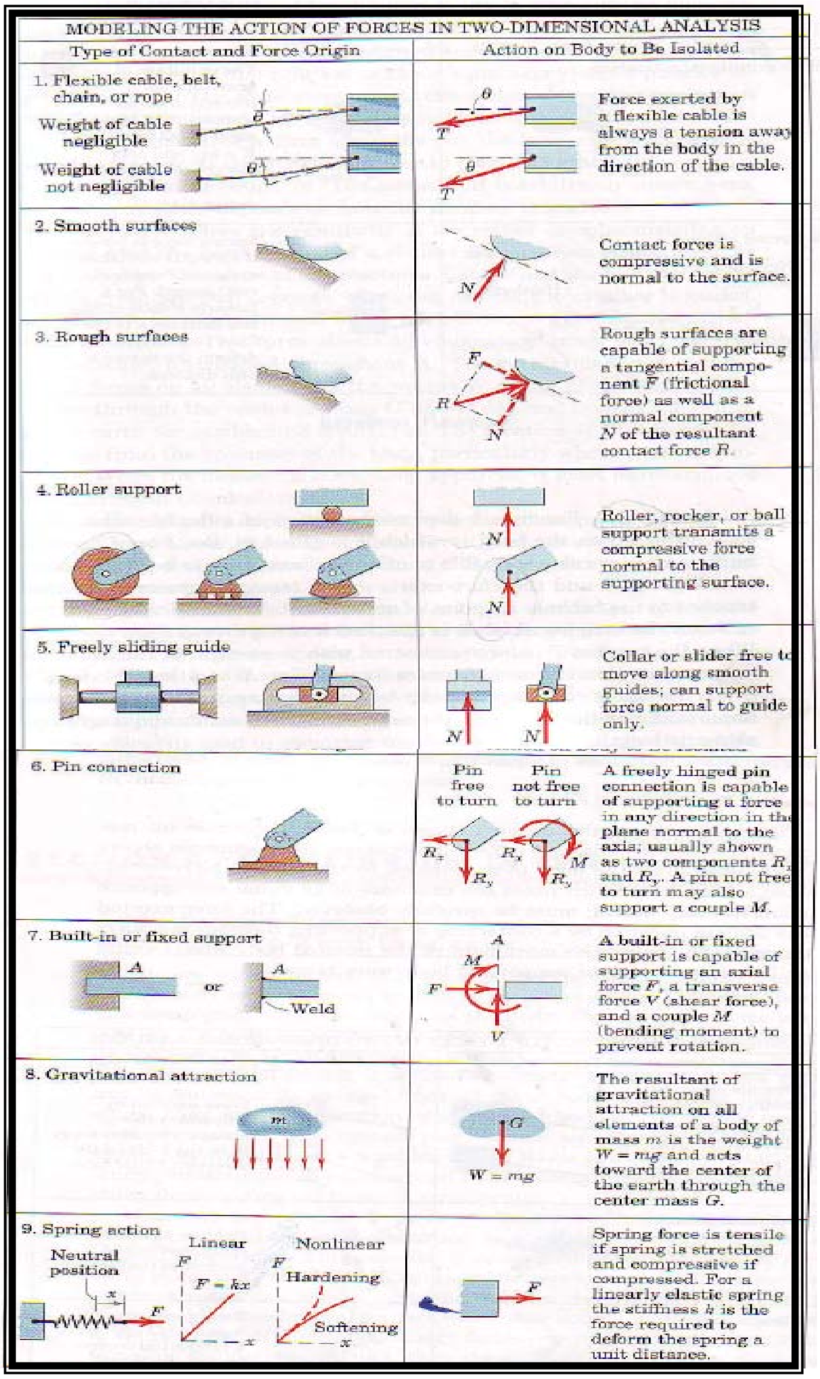
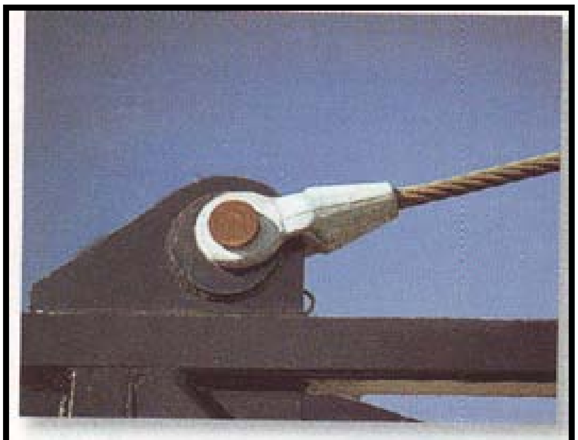


Figure1

Typical examples of actual supports that are referenced to Fig.1 are shown in the following sequence of photo



The cable exerts a force on the bracket in the direction of the cable.

This utility building is pin supported at the top of the column. (8)

The floor beams of this building are welded together and thus form fixed connections. (10)

In Fig. 1, Example 1 depicts the action of a flexible cable, belt, rope, or chain on the body to which it is attached. Because of its flexibility, a rope or cable is unable to offer any resistance to bending, shear, or compression and therefore exerts only a tension force in a direction tangent to the cable at its point of attachment. The force exerted by the cable on the body to which it is attached is always away from the body. When the tension T is large compared with the weight of the cable, we may assume that the cable forms a straight line. When the cable weight is not negligible compared with its tension, the sag of the cable becomes, important, and the tension in the cable changes direction and magnitude along its length.

When the smooth surfaces of two bodies are in contact. as in Example2 The force exerted by one on the other is normal to the tangent

to the surfaces and is compressive, Although no actual surfaces are perfectly smooth, we can assume this to be so for practical purposes in many instances.

When mating surfaces of contacting bodies are rough, as in Example3 , the force of contact is not necessarily normal to the tangent to the surfaces, but may be resolved into a tangential or frictional component F and a normal component N.

Example 4 illustrates a number of forms of mechanical support which effectively eliminate tangential friction forces. ln these cases the net reaction is normal to the supporting surface Example 5 shows the action of a smooth guide on the body it supports. There cannot be any resistance parallel to the guide

Example 6 illustrates the action of a pin connection. Such a connection can support force in any direction normal to the axis of the pin We usually represent this action in terms of two rectangular components. The correct sense of these components in a specific problem depends on how the member is loaded. when not otherwise initially known, the sense is arbitrarily assigned and the equilibrium equation are then written. If the solution of these equations yields a positive algebraic sign for the force component, the assigned sense is correct. A negative sign indicates the sense is opposite to that initially assigned.

If the joint is free to turn about the pin, the connection can support only the force R. If the joint is not free to turn, the connection can also support a resisting couple M. The sense of M is arbitrarily shown here, but the true sense depends on how the member is loaded.

Example 7 shows the resultants of the rather complex distribution of force over the cross section of a slender bar or beam at a built-in or fixed support. The sense of the reactions F and V and the bending couple M in a given problem depends of course, o how the member is loaded.

One of the most common forces is that due to gravitational attraction, Example 8. This force affects all elements of mass in a body and is, therefore. distributed throughout it. The resultant of the gravitational forces on all elements is the weight W = mg of the body, which passes through the center of mass G and is directed toward the center of the earth for earthbound structures The location of G is frequently obvious from the geometry of the body, particularly where there is symmetry. When the location is not readily apparent, it must be determined by experiment or calculations.

Similar remarks apply to the remote action of magnetic and electric forces. These forces of remote action have the same overall effect on a rigid body as forces of equal magnitude and direction applied by direct.

Example 9 illustrates the action of a linear elastic spring and of a nonlinear spring with either hardening or softening characteristics. The force exerted by a linear spring, in tension or compression, is given by F = kx, where k is the stiffness of the spring and x is its deformation measured from the neutral or unreformed position.

The representations in Fig. 1 are not free-body diagrams, but are merely elements used to construct free body diagrams. Study these nine conditions and identify them in the problem work so that you can draw the correct free-body diagrams.

Construction of Free-Body Diagrams

The full procedure for drawings a free-body diagram which isolates a body or system consists of the following steps

Step 1. Decide which system to isolate The system chosen should usually involve one or more of the desired unknown quantities.

Step 2. Next isolate the chosen system by drawing a diagram which represent its complete external boundary. This boundary defines the isolation of the system from all other attracting or contacting bodies, which are considered removed This step is often the most crucial of all. Make certain that you have completely isolated the system before proceeding with the next step.

Step 3. Identify{y all forces which act oz the isolated system as applied by the removed contacting and attracting bodies, and represent them in their proper positions on the diagram of the isolated system Make a systematic traverse of the entire boundary to identify all contact forces. Include body forces such as weights, where appreciable. Represent . all known forces by vector arrow, each with its! Proper magnitude, direction, and sense indicated. Each unknown force should be represented by a vector arrow with the unknown magnitude or direction indicated by symbol. if the sense of the vector is also unknown, you must arbitrarily assign a sense. The subsequent calculations with the equilibrium equations will yield a positive quantity if the incorrect sense was assumed and a negative quantity if the incorrect sense was assumed. it is necessary to be consistent with the assigned characteristics of unknown forces throughout all of the calculations. If you are consistent, the solution of the equilibrium equations will reveal the correct senses.

Step 4. Show the choice of coordinate axes directly on the diagram Pertinent dimensions may also be represented for convenience. Note, however., that the free-body diagram serves the purpose of focusing attention on the action of the external forces, and therefore the diagram should not be cluttered with excessive extraneous information. Clearly distinguish force arrows from arrows representing quantities other than forces. for this purpose a colored pencil may be used.

Completion of the foregoing four steps will produce a correct free-body diagram to use in applying the governing equations, both in statics and in dynamics. Be careful not to omit from the free-body diagram certain forces which may not appear at first glance to be needed in the calculations. lt is only through complete isolation and a systematic representation of all eternal forces that a reliable accounting of the effects of all applied and reactive forces can be made. very often a force which at first glance may not appear to influence a desired result does indeed have an influence. Thus. the only safe procedure is to include on the free-body diagram all forces whose magnitudes are not obviously negligible. The free-body method is extremely important in mechanics because it ensures an accurate definition of a mechanical system and focuses

attention on the exact meaning and application of the force laws of statics and dynamics. Review the foregoing four steps for constructing a free-body diagram while studying the sample free-body diagrams shown in Fig. 2 .

Examples of Free-Body Diagrams

Figure 2 gives four examples of mechanisms and structures together with their correct free-body diagrams. Dimensions and magnitudes are omitted for clarity. In each case we treat the entire system as a single body, so that the internal forces are not shown The characteristics of the various types of contact forces illustrated in Fig 1 are used in the four examples as they apply

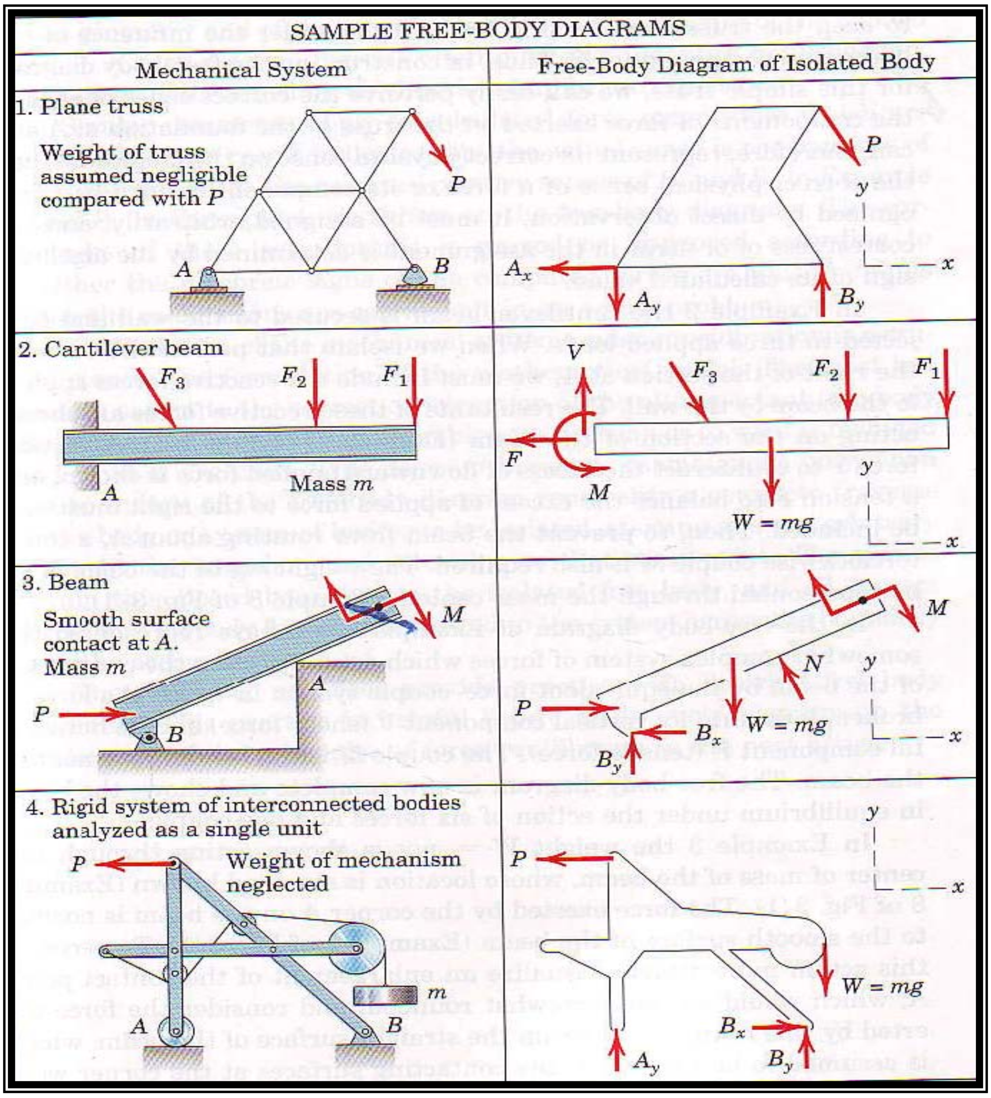
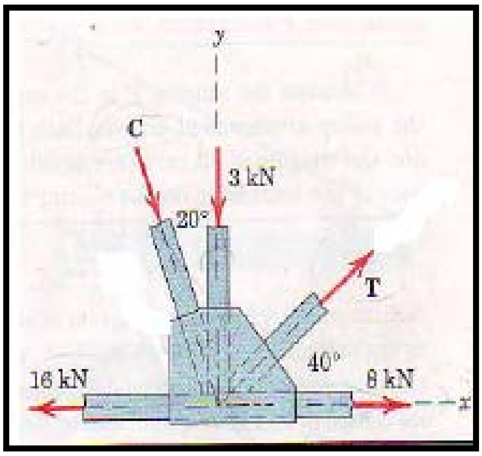


Figure 2

Examples

<u>Example 1</u>

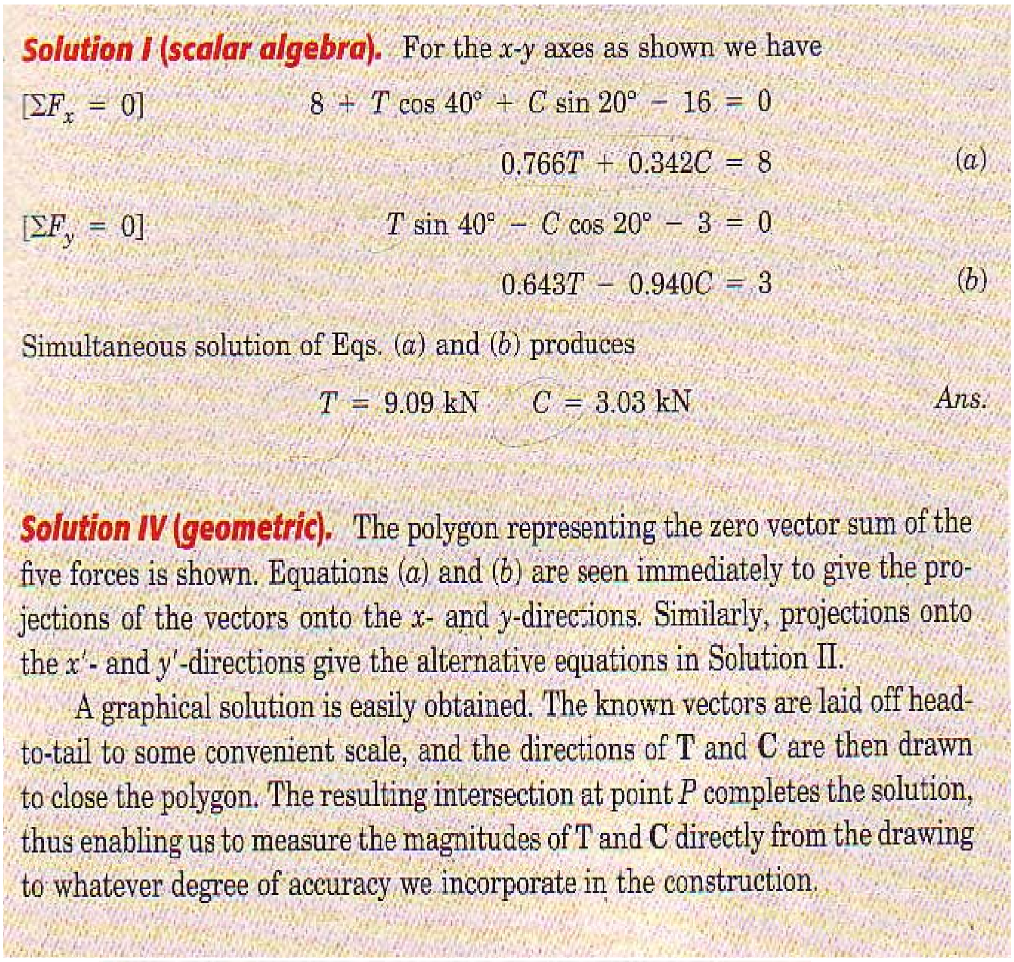
Determine the magnitudes of the forces C and T, which, along with the other Forces shown, act on the bridge-truss joint.



Solution

The given sketch constitutes the free-body diagram of the isolated section of the joint in question and shows the five forces which ere in equilibrium

Solution 1 (scalar algebra): for the x-y axes as shown we have



Solution I (scalar algebra). For the x-y axes as shown we have

$[\Sigma F_x = 0]$ $\quad 8 + T\cos 40° + C\sin 20° - 16 = 0$

$$0.766T + 0.342C = 8 \qquad (a)$$

$[\Sigma F_y = 0]$ $\quad T\sin 40° - C\cos 20° - 3 = 0$

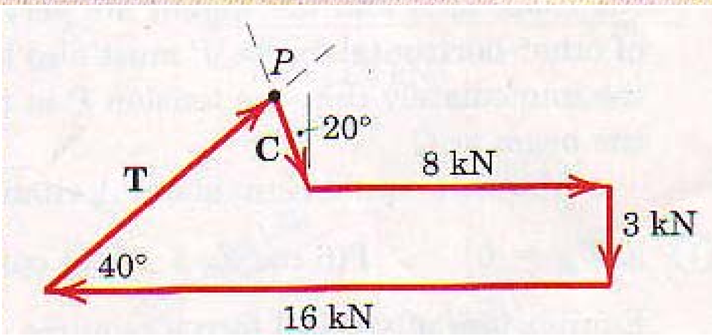
$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

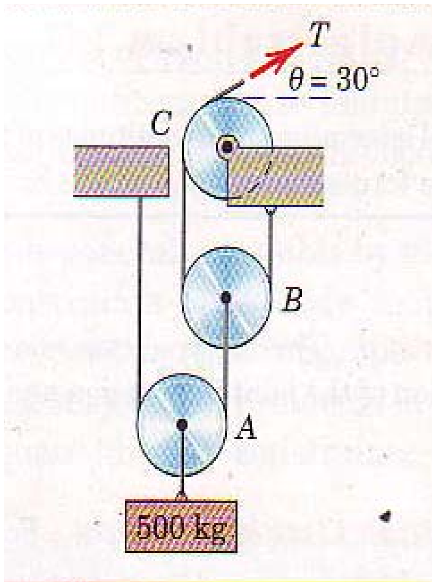
Solution IV (geometric). The polygon representing the zero vector sum of the five forces is shown. Equations (a) and (b) are seen immediately to give the projections of the vectors onto the x- and y-directions. Similarly, projections onto the x′- and y′-directions give the alternative equations in Solution II.

A graphical solution is easily obtained. The known vectors are laid off head-to-tail to some convenient scale, and the directions of **T** and **C** are then drawn to close the polygon. The resulting intersection at point P completes the solution, thus enabling us to measure the magnitudes of T and C directly from the drawing to whatever degree of accuracy we incorporate in the construction.



Example 2

Calculate the tension t in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the totl force on the bearing of pulley C.



Solution. The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley A, which includes the only known force. With the unspecified pulley radius designated by r, the equilibrium of moments about its center O and the equilibrium of forces in the vertical direction require

$[\Sigma M_O = 0]$ $\quad T_1 r - T_2 r = 0 \quad T_1 = T_2$

$[\Sigma F_y = 0]$ $\quad T_1 + T_2 - 500(9.81) = 0 \quad 2T_1 = 500(9.81) \quad T_1 = T_2 = 2450 \text{ N}$

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$$T_3 = T_4 = T_2/2 = 1226 \text{ N}$$

For pulley C the angle $\theta = 30°$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

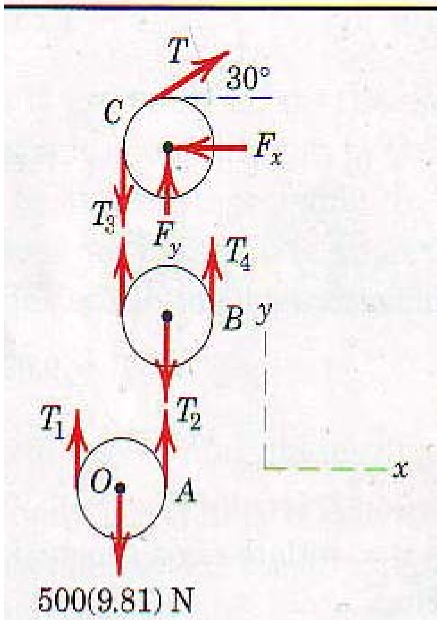
$$T = T_3 \quad \text{or} \quad T = 1226 \text{ N} \qquad \textit{Ans.}$$

Equilibrium of the pulley in the x- and y-directions requires

$[\Sigma F_x = 0]$ $\quad 1226 \cos 30° - F_x = 0 \qquad F_x = 1062 \text{ N}$

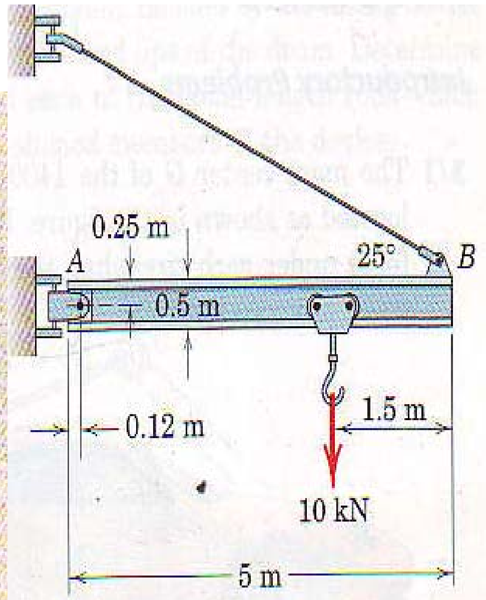
$[\Sigma F_y = 0]$ $\quad F_y + 1226 \sin 30° - 1226 = 0 \qquad F_y = 613 \text{ N}$

$[F = \sqrt{F_x^2 + F_y^2}]$ $\quad F = \sqrt{1062^2 + 613^2} = 1226 \text{ N} \qquad \textit{Ans.}$



Example 3

Determine the magnitude T of the tension in supporting cable and the magnitude of the force on pin at A for the jip crane shown. The beam AB is a standarad 0.5-m I-beam with a mass of 95 kg per meter of length.



Algebraic solution. The system is symmetrical about the vertical x-y plane through the center of the beam, so the problem may be analyzed as the equilibrium of a coplanar force system. The free-body diagram of the beam is shown in the figure with the pin reaction at A represented in terms of its two rectangular components. The weight of the beam is $95(10^{-3})(5)9.81 = 4.66$ kN and acts through its center. Note that there are three unknowns A_x, A_y, and T which may be found from the three equations of equilibrium. We begin with a moment equation about A, which eliminates two of the three unknowns from the equation. In applying the moment equation about A, it is simpler to consider the moments of the x- and y-components of $\mathbf{T}$ than it is to compute the perpendicular distance from $\mathbf{T}$ to A. Hence, with the counterclockwise sense as positive we write

$$[\Sigma M_A = 0] \qquad (T\cos 25°)0.25 + (T\sin 25°)(5 - 0.12) - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$$

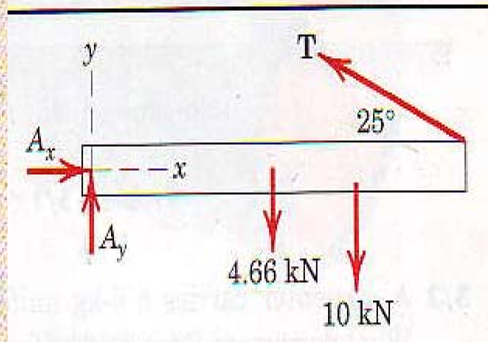
from which $\qquad T = 19.61$ kN $\qquad$ *Ans.*

Equating the sums of forces in the x- and y-directions to zero gives

$$[\Sigma F_x = 0] \qquad A_x - 19.61\cos 25° = 0 \qquad A_x = 17.77 \text{ kN}$$

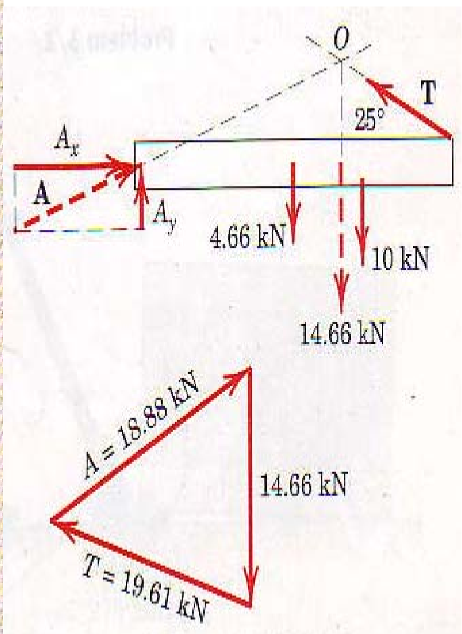
$$[\Sigma F_y = 0] \qquad A_y + 19.61\sin 25° - 4.66 - 10 = 0 \qquad A_y = 6.37 \text{ kN}$$

$$\left[A = \sqrt{A_x^{\,2} + A_y^{\,2}}\right] \quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN} \qquad \textit{Ans.}$$



Free-body diagram

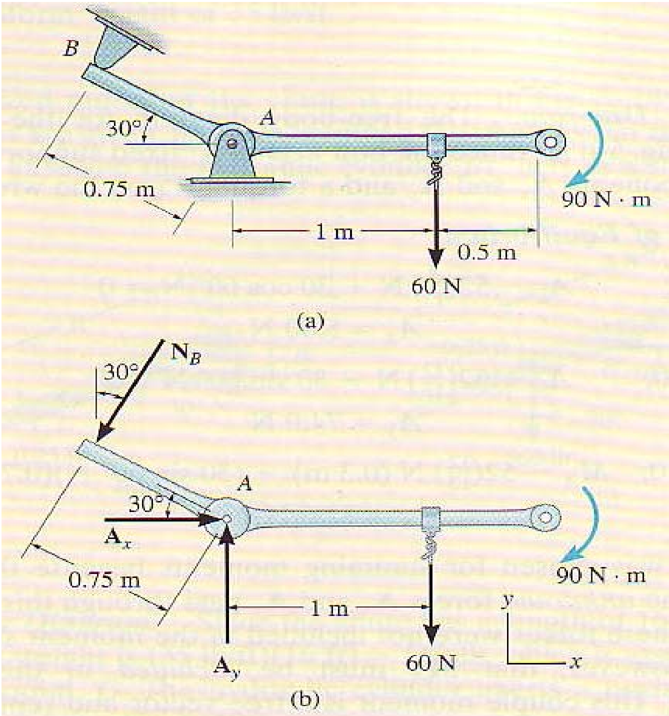
Graphical solution. The principle that three forces in equilibrium must be concurrent is utilized for a graphical solution by combining the two known vertical forces of 4.66 and 10 kN into a single 14.66-kN force, located as shown on the modified free-body diagram of the beam in the lower figure. The position of this resultant load may easily be determined graphically or algebraically. The intersection of the 14.66-kN force with the line of action of the unknown tension $\mathbf{T}$ defines the point of concurrency O through which the pin reaction $\mathbf{A}$ must pass. The unknown magnitudes of $\mathbf{T}$ and $\mathbf{A}$ may now be found by adding the forces head-to-tail to form the closed equilibrium polygon of forces, thus satisfying their zero vector sum. After the known vertical load is laid off to a convenient scale, as shown in the lower part of the figure, a line representing the given direction of the tension $\mathbf{T}$ is drawn through the tip of the 14.66-kN vector. Likewise a line representing the direction of the pin reaction $\mathbf{A}$, determined from the concurrency established with the free-body diagram, is drawn through the tail of the 14.66-kN vector. The intersection of the lines representing vectors $\mathbf{T}$ and $\mathbf{A}$ establishes the magnitudes T and A necessary to make the vector sum of the forces equal to zero. These magnitudes are scaled from the diagram. The x- and y-components of $\mathbf{A}$ may be constructed on the force polygon if desired.



Graphical solution

Example 4

The link shown in Fig. a is pin-connected at A and rests against a smooth support at B. Compute the horizontal and vertical components of reaction at pin A.



Solution

Equations of Equilibrium. Summing moments about A, we obtain a direct solution for N_B,

$$\circlearrowleft + \Sigma M_A = 0; \quad -90\ \text{N}\cdot\text{m} - 60\ \text{N}(1\ \text{m}) + N_B(0.75\ \text{m}) = 0$$

$$N_B = 200\ \text{N}$$

Using this result,

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x - 200 \sin 30^\circ\ \text{N} = 0$$

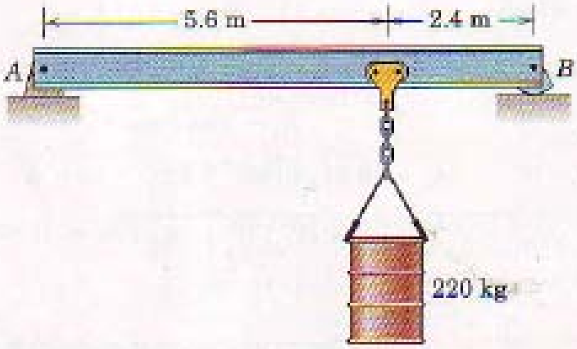
$$A_x = 100\ \text{N} \qquad \textit{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y - 200 \cos 30^\circ\ \text{N} - 60\ \text{N} = 0$$

$$A_y = 233\ \text{N} \qquad \textit{Ans.}$$

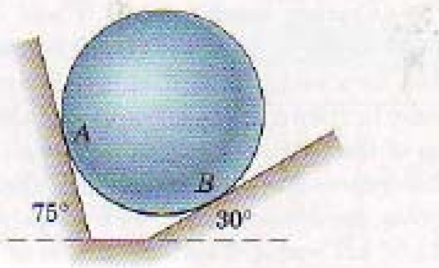
Problems

The 450-kg uniform I-beam supports the load shown. Determine the reactions at the supports.



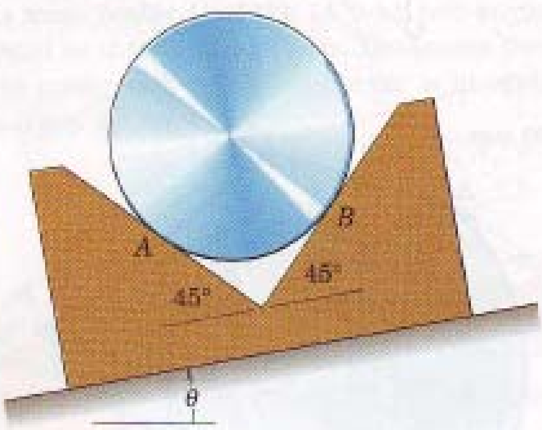
The 20-kg homogeneous smooth sphere rests on the two inclines as shown. Determine the contact forces at A and B.

Ans. $N_A = 101.6$ N, $N_B = 196.2$ N

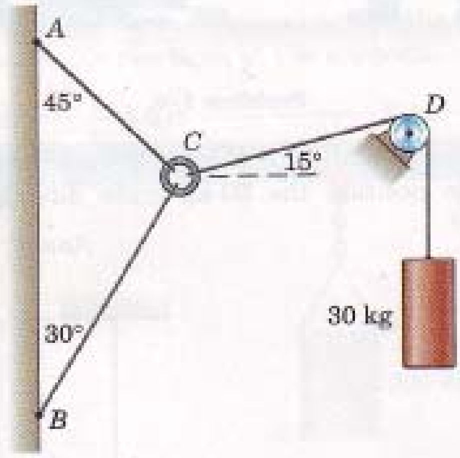


Find the angle of tilt θ with the horizontal so that the contact force at B will be one-half that at A for the smooth cylinder.

Ans. $\theta = 18.43°$

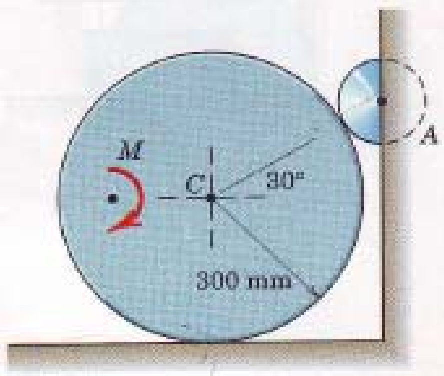


Three cables are joined at the junction ring C. Determine the tensions in cables AC and BC caused by the weight of the 30-kg cylinder.

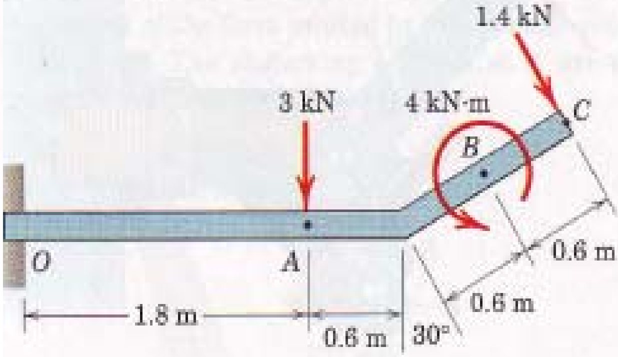


The 100-kg wheel rests on a rough surface and bears against the roller A when the couple M is applied. If $M = 60$ N·m and the wheel does not slip, compute the reaction on the roller A.

Ans. $F_A = 231$ N

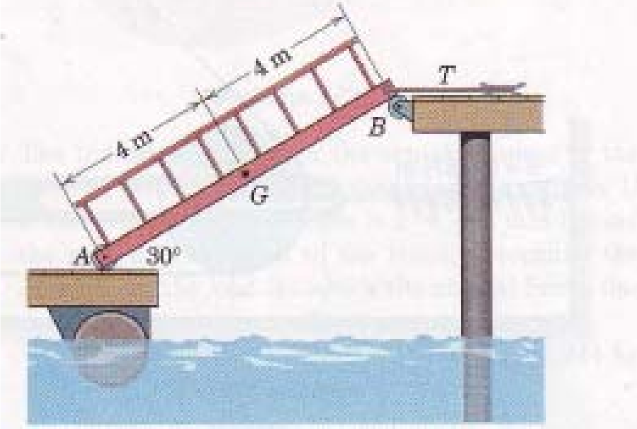


The uniform beam has a mass of 50 kg per meter of length. Compute the reactions at the support O. The force loads shown lie in a vertical plane.



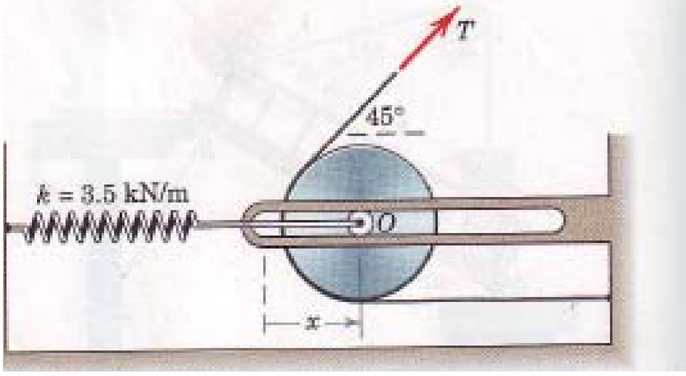
To accommodate the rise and fall of the tide, a walkway from a pier to a float is supported by two rollers as shown. If the mass center of the 300-kg walkway is at G, calculate the tension T in the horizontal cable which is attached to the cleat and find the force under the roller at A.

Ans. $T = 850$ N, $A = 1472$ N



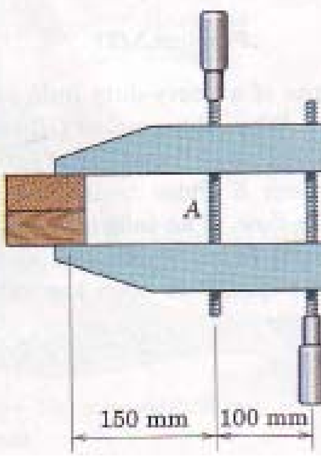
The spring of modulus $k = 3.5$ kN/m is stretched 10 mm when the disk center O is in the leftmost position $x = 0$. Determine the tension T required to position the disk center at $x = 150$ mm. At that position, what force N is exerted on the horizontal slotted guide? The mass of the disk is 3 kg.

Ans. $T = 328$ N, $N = 203$ N up

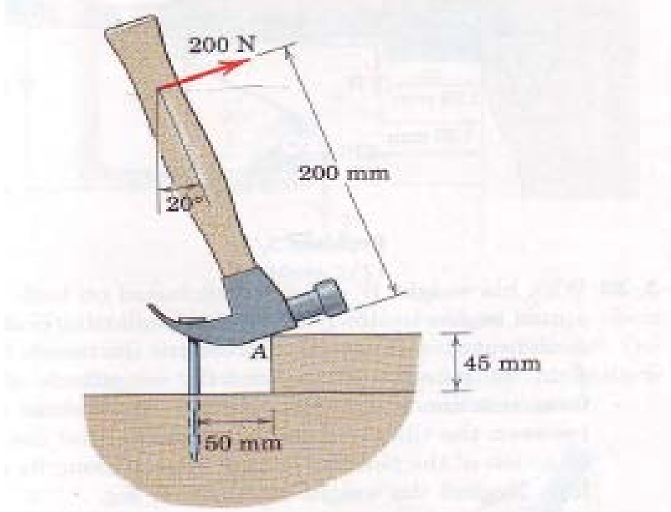


If the screw B of the wood clamp is tightened so that the two blocks are under a compression of 500 N, determine the force in screw A. (*Note:* The force supported by each screw may be taken in the direction of the screw.)

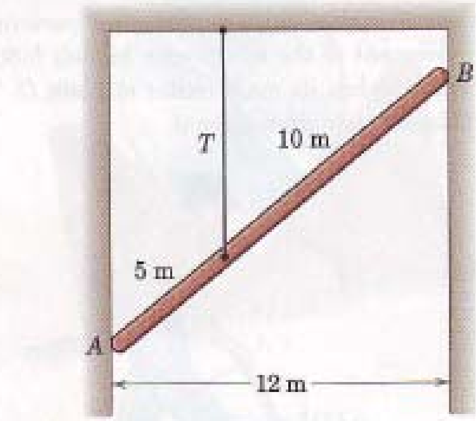
Ans. $A = 1250$ N



A block placed under the head of the claw hammer as shown greatly facilitates the extraction of the nail. If a 200-N pull on the handle is required to pull the nail, calculate the tension T in the nail and the magnitude A of the force exerted by the hammer head on the block. The contacting surfaces at A are sufficiently rough to prevent slipping.

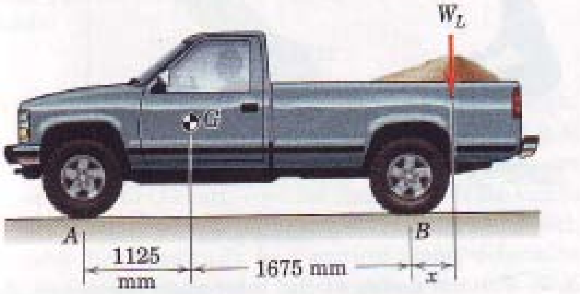


The uniform 15-m pole has a mass of 150 kg and is supported by its smooth ends against the vertical walls and by the tension T in the vertical cable. Compute the reactions at A and B.

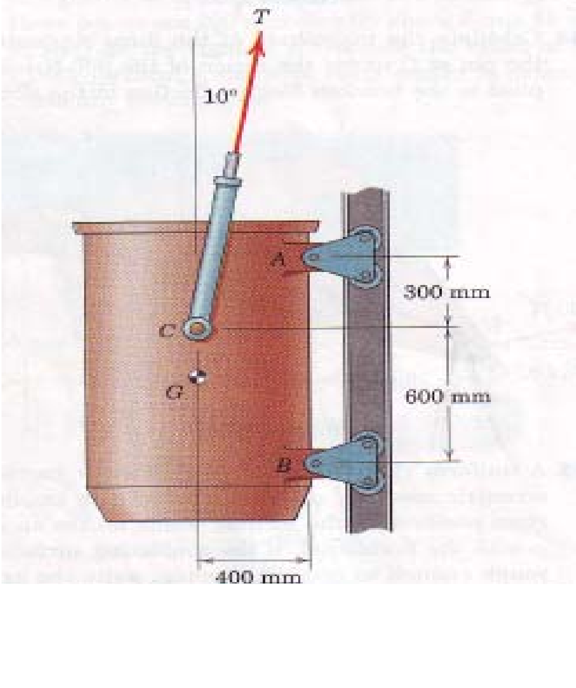


5 The indicated location of the center of mass of the 1600-kg pickup truck is for the unladen condition. If a load whose center of mass is x = 400 mm behind the rear axle is added to the truck, determine the mass m_L of the load for which the normal forces under the front and rear wheels are equal.

Ans. m_L = 244 kg

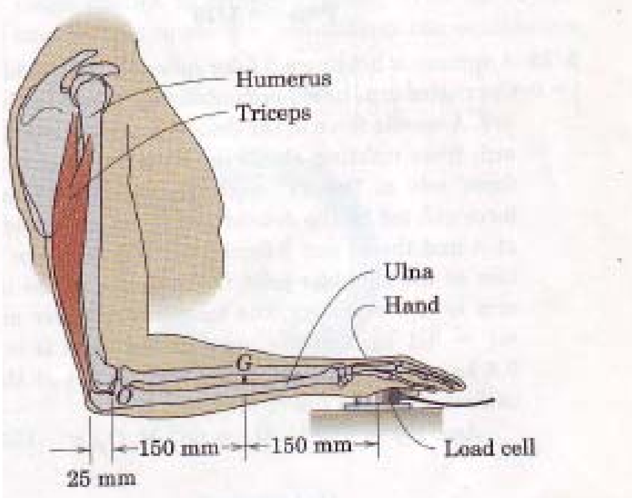


The concrete hopper and its load have a combi[ned] mass of 4 metric tons (1 metric ton equals 1000 [kg]) with mass center at G and is being elevated at co[n]stant velocity along its vertical guide by the ca[ble] tension T. The design calls for two sets of guide r[oll]ers at A, one on each side of the hopper, and two s[ets] at B. Determine the force supported by each of [the] two pins at A and by each of the two pins at B.

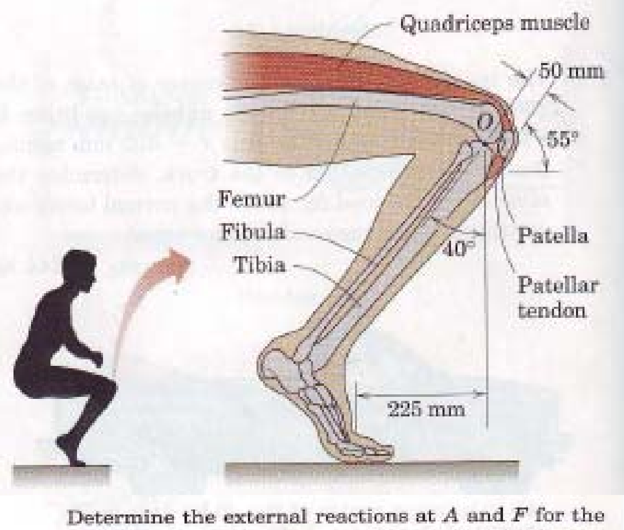


In a procedure to evaluate the strength of the triceps muscle, a person pushes down on a load cell with the palm of his hand as indicated in the figure. If the load-cell reading is 160 N, determine the vertical tensile force F generated by the triceps muscle. The mass of the lower arm is 1.5 kg with mass center at G. State any assumptions.

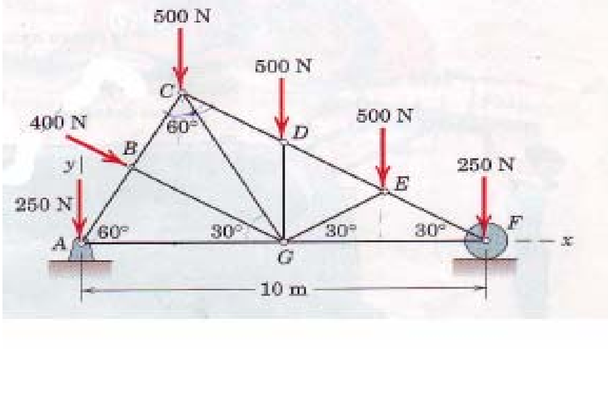
Ans. F = 1832 N



With his weight W equally distributed on both feet, a man begins to slowly rise from a squatting position as indicated in the figure. Determine the tensile force F in the patellar tendon and the magnitude of the force reaction at point O, which is the contact area between the tibia and the femur. Note that the line of action of the patellar tendon force is along its midline. Neglect the weight of the lower leg.



Determine the external reactions at A and F for the roof truss loaded as shown. The vertical loads represent the effect of the supported roofing materials, while the 400-N force represents a wind load.



Lecture 6
Friction

Tangential forces generated between contacting surfaces are called friction forces and occur to some degree in the interaction between all real surfaces. whenever a tendency exists for one contacting surface to slide along another surface, the friction forces developed are always in a direction to oppose this tendency In some types of machines and processes we want to minimize the retarding effect of friction forces. Examples are bearings of all types, power screws, gears, the flow of fluids in pipes, and the propulsion of aircraft and missiles through the atmosphere. In other situations we wish to maximize the effects of friction, as in brakes, clutches, belt drives, and wedges. Wheeled vehicles depend on friction for both starting and stopping, and ordinary walking depends on friction between the shoe and the ground.

Friction forces are present throughout nature and exist in all machine so matter how accurately constructed or carefully lubricated. A machine or process in which friction is small enough to be neglected is said to be ideal. When friction must be taken into account, the machine or process is termed real. In all real cases where there is sliding motion between parts, the friction forces result in a loss of energy which is dissipated in the form of heat. Wear is another effect of friction.

Friction Phenomena

Types of Friction

(a) Dry Friction. Dry friction occurs when the unlubricated surfaces of two solids are in contact under a condition of sliding or a tendency to slide. A friction force tangent to the surfaces of contact occurs both during the interval leading up to impending slippage and while slippage takes place. The direction of this friction force always opposes the motion or impending motion. This type of friction is also called Coulomb friction. The principles of dry or Coulomb friction were developed largely from the experiments of Coulomb in 1781 and from the work of Morin from 1831 to 1834. Although we do not yet have a comprehensive theory of dry friction, in Art. 6/3 we describe an analytical model sufficient to handle the vast majority of problems involving dry friction.

(b) Fluid Friction. Fluid friction occurs when adjacent layers fluid (liquid or gas) are moving at different velocities. This motion causes frictional forces between fluid elements, and these forces depend on the relative velocity between layers. When there is no relative velocity, there is no fluid friction. Fluid friction depends not only on the velocity gradients within the fluid but also on the viscosity of the fluid, which is a measure of its resistance to shearing action between fluid layers. Fluid friction is treated in the study of fluid mechanics and will not be discussed further in this book.

(c) Internal Friction. Internal friction occurs in all solid materials which are subjected to cyclical loading. For highly elastic materials the recovery from deformation occurs with very little loss of energy due to internal friction. For materials which have low limits of elasticity and which undergo appreciable plastic deformation during loading, a considerable amount of internal friction may accompany this deformation. The mechanism of internal friction is associated with the action of shear deformation, which is discussed in references on materials science.

Dry Friction

Mechanism of Dry Friction

Consider a solid block of mass m resting on a horizontal surface, as shown in Fig. 1a.We assume that the contacting surfaces have some roughness. The experiment involves the application of a horizontal force P which continuously increases from zero to a value sufficient to move the block and give it an appreciable velocity. The free-body diagram of the block for any value of P is shown in Fig.1b, where the tangential friction force exerted by the plane on the block is labeled "F'. This friction force acting on the body will always be in a direction to oppose motion or the tendency toward motion of the body. There is also a normal force N which in this case equals mg, and the total force R exerted by the supporting surface on the block is the resultant of N and F.

A magnified view of the irregularities of the mating surfaces, Fig.1c, helps us to visualize the mechanical action of friction. Support is necessarily intermittent and exists at the mating humps, The direction of each of the reactions on the block, R1, R2, R3, etc. depends not only

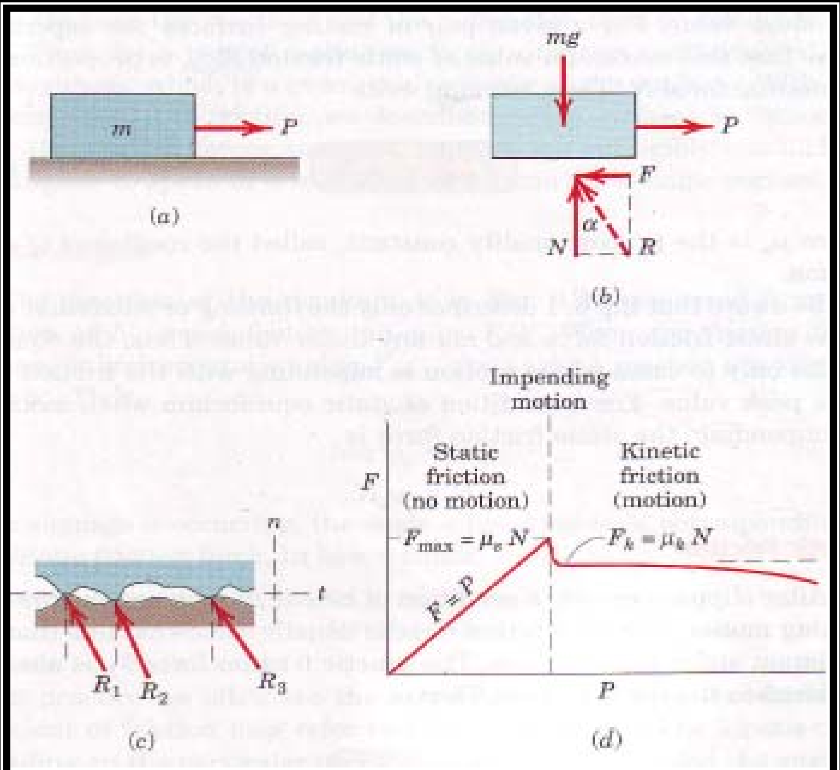


Figure 1

on the geometric profile of the irregularities but also on the extent of local deformation at each contact point. The total normal force N is the sum of the n-components of the R's, and the total frictional force F is the sum of the t-components of the R's. when the surfaces are in relative motion, the contacts are more nearly along the tops of the humps, and the t-components of the R's are smaller than when the surfaces are at rest relative to one another. This observation helps to explain the well known fact that the force P necessary to maintain motion is generally less than that required to start the block when the irregularities are more nearly in mesh.

If we perform the experiment and record the friction force F as a function of P, we obtain the relation shown in Fig. 1d. when P is zero, equilibrium requires that there. be no friction force. As p is increased the friction force must be equal and opposite to p as long as the block does not slip. During this period the block is in equilibrium, and all forces acting on the block must satisfy the equilibrium equations. Finally, we reach a value of P which causes the block to slip and to move in the direction of the applied force. At this same time the friction force decreases slightly and abruptly. It then remains essentially constant for a time but then decreases still more as the velocity increases.

Static Friction

The region in Fig. 1d up to the point of slippage or impending motion is called the range of static friction, and in this range the value of the friction force is determined by the equations of equilibrium. This friction force may have any value from zero up to and including the maximum value. For a given pair of mating surfaces the experiment shows that this maximum value of static friction F_{max} is proportional the normal force N. Thus. we may write

$$\boxed{F_{\max} = \mu_s N} \qquad \text{….. equ. 1}$$

where μ_s is the proportionality constant, called the coefficient of static friction. Be aware that Eq. 1 describes only the limiting or maximum value Of the static friction force and not any lesser value. Thus, the equation applies only to cases where motion is impending with the friction force at its peak value. For a condition of static equilibrium when motion is not impending, the static friction force is

$$F < \mu_s N$$

Kinetic Friction

After slippage occurs, a condition of kinetic friction accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. The kinetic friction force F_k, is also proportional to the normal force. Thus.

$$\boxed{F_k = \mu_k N} \qquad \text{….. equ. 2}$$

where μ_k is the coefficient of kinetic friction. It follows that μ_k is generally less than μ_s. As the velocity of the block increases, the kinetic friction decreases somewhat, and at high velocities, this decrease may be significant. Coefficients of friction depend greatly on the exact condition of the surfaces, as well as on the relative velocity, and are subject to considerable uncertainty.

Because of the variability of the conditions governing the action friction, in engineering practice it is frequently difficult to distinguish between a static and a kinetic coefficient, especially in the region of transition between impending motion and motion. Well-greased screw threads under mild loads, for example, often exhibit comparable frictional resistance whether they are on the verge of turning or whether they are in motion.

In the engineering literature we frequently find expressions for maximum static friction and for kinetic friction written simply as ,$F=\mu N$. It is understood from the problem at hand whether maximum static friction or kinetic friction is described. Although we will frequently distinguish between the static and kinetic coefficients, in other cases no distinction will be made, and the friction coefficient will be written simply as p,. In those cases you must decide which of the friction conditions, maximum static friction for impending motion or kinetic friction, is involved. We emphasize again that many problems involve a static friction force which is less than the maximum value at impending motion, and therefore under these conditions the friction relation Eq. 1 cannot be used.

Figure 1c shows that rough surfaces are more likely to have larger angles between the reactions and the n-direction than do smoother surfaces. Thus, for a pair of mating surfaces, a friction coefficient reflects the roughness, which is a geometric property of the surfaces. With this geometric model of friction, we describe mating surfaces as "smooth" when the friction forces they can support are negligibly small. It is meaningless to speak of a coefficient of friction for a single surface.

Factors Affecting Friction

Further experiment shows that the friction force is essentially independent of the apparent or projected area of contact. The true contact area is much smaller than the projected va1ue, since only the peaks of the contacting surface irregularities support the load. Even relatively small normal loads result in high stresses at these contact points. As the normal force increases, the true contact area also increases as the material undergoes yielding, crushing, or tearing at the points of contact.

A comprehensive theory of dry friction must go beyond the mechanical explanation presented here. For example, there is evidence that molecular attraction may be an important cause of friction under conditions where the mating surfaces are in very close contact. Other factors which influence dry friction are the generation of high local temperatures and adhesion at contact points, relative hardness of mating surfaces, and the presence of thin surface films of oxide, oil, dirt, or other substances,

Types of Friction Problems

We can now recognize tine following three types of problems encountered in applications involving dry friction. The first step in solving a friction problem is to identify its type.

(1) In the first type of problem, the condition of impending motion is known to exist. Here a body which is in equilibrium is on. the verge of slipping. and the friction force equals the limiting static friction $F_{max} = \mu_s N$. the equations of equilibrium will, of course, also hold.

(2) In the second, type of problem, neither the condition of impending motion nor the condition of motion is known to exist. To determine the actual friction conditions, we first assume static equilibrium and then solve for the friction force F necessary for equilibrium. Three outcomes are possible:

(a) $F < (F_{max} = \mu_s N)$: Here the friction force necessary for equilibrium can be supported, and therefore the body is in static equilibrium as assumed. We emphasize that the actual friction force F is less than the limiting value F_{max} given by Eq. 1 and that F is determined solely by the equations of equilibrium.

(b) $F = (F_{max} = \mu_s N)$: Since the friction force F is at its maximum value F_{max} motion impends, as discussed in problem type (1). The assumption of static equilibrium is valid.

(c) $F > (F_{max} = \mu_s N)$: Clearly this condition is impossible, because the surfaces cannot support more force than the maximum $\mu_s N$. The assumption of equilibrium is therefore invalid, and motion occurs. The friction force F is equal to $\mu_s N$ from Eq. 2.

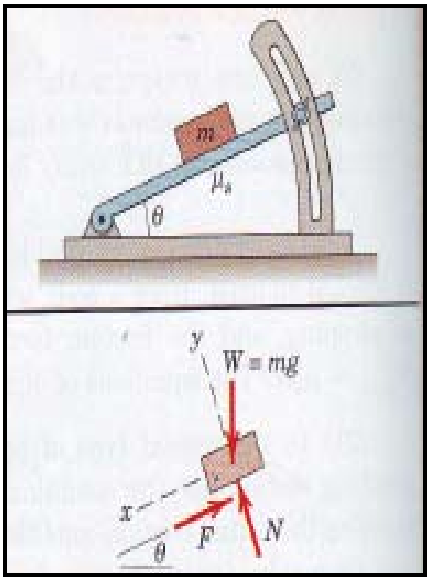
(3) In the third type of problem, relative motion is known to exist between the contacting surfaces, and thus the kinetic coefficient of friction clearly applies. For this problem type, Eq.2 always gives the kinetic friction force directly.

The foregoing discussion applies to all dry contacting surfaces and to a limited extent, to moving surfaces which are partially lubricated.

Examples

Example 1

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s.



Solution

The free-body diagram of the block shows its weight W = mg, the normal force N, and the friction force F exerted by the incline on the block. The friction force acts in the direction to oppose the slipping which would occur if no friction were present. Equilibrium in the x- and y-directions requires

$$[\Sigma F_x = 0] \qquad mg \sin\theta - F = 0 \qquad F = mg\sin\theta$$

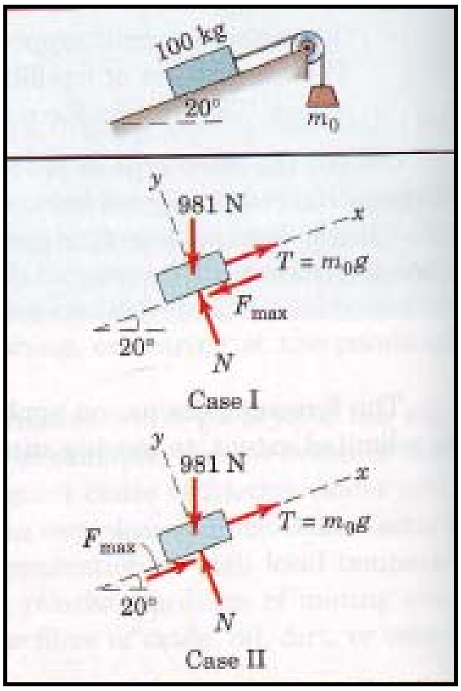
$$[\Sigma F_y = 0] \qquad -mg\cos\theta + N = 0 \qquad N = mg\cos\theta$$

Dividing the first equation by the second gives $F/N = \tan\theta$. Since the maximum angle occurs when $F = F_{max} = \mu_s N$, for impending motion we have

$$\mu_s = \tan\theta_{max} \qquad \text{or} \qquad \theta_{max} = \tan^{-1}\mu_s \qquad \textit{Ans.}$$

Example 2

Determine the range of values which the mass m_o may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30.



Solution. The maximum value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free-body diagram of the block for Case I in the figure. With the weight $mg = 100(9.81) = 981$ N, the equations of equilibrium give

$$[\Sigma F_y = 0] \qquad N - 981\cos 20^\circ = 0 \qquad N = 922 \text{ N}$$

$$[F_{max} = \mu_s N] \qquad F_{max} = 0.30(922) = 277 \text{ N}$$

$$[\Sigma F_x = 0] \qquad m_0(9.81) - 277 - 981\sin 20^\circ = 0 \qquad m_0 = 62.4 \text{ kg} \qquad \textit{Ans.}$$

The minimum value of m_0 is determined when motion is impending down the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free-body diagram for Case II. Equilibrium in the x-direction requires

$$[\Sigma F_x = 0] \qquad m_0(9.81) + 277 - 981\sin 20^\circ = 0 \qquad m_0 = 6.01 \text{ kg} \qquad \textit{Ans.}$$

Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

In both cases equilibrium requires that the resultant of F_{max} and N be concurrent with the 981-N weight and the tension T.

Example 3

The three flat blocks are positioned on the 30° incline as shown, and a force P parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surface. is shown. Determine the maximum value which P may have before any slipping takes place.

Solution. The free-body diagram of each block is drawn. The friction forces are assigned in the directions to oppose the relative motion which would occur if no friction were present. There are two possible conditions for impending motion. Either the 50-kg block slips and the 40-kg block remains in place, or the 50- and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline.

The normal forces, which are in the y-direction, may be determined without reference to the friction forces, which are all in the x-direction. Thus,

$[\Sigma F_y = 0]$ (30-kg) $N_1 - 30(9.81)\cos 30° = 0$ $\quad N_1 = 255$ N

(50-kg) $N_2 - 50(9.81)\cos 30° - 255 = 0$ $\quad N_2 = 680$ N

(40-kg) $N_3 - 40(9.81)\cos 30° - 680 = 0$ $\quad N_3 = 1019$ N

We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place. Thus, for impending slippage at both surfaces of the 50-kg block, we have

$[F_{max} = \mu_s N]$ $\quad F_1 = 0.30(255) = 76.5$ N $\quad F_2 = 0.40(680) = 272$ N

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$[\Sigma F_x = 0]$ $\quad P - 76.5 - 272 + 50(9.81)\sin 30° = 0$ $\quad P = 103.1$ N

We now check on the validity of our initial assumption. For the 40-kg block with $F_2 = 272$ N the friction force F_3 would be given by

$[\Sigma F_x = 0]$ $\quad 272 + 40(9.81)\sin 30° - F_3 = 0$ $\quad F_3 = 468$ N

But the maximum possible value of F_3 is $F_3 = \mu_s N_3 = 0.45(1019) = 459$ N. Thus, 468 N cannot be supported and our initial assumption was wrong. We conclude, therefore, that slipping occurs first between the 40-kg block and the incline. With the corrected value $F_3 = 459$ N, equilibrium of the 40-kg block for its impending motion requires

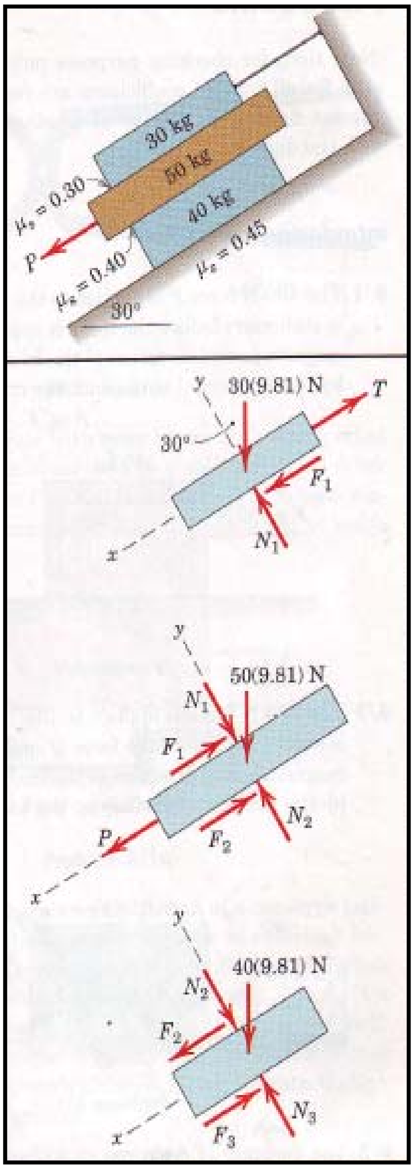
$[\Sigma F_x = 0]$ $\quad F_2 + 40(9.81)\sin 30° - 459 = 0$ $\quad F_2 = 263$ N

Equilibrium of the 50-kg block gives, finally,

$[\Sigma F_x = 0]$ $\quad P + 50(9.81)\sin 30° - 263 - 76.5 = 0$

$P = 93.8$ N *Ans.*

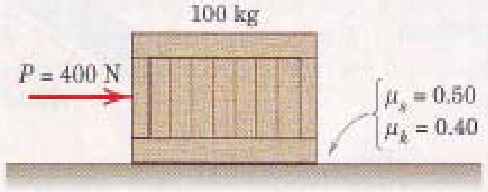
Thus, with $P = 93.8$ N, motion impends for the 50-kg and 40-kg blocks as a unit.



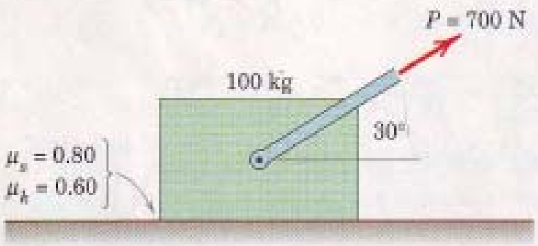
Problems

The 400-N force P is applied to the 100-kg crate, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the crate.

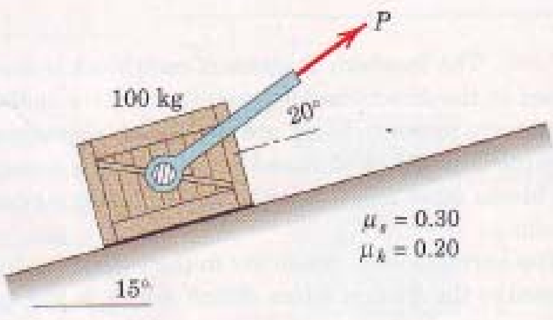
Ans. F = 400 N to the left



The 700-N force is applied to the 100-kg block, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the block.

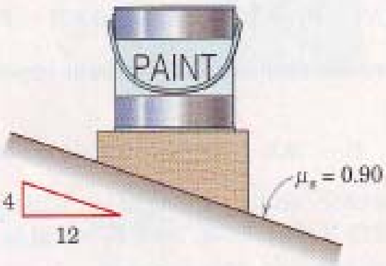


The coefficients of static and kinetic friction between the 100-kg block and the inclined plane are 0.30 and 0.20, respectively. Determine (*a*) the friction force F acting on the block when P is applied with a magnitude of 200 N to the block at rest, (*b*) the force P required to initiate motion up the incline from rest, and (*c*) the friction force F acting on the block if P = 600 N.

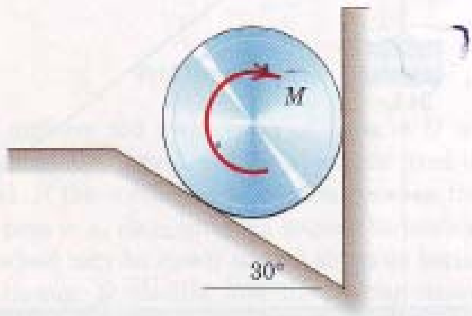


The 1.2-kg wooden block is used for level support of the 9-kg can of paint. Determine the magnitude and direction of (*a*) the friction force exerted by the roof surface on the wooden block and (*b*) the total force exerted by the roof surface on the wooden block.

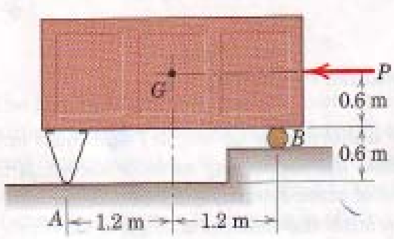
Ans. (*a*) F = 31.6 N, (*b*) P = 100.1 N up



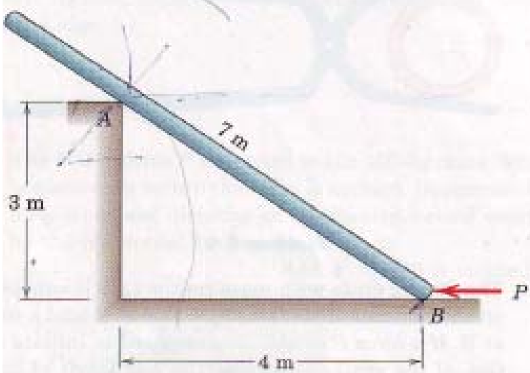
The 30-kg homogeneous cylinder of 400-mm diameter rests against the vertical and inclined surfaces as shown. If the coefficient of static friction between the cylinder and the surfaces is 0.30, calculate the applied clockwise couple M which would cause the cylinder to slip.



The 200-kg crate with mass center at G is supported on the horizontal surfaces by a skid at A and a roller at B. If a force P of 400 N is required to initiate motion of the crate, determine the coefficient of static friction at A.

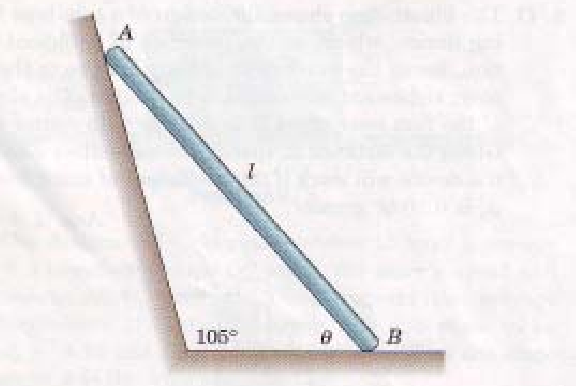


2 The uniform 7-m pole has a mass of 100 kg and is supported as shown. Calculate the force P required to move the pole if the coefficient of static friction for each contact location is 0.40.

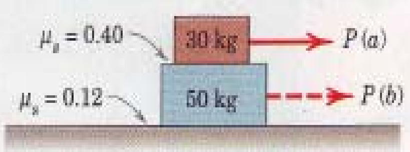


The uniform pole of length l and mass m is placed against the supporting surfaces shown. If the coefficient of static friction is $\mu_s = 0.25$ at both A and B, determine the maximum angle θ at which the pole can be placed before it begins to slip.

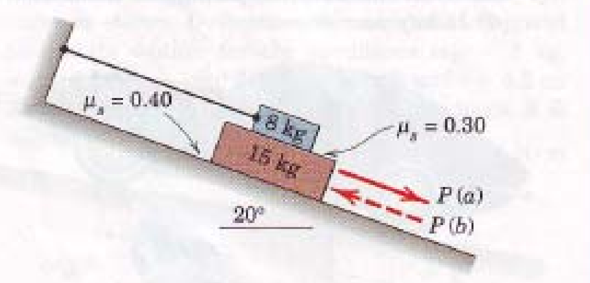
Ans. $\theta = 59.9°$



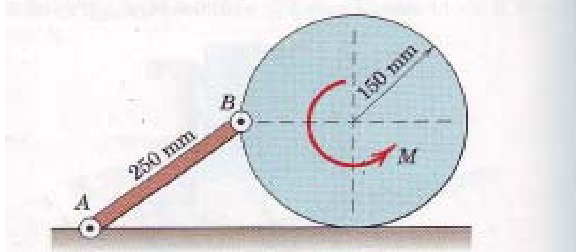
The force P is applied to (a) the 30-kg block and (b) the 50-kg block. For each case, determine the magnitude of P required to initiate motion.



The two blocks are placed on the incline with the cable taut. (a) Determine the force P required to initiate motion of the 15-kg block if P is applied down the incline. (b) If P is applied up the incline and slowly increased from zero, determine the value of P which will cause motion and describe that motion.



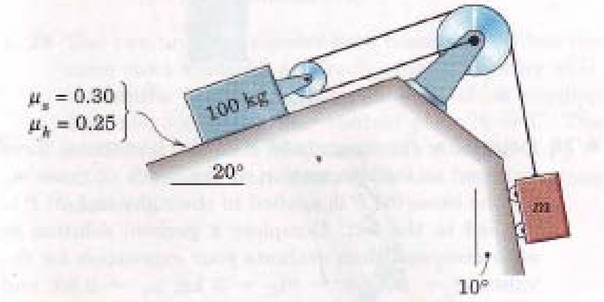
The strut AB of negligible mass is hinged to the horizontal surface at A and to the uniform 25-kg wheel at B. Determine the minimum couple M applied to the wheel which will cause it to slip if the coefficient of static friction between the wheel and the surface is 0.40.



The system of two blocks, cable, and fixed pulley is initially at rest. Determine the horizontal force P necessary to cause motion when (a) P is applied to the 5-kg block and (b) P is applied to the 10-kg block. Determine the corresponding tension T in the cable for each case.

Ans. (a) $P = 137.3$ N, $T = 112.8$ N
(b) $P = 137.3$ N, $T = 24.5$ N

Determine the range of mass m for which the 100-kg block is in equilibrium. All wheels and pulleys have negligible friction.



Determine the magnitude P of the horizontal force required to initiate motion of the block of mass m_0 for the cases (a) P is applied to the right and (b) P is applied to the left. Complete a general solution in each case, and then evaluate your expression for the values $\theta = 30°$, $m = m_0 = 3$ kg, $\mu_s = 0.60$, and $\mu_k = 0.50$.